# Closed-loop Identification of Transfer Functions of a Glass Furnace

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#### Abstract

The identification of transfer functions of linear systems operating in closed-loop has been the subject of much research in recent years (1)~(4). This paper presents an identification method of switching two independent lower order regulators. By using this method we estimated the transfer function of the furnace temperature changes to the variations of fuel oil flow rate and the variations of the combustion air flow rate in the glass furnace. Results are verified by open and closed-loop tests.

Guangzhou Electric Light Lamp Factory is going to introduce the computer control to the glass furnace for substituting its conventional control and we are dealing with the technical problems of this project. one of the major problems we encountered is the identification of transfer functions of the furnace temperature changes to the variations of fuel oil flow rate and the variations of the combustion air flow rate. Fig. 1 shows the simplified scheme of the burning process. By means of the pressurized air, the fuel oil flows through the left (or right) oil injector and is sprayed into the melter where it is burned with the combustion air coming from one of the recuperation chambers, producing the required temperature for melting the glass. Firing direction is changed from one side of the melter to the other under certain conditions of temperature and firing duration.

The furnace temperature is constantly monitored and regulated in closed-loop by controlling the fuel oil flow rate with a conventional regulator, combining with a manual adjustment of the

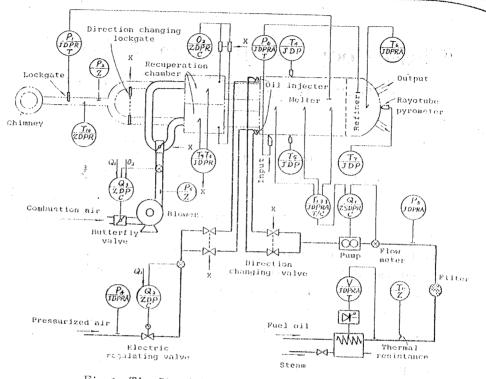


Fig.1 The Simplified Scheme of Burning Process

combustion air flow rate for maintaining the good burning condition. To ensure the safety of production and the quality of products, this regulator is not allowed to be removed from the closed-loop during the process of identification experiment. This entails the experiment to be carried out under the closed-loop operating condition.

Fig.2 shows the schematic diagram of the closed-loop control of the furnace temperature, where y(k) is the changes of furnace

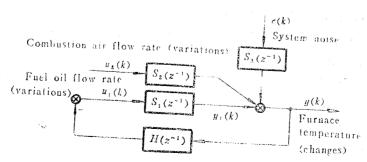


Fig.2 The Schematic Diagram of the Closed-loop Control for the Furnace Temperature

of a Glass Furnace

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temperature,  $u_1(k)$  and  $u_2(k)$  are the variations of fuel oil flow rate and combustion air flow rate respectively, e(k) is the system noise which is assumed to be normally distributed.  $S_1(z^{-1})$ ,  $S_2(z^{-1})$  and  $S_3(z^{-1})$  are z-transfer functions which are to be estimated.  $H(z^{-1})$  is z-transfer function of PID or PD controller.

(A) Identification of the transfer function  $S_1$  ( $z^{-1}$ ).

To simplify the procedure in identifying the transfer function  $s_1(z^{-1})$ , the combustion air flow rate is maintained at a fixed level by manually adjusting the batterfly valve to eliminate its effect on the changes of furnace temperature. Consequently, we can write:

$$y(k) = S_1(z^{-1})u_1(k) + S_3(z^{-1})e(k)$$
 (1)

$$u_1(k) = -H(z^{-1})y(k)$$
 (2)

where

$$S_1(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{b_1 z^{-1} + \dots + b_n z^{-n}}{1 + a_1 z^{-1} + \dots + a_n z^{-n}}$$
(3)

$$S_3(z^{-1}) = \frac{C(z^{-1})}{A(z^{-1})} = \frac{1 + c_1 z^{-1} + \dots + c_n z^{-n}}{1 + a_1 z^{-1} + \dots + a_n z^{-n}}$$
(4)

$$H(z^{-1}) = \frac{F(z^{-1})}{G(z^{-1})} = \frac{f_0 + f_1 z^{-1} + \dots + f_m z^{-m}}{1 + g_1 z^{-1} + \dots + g_m z^{-m}}$$
 (5)

Eliminating  $u_1(k)$  between (1) and (2) gives:

$$y(k) = \frac{G(z^{-1})C(z^{-1})}{A(z^{-1})G(z^{-1}) + B(z^{-1})F(z^{-1})}e(k)$$
 (6)

Let 
$$Q(z^{-1}) = G(z^{-1})C(z^{-1}) = 1 + q_1 z^{-1} + \dots + q_{n+m} z^{-(n+m)}$$
 (7)

$$P(z^{-1}) = A(z^{-1})G(z^{-1}) + B(z^{-1})F(z^{-1}) = 1 + p_1 z^{-1} + \dots + p_{n+m} z^{-(n+m)}$$
(8)

then Eq. (6) is equivalent to the following ARMA model:  $y(k) + p_1 y(k-1) + \dots + p_{n+m} y(k-n-m) = e(k) + q_1 e(k-1) + \dots + q_{n+m} e(k-n-m)$ (9)

Parameters p's and q's in the ARMA model were estimated by the Extended Matrix Method together with an F-testing criterion for determining the order of Eq. (9):

$$p_1 = 0.7966$$
,  $p_2 = 0.0739$ ,  $p_3 = 0.3451$ ,  $p_4 = 0.6423$ ,  $p_5 = 0.6576$   $q_1 = -0.0439$ ,  $q_2 = -0.2598$ ,  $q_3 = -0.1001$ ,  $q_4 = -0.0019$  and the mean value of  $e(k)$  is 0.0017, its variance is 0.9250.

By using the Recursive Least Squares Estimation (RLSE) method the transfer function of the PID controller was identified

as:

$$H(z^{-1}) = \frac{0.9782 - 1.533z^{-1} + 0.7314z^{-2}}{1 - z^{-1}}$$

In order to obtain a's and b's we go back to Eq. (8) and equate the coefficients of the same power of  $z^{-1}$  on the both sides, this gives

$$\begin{pmatrix}
1 & 0 & 0 & 0.9782 & 0.0 & 0.0 \\
-1 & 1 & 0 & -1.5033 & 0.9782 & 0.0 \\
0 & -1 & 1 & 0.7314 & -1.5033 & 0.9782 \\
0 & 0 & -1 & 0.0 & 0.7314 & -1.5033 \\
0 & 0 & 0 & 0.0 & 0.0 & 0.7314
\end{pmatrix}
\begin{pmatrix}
a_1 \\
a_2 \\
a_3 \\
b_1 \\
b_2 \\
b_3
\end{pmatrix} = \begin{pmatrix}
1.7966 \\
0.0739 \\
0.3451 \\
0.6423 \\
0.6576
\end{pmatrix}$$

(10)

Obviously Eq. (10) does not have an unique solution because the left matrix is not of the full column rank. To circumvent this, we switch the PID controller to a PD controller, the transfer function of which was estimated as:

$$H'(z^{-1}) = 2.5041 - 1.1225z^{-1}$$

Repeating the procedure for estimating the parameters in Eq.(9) gives

$$p'_1 = 1.1783$$
,  $p'_2 = 0.6132$ ,  $p'_3 = 0.6410$ ,  $p'_4 = 0.4155$ 

$$q'_1 = 0.0026$$
,  $q'_2 = -0.0133$ ,  $q'_3 = -0.1184$ 

The equation which is similar to Eq. (10) is

$$\begin{pmatrix}
1 & 0 & 0 & 2.5041 & 0.0 & 0.0 \\
0 & 1 & 0 & -1.1225 & 2.5041 & 0.0 \\
0 & 0 & 1 & 0.0 & -1.1225 & 2.5041 \\
0 & 0 & 0 & 0.0 & 0.0 & -1.1225
\end{pmatrix}
\begin{pmatrix}
a_1 \\
a_2 \\
a_3 \\
b_1 \\
b_2 \\
b_3
\end{pmatrix} = \begin{pmatrix}
1.1783 \\
0.6132 \\
0.6410 \\
0.4155
\end{pmatrix}$$
(11)

Combining (10) and (11), we obtain the parameter estimates by RLSE.

$$a_1 = 1.0214$$
,  $a_2 = 0.5843$ ,  $a_3 = 0.3125$   
 $b_1 = 0.0947$ ,  $b_2 = 0.0521$ ,  $b_3 = 0.0193$ 

To determine c's, we equate the coefficients of the same power of  $z^{-1}$  on the both sides of Eq.(7) to obtain a set of linear equations similar to Eq.(10), The LS solution of this set of linear equations

is:

$$c_1 = 0.4291$$
,  $c_2 = 0.0689$ ,  $c_3 = -0.0492$  (1) and  $c_4 = 0.0492$ 

(B) Identification of transfer function S2: (271) in the base would

Having finished the identification of transfer function  $S_1(z^{-1})$  we proceed to the experiment for identifying transfer function  $S_2(z^{-1})$ . In this experiment, the combustion air flow rate was varied continuously by manually adjusting the butterfly valve. In this case we can write:

$$y(k) = S_1(z^{-1})u_1(k) + S_2(z^{-1})u_2(k) + S_3(z^{-1})e(k)$$
Let 
$$y_2(k) = y(k) - S_1(z^{-1})u_1(k), e'(k) = S_3(z^{-1})e(k)$$
then 
$$y_2(k) = S_2(z^{-1})u_2(k) + e'(k)$$
(12)

Denoting 
$$S_2(z^{-1}) = \frac{B'(z^{-1})}{A'(z^{-1})} = \frac{b'_1 z^{-1} + b'_2 z^{-2} + b'_3 z^{-3}}{1 + a'_1 z^{-1} + a'_2 z^{-2} + a'_3 z^{-3}}$$

Expanding it into an infinite series and reserving the first 10 terms, (12) can be written as follows:

$$y_2(k) = (s_1 z^{-1} + \dots + s_{10} z^{-10}) u_2(k) + e''(k)$$

where  $s_1$ , ...,  $s_{10}$  are the pulse response sequence and e''(k) includes the measurement noise as well as the truncated error in the expansion.  $s_1$ ,...,  $s_{10}$ , were estimated by LSE:

$$s_1 = -0.0397$$
,  $s_2 = -0.0147$ ,  $s_3 = 0.0353$ ,  $s_4 = -0.0105$ ,  $s_5 = -0.0158$ ,  $s_6 = -0.0267$ ,  $s_7 = 0.0112$ ,  $s_8 = 0.0134$ ,  $s_9 = 0.0101$ ,  $s_{10} = 0.0040$ .

After obtaining the pulse response sequence we determined the coefficients of the transfer function  $S_2(z^{-1})$  by the method proposed in [2].

$$a'_1 = 0.8512$$
,  $a'_2 = 0.5184$ ,  $a'_3 = 0.5537$   
 $b'_1 = -0.0397$ ,  $b'_2 = -0.0490$ ,  $b'_3 = 0.0022$ 

Summarizing the results in (A) and (B), we have the dynamic equation for the furnace temperature:

$$y(k) = \frac{(0.0947 + 0.0521z^{-1} + 0.0193z^{-2})z^{-1}}{1 + 1.0214z^{-1} + 0.5843z^{-2} + 0.3125z^{-3}} u_1(k)$$

$$+ \frac{(-0.0397 - 0.0490z^{-1} + 0.0022z^{-2})z^{-1}}{1 + 0.8512z^{-1} + 0.5184z^{-2} + 0.5537z^{-3}} u_2(k)$$

$$+ \frac{1 + 0.4291z^{-1} + 0.0689z^{-2} - 0.0492z^{-3}}{1 + 1.0214z^{-1} + 0.5843z^{-2} + 0.3125z^{-3}} e(k)$$
(13)

## (C) Verification.

The estimated transfer functions were verified first in closedloop operation by using the values of  $u_1(k)$  and  $u_2(k)$  collected by a monitoring computer and the values of e(k) generated by a simulation program, the temperature changes y(k) were computed according to Eq. (13) and were compared in Fig. 3 with

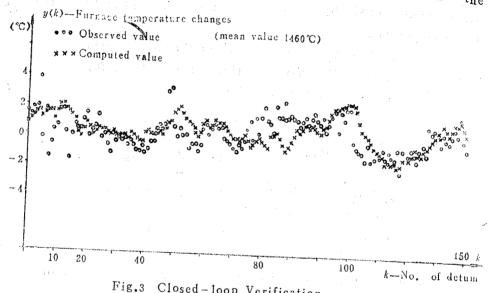


Fig.3 Closed-loop Verification

observed ones. It is seem that they are close enough to each other. In the second experiment of verification, the furnace operator was persuaded to open the control loop of the furnace temperature

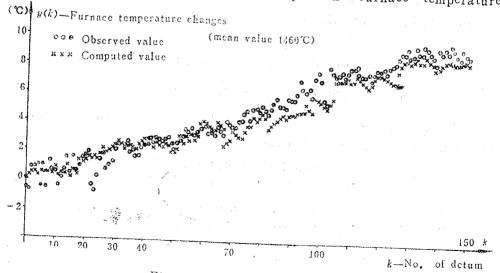


Fig.4 Open-loop Verification

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for a short time at the risk of abnormal production, and to increase steadily the fuel oil flow rate from 340.0 kg/hr to 497.5kg/hr while keeping the combustion air flow rate at a fixed level of 3500.0 m<sup>3</sup>/hr. The final observed value of temperature was nearly 9°C which is consistent with the computed value:

$$\lim_{k \to \infty} y(k) = \lim_{k \to \infty} s_1(z^{-1}) \cdot (497.5 - 340.0) = 0.057 \cdot 157.5 = 8.98^{\circ}C$$

### (D) Conclusion.

The closed-loop identification method by switching two independent lower order controllers was discussed in detail in [4]. our experiment for estimating the transfer functions of a glass furnace demonstrated the feasibility of this method.

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# 玻璃窑炉传递函数的闭环辨识

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### 摘 要

本文讨论了切换两个独立的低阶调节器进行闭环辨识的方法,并利用该方法辨识了玻璃 窑炉中重油流量对炉温以及二次风流量对炉温的传递函数。最后,进行了闭环和开环验证。