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The Consistent Condition of Disturbance Decoupling of Decentralized Control

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Abstract

The mistakes, which have existed up till now, in the field of decentralized disturbance decoupling are pointed out in this paper. The causes of making such mistakes are explained by examples. To solve this problem, a new definition, the consistent condition of structure $(A\ B_i)$ invariant subspace, is put forward. Furthermore a necessary and sufficient condition about structure $(A\ B_i)$ invariant subspace with 3 stations is given.

1. Introduction

In the last decade several reseach papers were submitted dealing with the problem of decentralized disturbance decoupling (DDDP), i. e. Hamano and Furuta[1], Moog and Cury[2], Cury et. al.[3] and Leite[4,5]. It seems that all of them made mistakes in the proof of necessary and/or sufficient conditions of structure $(A\ B_i)$ invariant subapace ($(A\ B_i)$ inv. s.) which paid a key role in their discussion.

In this paper we shall clarify some facts, first, that several counterexamples are worked out to the conditions of $(A\ B_i)$ inv. s. given in [3] and [4] (These conclusions are picked up in the Appendix), second, that a consistent condition of $(A\ B_i)$ inv. s. is presented firstly, and a necessary and sufficient condition of DDDP with 3 stations is also advanced.

2. Counterexamples

The theorem 2.1 of[1], which was subjected to censure of Moog

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and Cury[2], is correct and can be reproved by an accurate means, but it can not generalize by simple imitation, and, up to now no one has given a correct condition of $(A\ B_i)$ inv. s. for following 3 stations decentralized system

$$\dot{x}(t)=Ax(t)+B_1u_1(t)+B_2u_2(t)+B_3u_3(t) \tag{1}$$

Example 1 Let $\mathsf{X}=R^5$.

$$\begin{aligned} A:\ \mathsf{X}\longrightarrow\mathsf{X}.\quad & e_1+e_2\longrightarrow e_4+2e_5,\\ & e_1+e_3\longmapsto e_4-e_5,\\ & \text{zero}\quad \text{otherwise,}\end{aligned}$$

where $e_i=(0\ldots0\ \underset{i}{1}\ \ 0\ldots0)'$, for example, the matrix expression of A may be

$$A=\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&1&1&0&0\\0&2&-1&0&0\end{pmatrix}.$$

And

$$B_1=\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}\ B_2=\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}\ B_3=\begin{pmatrix}0\\0\\0\\1\\1\end{pmatrix}.$$

$\mathsf{X}_1=\langle e_1\rangle$, $\mathsf{X}_2=\langle e_2\rangle$, $\mathsf{X}_3=\langle e_3\rangle$ and $\mathsf{V}=\langle e_1+e_2,\ e_1+e_3\rangle$. It is easy to verify the following two facts:

I $(A+B_1F_1+B_2F_2+B_3F_3)\mathsf{V}\subset\langle e_4,e_5\rangle$,

II $A\mathsf{V}\subset\mathsf{V}+\mathsf{B}_1+\mathsf{B}_2+\mathsf{B}_3$ and $A(\mathsf{V}\cap\underset{i\in I}{\oplus}\mathsf{X}_i)\subset\mathsf{V}+\sum_{i\in I}\mathsf{B}_i$ for any I, nonempty subset of $\{1,2,3\}$.

If there were some F_i: $\mathsf{X}\longrightarrow\mathsf{U}_i$, $F_i|\underset{j\neq i}{\oplus}\mathsf{X}_j=0$ such that $(A+B_1F_1+B_2F_2+B_3F_3)\ \mathsf{V}\subset\mathsf{V}$, then, provided that $\mathsf{V}\cap\langle e_4,e_5\rangle=0$, we would get

$$(A+B_1F_1+B_2F_2+B_3F_3)(e_1+e_2)=0,$$

then

$$F_1e_1=-1,\ F_2e_2=-2 \tag{2}$$

and

$$(A+B_1F_1+B_2F_2+B_3F_3)(e_1+e_3)=0,$$

then

$$F_1e_1=-2,\ F_3e_3=1, \tag{3}$$

(3) contradicts (2). It implies that there are no F_i $(i=1,2,3)$ such that $F_i|\underset{j\neq i}{\oplus}\mathbf{X}_j=0$ and $(A+B_1F_1+B_2F_2+B_3F_3)\mathbf{V}\subset\mathbf{V}$, so $\mathbf{V}$ is not an $(A\ B_i)$ inv. s.. It shows the theorem 2.1 of [1] can't generalize.

Example 2 A, B_1, B_2 and B_3 are same matrices as those in Example 1.

$\mathbf{X}_1$, $\mathbf{X}_2$ and $\mathbf{V}$ are also the same subspaces as those in Example 1. $\mathbf{X}_3$ is taken as $\mathbf{X}_3=\langle e_3, e_4, e_5\rangle$.

It is easy to verify that all the conditions given by Cury et. al. are satisfied. but we can show that $\mathbf{V}$ is not an $(A\ B_i)$ inv. s. by the same way for Example 1. The detail is omitted due to the space limited.

Example 3 Let A, B_1, B_2, and B_3 are equal to those in Example 1. We put $\mathbf{S}_1=\langle e_2, e_3\rangle$ $\mathbf{S}_2=\langle e_1, e_3\rangle$ $\mathbf{S}_3=\langle e_1, e_2\rangle$, and $\mathbf{V}$ is still taken to be $\langle e_1+e_2, e_1+e_3\rangle$.

Although $\mathbf{V}$ satisfies with all Leite's conditions but it fails to be an $(A\ B_i)$ inv. s. by similar way which was used in Example 1.

3. Consistency

We are interested in analysing the reason why they fail. In this section we follow the definition of Hamano and Fufuta[1].

$\mathbf{V}\cap(\mathbf{X}_i\oplus\mathbf{X}_j)$ is decomposed into three parts

$$\mathbf{V}\cap(\mathbf{X}_i\oplus\mathbf{X}_j)=\mathbf{V}_{ij}\oplus\mathbf{V}\cap\mathbf{X}_i\oplus\mathbf{V}\cap\mathbf{X}_j; \tag{4}$$

For every $x\in\mathbf{V}_{ij}$, x can be decomposed, $x=x_i+x_j$, where $x_i\in\mathbf{X}_i$ $x_i\bar{\in}\mathbf{V}\cap\mathbf{X}_i$, and $x_j\in\mathbf{X}_j$ $x_j\bar{\in}\mathbf{V}\cap\mathbf{X}_j$. $P_{ij}^{(i)}$ denotes projective map from $\mathbf{V}_{ij}$ into $\mathbf{X}_i$, i. e. for any $x\in\mathbf{V}_{ij}$ $P_{ij}^{(i)}x=x_i$.

By the same way used in (4), $\mathbf{V}\subset\mathbf{X}$ may be divided into

$$\begin{aligned}\mathbf{V}&=\tilde{\mathbf{V}}\oplus\mathbf{V}\cap\mathbf{X}_1\oplus\mathbf{V}\cap\mathbf{X}_2\oplus\mathbf{V}\cap\mathbf{X}_3\oplus\mathbf{V}_{12}\oplus\mathbf{V}_{13}\oplus\hat{\mathbf{V}}_{23}\oplus\mathbf{V}_{123}\\&=\hat{\mathbf{V}}\oplus\mathbf{V}\cap\mathbf{X}_1\oplus\mathbf{V}\cap\mathbf{X}_2\oplus\mathbf{V}\cap\mathbf{X}_3\oplus\mathbf{V}_{12}\oplus\mathbf{V}_{13}\oplus\hat{\mathbf{V}}_{23},\end{aligned}$$

When $P_{ij}^{(i)}\mathbf{V}_{ij}\cap P_{ik}^{(i)}\mathbf{V}_{ik}\neq0$, there exists at least an $x_i\neq0$, $x_i\in P_{ij}^{(i)}\mathbf{V}_{ij}\cap P_{ik}^{(i)}\mathbf{V}_{ik}$, then, F_i must be defined as $F_ix_i=u_i$ for some $u_i\in U_i$ according to $A\mathbf{V}_{ij}\subset\mathbf{V}+B_i+B_j$, on the other hand according to $A\mathbf{V}_{ik}\subset\mathbf{V}+B_i+B_k$, F_i must be defined $F_ix_i=\bar{u}_i$. Often $u_i\neq\bar{u}_i$ and then the trouble comes. In Example 1 $e_1\in P_{12}^{(1)}\mathbf{V}_{12}\cap P_{13}^{(1)}\mathbf{V}_{13}$, $F_1e_1=-1$ follows equation (2) and $F_1e_1=-2$ by virture of equation (3).

This contradiction implies that V is not an $(A\ B_i)$ inv. s..

We now define consistent condition of DDDP as follows:

If $P_{ij}^{(i)}\mathsf{V}_{ij}\cap P_{ik}^{(i)}\mathsf{V}_{ik}\neq 0$, then we have

$$A(x_i+x_j)=v_{ij}+B_iu_i+B_ju_j,$$

$$A(x_i+x_k)=v_{ik}+B_i\bar{u}_i+B_ku_k,$$

where v_{ij} and $v_{ik}\in \mathsf{V}$, u_i and $\bar{u}_i\in \mathsf{U}_i$, $u_j\in \mathsf{U}_j$ and $u_k\in \mathsf{U}_k$.

Definition If there exist v_{ij} and v_{ik} such that $u_i=\bar{u}_i$ for every $x_i\in P_{ij}^{(i)}\mathsf{V}_{ij}\cap P_{ik}^{(i)}\mathsf{V}_{ik}$, then V is consistent relative to X_i.

In the light of this definition, there is no difficulty to verify the theorem given below and we omit its proof to save the space.

Theorem V is a $(A\ B_i)$ inv. s. if and only if the following conditions are satisfied:

I V is consistent relative to X_i for every $i\in\{1,2,3\}$;

II $A(\mathsf{V}\cap \bigoplus_{i\in I}\mathsf{X}_i)\subset \mathsf{V}+\sum_{i\in I}B_i$ for every $I\subset\{1,2,3\}$ and $A\mathsf{V}\subset\mathsf{V}+B_1+B_2+B_3$.

The definition and theorem can generalize and we work on it in another paper.

Appendix

In the Appendix we deal with general case, i. e. the model is

$$\dot{x}(t)=Ax(t)+\sum_{i=1}^{v}B_iu_i(t).$$

Firstly, we offer the definition given by Hamano and Furuta. It assumes that $\sum_{i=1}^{v}\mathsf{X}_i=\bigoplus_{i=1}^{v}\mathsf{X}_i$.

Definition V is called an $(A\ B_i)$ inv. s. if there exist F_i: $\mathsf{X}\to\mathsf{U}_i$ $F_i|\bigoplus_{j\neq i}\mathsf{X}_j=0$, $i=1,2,\cdots,v$, such that

$$(A+\sum_{i=1}^{v}B_iF_i)\mathsf{V}\subset\mathsf{V}.$$

We copy Curys' definition and conclusion as follows:

Definition Let A: $\mathsf{X}\longrightarrow\mathsf{X}$ and B_i: $\mathsf{U}_i\longrightarrow\mathsf{X}$ and consider a decomposition $\mathsf{X}=\bigoplus_{i=1}^{v}\mathsf{X}_i$, then V is called an $(A\ B_i)$ inv. s. if and only if there exist v linear maps

$$F_i\text{: }\mathsf{X}\longrightarrow\mathsf{U}_i,\ F_i|\bigoplus_{j\neq i}\mathsf{X}_j=0,\ \text{for } i=1,2,\cdots,v.$$

such that

$$(A+\sum_{i=1}^{v} B_iF_i)\mathrm{V}\subset \mathrm{V}.$$

Theorem V is an $(A\ B_i)$ inv. s. if and only if for all I, $I\subset\{1, 2,\cdots v\}$, $A(\mathrm{V}\cap \underset{i\in I}{\oplus} \mathrm{X}_i)\subset \mathrm{V}+\sum_{i\in I} \mathrm{B}_i$.

Lastly, let us recall what Leite gave. Leite defined $(A\ B_i)$ inv. s. in time domain, but an altertive way is the following Definition.

Definition S_i $i\in\{1,2,\cdots,v\}$ are any subspace of X. A subspace is called $(A\ B_i)$ inv. s. if there exist F_i: $\mathrm{X}\longrightarrow\mathrm{U}_i$, $i\in\{1,2,\cdots,v\}$ such that Ker $F_i\supset\mathrm{S}_i$ and $(A+\sum_{i=1}^{v} B_iF_i)\mathrm{V}\subset\mathrm{V}$.

Theorem V is an $(A\ B_i)$ inv. s. if and only if $A(\mathrm{V}\cap(\underset{i\in I}{\cap}\mathrm{S}_i))\subset \mathrm{V}+\sum_{i\in\bar{I}} B_i$ for any $I\subset\{1,2,\cdots,v\}$, where $\bar{I}$ is the complement of I in $\{1, 2, \cdots, v\}$.

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分散控制的干扰解耦的相容条件

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摘 要

本文指出目前在分散控制干扰解耦中的结论是错误的，并用例子说明了产生这种错误的原因。为此文章首次给出结构（$A\ B_i$）不变子空间的相容性条件。对三个控制站的分散系统给出了结构（$A\ B_i$）不变子空间的一个充要条件。