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A Note on the Perturbation Equations of General Classical Networks

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Abstract

In this paper the solution of the perturbation equations of general classical networks on some general conditions is given. Moreover, the possibility of transforming the original problem to a random walk question is discussed.

In [1] Prof. Ho and Doctor Cao presented a new approach to Discrete Event Dynamic Systems(DEDS) and provided with various experimental results validating this approach to a general queueing network with a single customer class. Both classical and nonclassical problems were treated. The appendices of [1] also contained analytical development associated with this approach. In remark 5.2 of [1] they presented the perturbation equations for general classical networks:

$$\Big(\sum_{i=1}^{K}\varepsilon(n_i)\mu_i\Big)f(n_1,\cdots,\underline{n_l},\cdots,n_K)$$
$$=\sum_{i=1}^{K}\sum_{j=1}^{K}\varepsilon(n_i)\mu_i q_{ij} f(\cdots,n_i-1,\cdots,\underline{n_l},\cdots n_j+1,\cdots)$$
$$+\sum_{j=1}^{K}(1-\varepsilon(n_j))\mu_l q_{lj} f(\cdots,n_j,\cdots,n_l-1,\cdots,\underline{1},\cdots)$$

$(0<n_l<N,\ 1\le l\le K)$,

where
$$\varepsilon(n_i)=\begin{cases}1, & n_i>0,\\ 0, & n_i=0.\end{cases}$$

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($\mu_l p_{lj}$ were missing from the second term on the r. h. s. in [1]) In Appendix B of [1], it was shown that the solution of the equations under conditions $\mu_1=\mu_2=\cdots=\mu_K=\mu$ and

$$q_{ij}=\begin{cases}\dfrac{1}{K-1}, & \text{if } i\neq j,\\ 0, & \text{if } i=j,\end{cases} \tag{*}$$

is $f(n_1,\cdots,\underline{n_l},\cdots,n_K)=\dfrac{n_l}{N}$.

We will make some discussions on the solution of the equations under more general conditions.

We assume that

$$\sum_{j=1}^{K} q_{ij}=1,\quad \sum_{i=1}^{K} q_{ij}=1,\quad q_{ii}=0,\quad 1\leq i,j\leq K, \tag{**}$$

and employ the same notations used in [1].

Theorem 1 If $\mu_1=\mu_2=\cdots=\mu_K=\mu$, then the solutions to equations are $f(n_1,\cdots,\underline{n_l},\cdots,n_K)=\dfrac{n_l}{N}(1\leq l\leq K)$ if and only if $Q=(q_{ij})_{K\times K}$ is symmetric matrix, i. e. $q_{ij}=q_{ji}$ $1\leq i,j\leq K$.

Proof First, these solutions obviously satisfy Eqs (8) – (11) of lemma 4.2 in [1].

i) If all $n_i>0$, $i=1,2,\cdots,K$, then

$$l.h.s.=\mu K\frac{n_l}{N},$$

$$r.h.s.=\sum_{\substack{i=1\\ i\neq l}}^{K}\sum_{\substack{j=1\\ j\neq l\\ j\neq i}}^{K}\mu q_{ij}\frac{n_l}{N}+\sum_{\substack{i=1\\ i\neq l}}^{K}\mu q_{il}\frac{n_l+1}{N}+\sum_{\substack{j=1\\ j\neq l}}^{K}\mu q_{lj}\frac{n_l-1}{N}$$

$$=\sum_{i=1}^{K}\sum_{\substack{j=1\\ j\neq i}}^{K}\mu q_{ij}\frac{n_l}{N}+\frac{\mu}{N}\Big(\sum_{\substack{i=1\\ i\neq l}}^{K}q_{il}-\sum_{\substack{j=1\\ j\neq l}}^{K}q_{lj}\Big)$$

$$=\mu\frac{n_l}{N}\sum_{i=1}^{K}\sum_{\substack{j=1\\ j\neq i}}^{K}q_{ij}+\frac{\mu}{N}(1-1)$$

$$= \mu K \frac{n_l}{N},$$

$\therefore l.h.s. = r.h.S..$

ii) If there are K_0 $n_l = 0$, to simplify notations we assume that $n_1 = \cdots = n_{K_0} = 0$ and $l > K_0$ (If $l \leq K_0$, it is trivial).

$$l.h.s. = (K - K_0)\mu \frac{n_l}{N},$$

$$r.h.s. = \mu \sum_{\substack{i = K_0+1 \\ i \neq l}}^{K} \sum_{\substack{j=1 \\ j \neq l \\ j \neq i}}^{K} q_{ij} \frac{n_l}{N} + \sum_{\substack{i = K_0+1 \\ i \neq l}}^{K} \mu q_{il} \frac{n_l + 1}{N} + \sum_{\substack{j=1 \\ j \neq l}}^{K} \mu q_{lj} \frac{n_l - 1}{N}$$

$$+ \sum_{j=1}^{K_0} \mu q_{lj} \frac{1}{N}$$

$$= \mu \frac{n_l}{N} \sum_{i = K_0+1}^{K} \sum_{\substack{j=1 \\ j \neq i}}^{K} q_{ij} + \frac{\mu}{N} \left(\sum_{\substack{i = K_0+1 \\ i \neq l}}^{K} q_{il} - \sum_{\substack{j=1 \\ j \neq l}}^{K} q_{lj} + \sum_{j=1}^{K_0} q_{lj} \right)$$

$$= (K - K_0)\mu \frac{n_l}{N} + \frac{\mu}{N} \left(\sum_{\substack{i = K_0+1 \\ i \neq l}}^{K} q_{il} - \sum_{\substack{j = K_0+1 \\ j \neq l}}^{K} q_{lj} \right).$$

Hence $\forall\ 1 \leq l \leq K,\ 1 \leq K_0 \leq K,$

$l.h.s. = r.h.s.$ iff $q_{ij} = q_{ji}$ $1 \leq i, j \leq K.$

It is quite evident that $Q = (q_{ij})_{K \times K}$ satisfying (*) is symmetric matrix. On the other hand, if we choose

$$(q_{ij})_{4\times 4} = \begin{pmatrix} 0 & 0.3 & 0.2 & 0.5 \\ 0.3 & 0 & 0.5 & 0.2 \\ 0.2 & 0.5 & 0 & 0.3 \\ 0.5 & 0.2 & 0.3 & 0 \end{pmatrix},$$

then it is symmetric matrix, but it doesn't satisfy(*).

Remark 1 The above statement is also ture without assuming(**).

Theorem 2 If $K = 2$, $\mu_1 \neq \mu_2$, then

$$f(\underline{n_1},\ n_2)=\frac{\mu_2^{n_2}(\mu_1^{n_1}-\mu_2^{n_1})}{\mu_1^N-\mu_2^N},\quad f(n_1,\ \underline{n_2})=\frac{\mu_1^{n_1}(\mu_1^{n_2}-\mu_2^{n_2})}{\mu_1^N-\mu_2^N}.$$

Proof When $K=2$, the equation is simplified:
$(\mu_1+\mu_2)f(\underline{n_1},\ n_2)=\mu_1 f(\underline{n_1-1},\ n_2+1)+\mu_2 f(\underline{n_1+1},n_2-1),0<n_1<N,$

meanwhile, $f(\underline{0},\ N)=0$, $f(\underline{N},\ 0)=1$.

We define $a_n=f(\underline{n},\ N-n)$, then

$$\begin{cases}\mu_2(a_{n+1}-a_n)=\mu_1(a_n-a_{n-1}),0<n<N,\\ a_0=0,\ a_N=1.\end{cases}$$

It is not difficult for us to solve this equation:

$$a_n=\frac{\mu_2^{N-n}(\mu_1^n-\mu_2^n)}{\mu_1^N-\mu_2^N}. \qquad (\mu_1\neq\mu_2)$$

$$\therefore\ \text{if}\ \mu_1\neq\mu_2,\ f(\underline{n_1},n_2)=\frac{\mu_2^{n_2}(\mu_1^{n_1}-\mu_2^{n_1})}{\mu_1^N-\mu_2^N},$$

$$f(n_1,\underline{n_2})=\frac{\mu_1^{n_1}(\mu_1^{n_2}-\mu_2^{n_2})}{\mu_1^N-\mu_2^N}.$$

Remark 2 Letting $\mu_1\to\mu_2$, we obtain for $\mu_1=\mu_2$

$$f(\underline{n_1},\ n_2)=\frac{n_1}{N},\quad f(n_1,\underline{n_2})=\frac{n_2}{N}.$$

Remark 3 We notice that the above-mentioned problem is the same as the following one:
There is an integral point on line segment $[0,\ N]$, moving one unit towards the left with probability p_1 and one unit towards the right with $p_2(p_1+p_2=1)$. What is the probability with which it reaches N before reaching 0. This is a problem of random walk on line segment. After this transformation of problem, it is hoped that results on random walk may be applied.

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References

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关于一般经典网络系统的干扰方程的讨论

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摘 要

本文在较一般的条件下给出了一般经典网络系统的干扰方程的解，另外讨论了把原问题转化成随机游动问题的可能性。