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Parameter Estimation of Time-varying Systems with Observation Noise

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Abstract

This paper presents a new approach to estimate parameter of time-varying systems with observation noise[1]. It consists of two-stage coupled adaptive estimators. The first stage is state estimator by an adaptive recursive filter for systems with multiple time delays[2]; The second stage is parameter estimator by an adaptive Kalman filter[3]. Introducing the fictitious noise, we combine the model errors caused in coupled algorithms into the fictitious noise, so that model errors are compensated and effectively overcome filtering divergence. The simulated example is given to show the usefulness of the newly proposed approach.

Problem Formulation and Results

We consider univariate linear discrete timevarying systems modelled by following

$$x(k+1)=\sum_{i=0}^{n} a_i(k)x(k-i)+b_i(k)u(k-i)+w(k) \tag{1}$$

where $x(k)$ is state variable, $u(k)$ is control variable, and $w(k)$ is unknown Gaussian white noise, i. e.,

$$E(w(k))=q, \quad \mathrm{cov}[w(k),\ w(j)]=Q\delta_{kj}$$

where, E denotes the mathematical expectation and cov is the covariance symbol, and δ_{kj} is the Kronecker function.

Introducing vector,

$$\theta(k)=[a_0(k),\cdots,a_n(k);\ b_0(k),\cdots,b_n(k)]^T$$

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where T denotes the transpose.

It is assumed that the parameter is modelled by following random walk

$$\theta(k+1)=\theta(k)+\alpha(k) \tag{2}$$

and the observation equation is described by

$$y(k)=x(k)+v(k) \tag{3}$$

In Eqs. (2), (3), $\alpha(k)$ and $v(k)$ are unknown Gaussian white noise, i. e. ,

$$E(\alpha(k))=s,\ \operatorname{cov}(\alpha(k),\ \alpha(j))=S\delta_{kj} \tag{4}$$

$$E(v(k))=r,\ \operatorname{cov}(v(k),\ v(j))=R\delta_{kj} \tag{5}$$

and $v(k)$ is uncorrelated to $w(k)$ and $\alpha(k)$ is uncorrelated to $v(k)$.

The problem is to estimate the timevarying parameters and to estimate the state at each k, based on observation data up to time k, $y(1),\cdots,y(k)$.

It is interesting to notice that model (1) can be considered as an univariate multiple time delays model [2]. Therefore, we can perform directly suboptimal recursive filter instead of extended state Kalman filter [1]. Assuming that the estimates $\hat{\theta}(k/k)$ of parameters are given, substituing them into Eq. (1), we have

$$x(k+1)=\sum_{i=0}^{n}\hat{a}_i(k/k)x(k-i)+\hat{b}_i(k/k)u(k-i)+\omega(k) \tag{6}$$

where $\omega(k)=w(k)+$model error term, $\omega(k)$ is called the fictitious state noise which combines the state model error into the noise statistics. Obviously, the fictitious noise has unknown timevarying mean and variance and can be considered approximately as Gaussian white noise[3]. Let

$$E(\omega(k))=q(k),\ \operatorname{cov}(\omega(k),\ \omega(j))=Q(k)\delta_{kj} \tag{7}$$

For system (6), (3) with unknown timevarying noise statistics, we use following suboptimal recursive filter equations [2]:

$$\hat{x}(k+1/k+1)=\hat{x}(k+1/k)+K_x(k+1)e_x(k+1) \tag{8}$$

$$\hat{x}(k+1/k)=\sum_{j=0}^{n}[\hat{a}_i(k/k)\hat{x}(k-j/k-j)+\hat{b}_i(k/k)u(k-j)]+\hat{q}(k) \tag{9}$$

$$e_x(k+1) = y(k+1) - \hat{x}(k+1/k) - \hat{r}(k) \tag{10}$$

$$K_x(k+1) = P_x(k+1/k)[P_x(k+1/k) + \hat{R}(k)]^{-1} \tag{11}$$

$$P_x(k+1/k+1) = [I - K_x(k+1)]P_x(k+1/k) \tag{12}$$

$$P_x(k+1/k) = \sum_{j=0}^{n} [\hat{a}_j^2(k/k)P_x(k-j/k-j)] + \hat{Q}(k) \tag{13}$$

$$\hat{q}(k+1) = (1-d_k)\hat{q}(k) + d_k\Big\{\hat{x}(k+1/k+1) - \sum_{j=0}^{n}[\hat{a}_i(k/k)\hat{x}(k-j/k-j) + \hat{b}_i(k/k)u(k-j)]\Big\} \tag{14}$$

$$\hat{Q}(k+1) = (1-d_k)\hat{Q}(k) + d_k\Big[K_x^2(k+1)e_x^2(k+1) + P_x(k+1/k+1) - \sum_{j=0}^{n}\hat{a}_j^2(k/k)P_x(k-j/k-j)\Big] \tag{15}$$

$$\hat{r}(k+1) = (1-d_k)\hat{r}(k) + d_k[y(k+1) - \hat{x}(k+1/k)] \tag{16}$$

$$\hat{R}(k+1) = (1-d_k)\hat{R}(k) + d_k[e_x^2(k+1) - P_x(k+1/k)] \tag{17}$$

where $d_k = (1-b)/(1-b^{k+1})$, $0<b<1$, b is called the forgetting factor.

Now, substituting Eq. (1) into Eq. (3), we have

$$y(k+1) = h^T(k)\theta(k) + w(k) + v(k+1) \tag{18}$$

where $h^T(k) = [x(k),\ x(k-1),\cdots,x(k-n);\ u(k),\ u(k-1),\cdots,u(k-n)]$

Once the estimate $\hat{x}(k/k)$ is given, we have

$$\hat{h}^T(k) = [\hat{x}(k/k),\cdots,\hat{x}(k-n/k-n);\ u(k),\ u(k-1),\cdots,u(k-n)] \tag{19}$$

Eq. (18) can be rewritten as

$$y(k+1) = \hat{h}^T(k)\theta(k) + \eta(k) \tag{20}$$

where $\eta(k) = w(k) + v(k+1) + (h(k) - \hat{h}(k))^T\theta(k)$ is the fictitious noise which combines the observation model error into the noise statistics so that model error is compensated. Clearly, $\eta(k)$ is timevarying noise statistics. Let

$$E(\eta(k)) = n(k), \quad \mathrm{cov}[\eta(k), \ \eta(j)] = N(k)\delta_{kj} \tag{21}$$

From Eq. (20), we see that $\eta(k)$ can be considered approximately as Gaussian white noise. Hence, for system (2) and (20), we obtain following adaptive Kalman filter equations[3]

$$\hat{\theta}(k+1/k+1) = \hat{\theta}(k+1/k) + K_\theta(k+1)e_\theta \ (k+1) \tag{22}$$

$$\hat{\theta}(k+1/k) = \hat{\theta}(k/k) + \hat{S}(k) \tag{23}$$

$$e_\theta(k+1) = y(k+2) - \hat{h}^T(k+1)\hat{\theta}(k+1/k) - \hat{n}(k) \tag{24}$$

$$K_\theta(k+1) = P_\theta(k+1/k)\hat{h}(k+1) \cdot [\hat{h}^T(k+1)P_\theta(k+1/k)\hat{h}(k+1) + \hat{N}(k)]^{-1} \tag{25}$$

$$P_\theta(k+1/k) = P_\theta(k/k) + \hat{S}(k) \tag{26}$$

$$P_\theta(k+1/k+1) = [I - K_\theta(k+1)\hat{h}^T(k+1)]P_\theta(k+1/k) \tag{27}$$

$$\hat{s}(k+1) = (1-\beta_k)\hat{s}(k) + \beta_k[\hat{\theta}(k+1/k+1) - \hat{\theta}(k/k)] \tag{28}$$

$$\hat{S}(k+1) = (1-\beta_k)\hat{S}(k) + \beta_k[K_\theta(k+1)e^2(k+1)K_\theta^T(k+1) + P_\theta(k+1/k+1) - P_\theta(k/k)] \tag{29}$$

$$\hat{n}(k+1) = (1-\beta_k)\hat{n}(k) + \beta_k[y(k+2) - \hat{h}^T(k+1)\hat{\theta}(k+1/k) \tag{30}$$

$$\hat{N}(k+1) = (1-\beta_k)\hat{N}(k) + \beta_k[e^2(k+1) - \hat{h}^T(k+1)P_\theta(k+1/k)\hat{h}(k+1)] \tag{31}$$

where $\beta_k = (1-\alpha)/(1-\alpha^{k+1})$, $0<\alpha<1$, α is the forgetting factor.

Eqs. (22)—(31) are called the second stage adaptive parameter estimators.

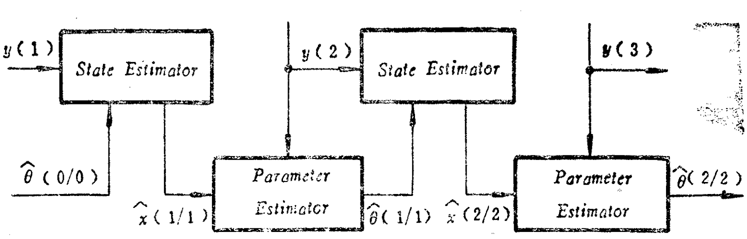

Fig. 1 The alternative use of the two-stage adaptive estimators.

If we choose sutiable initial values and forgetting factors α, b, and alternatively perform this two-stage coupled adaptive estimation, the state and parameter can be updated on-line simultaneously. Fig. 1 shows the alternative use of the two-stage adaptive estimators.

Simulation Example

To illustrate the usefulness of the proposed approach, the simulated example is given. For simplicity, we do not consider exogenous control variable $u(k)$.

Let us consider following discrete linear timevarying system with observation noise.

$$x(k+1)=a_0(k)x(k)+a_1(k)x(k-1)+w(k) \tag{32}$$

$$a_0(k+1)=a_0(k)=0.65 \tag{33}$$

$$a_1(k+1)=a_1(k)+\alpha(k) \tag{34}$$

$$y(k)=x(k)+v(k) \tag{35}$$

$$x(0)=0.01,\ x(-1)=0.1,\ a_1(0)=0 \tag{36}$$

where $w(k)$ is unknown zero-mean Gaussian noise, its variance was assumed to be $(0.04)^2$ in simulation; $\alpha(k)$ and $v(k)$ are given zero-mean Gaussian noise independent of each other and having variance $(0.1)^2$ and $(0.02)^2$ respectively.

We chose initial values,

$$\hat{x}(0/0)=\hat{x}(-1/-1)=0.1,\ P_x(0/0)=P_x(-1/-1)=1;$$

$$\hat{a}_0(0/0)=1,\ \hat{a}_1(0/0)=0;\ P_\theta(0/0)=I$$

In two-stage coupled adaptive estimators, forgetting factors α and b are taken as 0.98 and 0.975 respectively; Noise statistics are

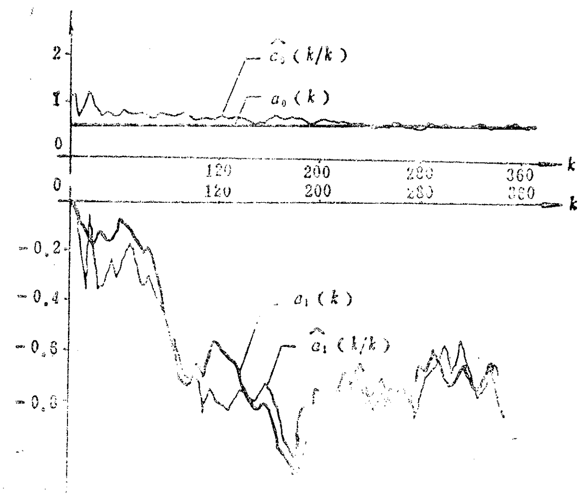

Fig. 2 Tracking parameters by the technique proposed.

$$\hat{q}(0)=0.001,\quad \hat{Q}(0)=0.2,\quad \hat{n}(0)=0.001,\quad \hat{N}(0)=0.1.$$

On TRS—80 computer simulations were performed, results are shown in Fig. 2 in which thick lines denote the true parameters, estimates were updated very well after 200 steps by using the suboptimal filter given in reference [2].

References

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有观测噪声的时变系统的参数估计

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摘　要

本文给出了有观测噪声、线性离散时变系统的参数估计新方法。它由两段互耦的自适应状态估计器和自适应参数估计器组成。通过引入虚拟时变噪声，我们结合在互耦算法中产生的模型误差到虚拟噪声统计，使模型误差得到有效地补偿和克服滤波发散。模拟例子说明了本文方法的有效性。