第6卷 第3期
1989年9月
控制理论与应用
CONTROL THEORY AND APPLICATIONS
Vol.6, No.3
Sept., 1989

The Lowest Degree Polynomial Matrix and Multivariable Singular System Identification

Chen Dong, Tu Fengsheng

(Department of Computer and System Sciences, Nankai University, Tianjin)

Abstract

The major work of this paper is the introduction of the concept of the lowest degree polynomial matrix as a tool to study the uniqueness of the singular systems satisfying the same input-output data, and the extension of the previous identification algorithm to singular multivariable systems.

Key words——Singular systems; The lowest degree polynomial matrix ; Equivalence; Identification; Canonical forms

1. Input output Desciptions of Multivariable Singular Systems

Consider the singular system of the form

$$Ex(k+1) = Ax(k) + Bu(k) \tag{1.1a}$$

$$y(k) = Cx(k) \tag{1.1b}$$

with

$$\det(zE - A) \neq 0 \tag{1.1c}$$

where E, A are n×n matrices, B, C are n×m, p×n matrices, respectively.

It is well known that singular system (1.1) is restrict system equivalent (r. s. e.) to the following decomposed system[1]

$$x_1(k+1) = A_1 x_1(k) + B_1 u(k) \tag{1.2a}$$

$$E_2 x_2(k+1) = x_2(k) + B_2 u(k) \tag{1.2b}$$

$$y(k) = C_1 x_1(k) + C_2 x_2(k) \tag{1.2c}$$

where A_1 is $n_1 \times n_1$ matrix, E_2 is $n_2 \times n_2$ nilpotent matrix, others are appropriate dimensional matrices, $n_1 + n_2 = n$.

Note that E_2 is nilpotent, then the transfer function matrix of

Manuscript received Feb. 11, 1988, revised Mar. 14, 1989.

singular system (1.2) (also system (1.1)) can be expressed as

$$G(z)=C(zE-A)^{-1}B=G_1(z)+W(z) \tag{1.3a}$$

where

$$G_1(z)=C_1(zI-A_1)^{-1}B_1 \tag{1.3b}$$

is strictly proper fraction matrix,

$$W(z)=C_2(zE_2-I)^{-1}B_2=C_2(I+zE_2+\cdots+z^{n_2}E_2^{n_2})B_2$$
$$=W_0+zW_1+\cdots+z^vW_v \tag{1.3c}$$

is polynomial matrix where v is the largest nonnegative integer such that $C_2E_2^vB_2\neq 0$ if $W(z)\neq 0$.

[2] has shown that for observable matrix pair (C_1,A_1), there exist unique two polynomial matrices $P(z)\in F^{p\times p}[z]$ and $Q(z)\in F^{p\times m}[z]$ such that

$$C_1(zI-A_1)^{-1}B_1=P^{-1}(z)Q(z) \tag{1.4a}$$

where $P(z)$and $Q(z)$ have canonical structures as follows

$P(z)=(p_{ij}(z))$

$p_{ii}(z)=z^{v_i}-p_{ii,v_{ii}}z^{v_{ii}-1}-\cdots-p_{ii,1} \qquad 1\leqslant i\leqslant p$

$p_{ij}(z)=-p_{ij,v_{ij}}z^{v_{ij}-1}-p_{ij,v_{ij}-1}z^{v_{ij}-2}-\cdots-p_{ij,1} \qquad i\neq j,1\leqslant i,j\leqslant p$

$Q(z)=(q_{ij}(z))$

$q_{ij}(z)=q_{iv_i,j}z^{v_i-1}+q_{iv_i-1,j}z^{v_i-2}+\cdots+q_{i1,j} \qquad 1\leqslant i\leqslant p,1\leqslant j\leqslant m$

$v_{ii}=v_i, v_{ij}=\min(v_i+1,v_j)$ for $j<i, v_{ij}=\min(v_i,v_j)$ for $j\geqslant i$ (1.4b)

Substitute (1.4), (1.3c) into (1.3a), we obtain input-output description of singular system (1.2) with (C_1,A_1) observable as follows

$$P(z)y(k)=R(z)u(k) \tag{1.5a}$$

where $R(z)=P(z)W(z)+Q(z)$ can be written in the form

$R(z)=(r_{ij}(z))$

$$r_{ij}(z)=r_{iv_i+v+1,j}z^{v_i+v}+r_{iv_i+v,j}z^{v_i+v-1}+\cdots+r_{i1,j} \tag{1.5b}$$

where when $W(z)=0$, v is defined as -1.

2. Lowest Degree Polynomial Matrix of N Input Data

For given N data $\{u(k)\in R^m,\ k=1,2,\cdots,N\}$, obviously, there exist a nonzero polynomial matrix $F(z)\in F^{p\times m}[z]$ such that

$$F(z)u(k)=0,\ \text{for all}\ k=1,2,\cdots,N-1 \tag{2.1}$$

where $l=\deg F(z)$, p is positive integer. Let $S(F)$ denote the set of all nonzero polynomial matrices $F(z)$ such that (2.1) holds.

Definition 2.1: Let $F_N(z)$ be one of the lowest degree polynomial matrix in $S(F)$, $F_N(z)$ is called the lowest degree polynomial matrix of N input data $\{u(k)\in R^m,\ k=1,2,\cdots,N\}$.

Let

$$U^T(r)=\begin{pmatrix} u(1),u(2),\cdots,u(N-r+1) \\ u(2),u(3),\cdots,u(N-r+2) \\ \cdots \\ u(r),u(r+1),\cdots,u(N) \end{pmatrix} \qquad (2.2)$$

Theorem 2.1 If j is the least positive integer such that matrix $U^T(j)$ is not row full rank, then $\deg F_N(z)=j-1$.

Proof Let $F(z)=F_0+F_1z+\cdots F_{i-1}z^{i-1}$, then $F(z)u(k)=0$ for all $k=1,2,\ \cdots,N-i+1$ if and only if $(F_0,F_1,\cdots,F_{i-1})U^T(i)=0$. If the condition of Theorem 2.1 is satisfied, obviously, j is the least positive integer such that equation$(F_0,F_1,\cdots,F_{i-1})U^T(i)=0$ exists nonzero solutions. This implies that $\deg F_N(z)=j-1$.

Let two singular systems $P_i(z)y(k)=R_i(z)u(k),i=1,2$, which need not to be assumed to have canonical structures, satisfy the same input-output data $\{y(k)\in R^p,k=1,2,\cdots,N_i;\ u(k)\in R^m,k=1,2,\cdots,N\}$ and the lowest degree polynomial matrix of $\{u(k),k=1,2,\cdots,N\}$ is $F_N(z)$.

Theorem 2.2 If $\deg F_N(z)$ is greater than $\max(\deg P_1(z)+\deg R_2(z),\ \deg P_2(z)+\deg R_1(z))$, then the two singular systems $P_i(z)y(k)=R_i(z)u(k)$, $i=1,2$, are equivalent in the sense of polynomial matrices description, i. e., $P_1^{-1}(z)R_1(z)=P_2^{-1}(z)R_2(z)$.

Proof According to the result in [3], there exist two polynomial matrices $\widetilde{P}_i(z)\in F^{p\times p}[z]$, $i=1,2$, such that $\deg\widetilde{P}_i(z)=\deg P_i(z)$, $\det\widetilde{P}_i(z)=\det P_i(z)$, $i=1,2,\widetilde{P}_1(z)P_2(z)=\widetilde{P}_2(z)P_1(z)$ Obviously, we can deduce the following formula

$$(\widetilde{P}_1(z)R_2(z)-\widetilde{P}_2(z)R_1(z))u(k)=0,\ \text{for all } k=1,2,\cdots,N-l^*$$

where $l^*=\deg(\widetilde{P}_1(z)R_2(z)-\widetilde{P}_2(z)R_1(z))$, If the two singular systems $P_i(z)y(k)=R_i(z)u(k)$, $i=1,2$, are not equivalet, then it can be proved that $\widetilde{P}_1(z)R_2(z)-\widetilde{P}_2(z)R_1(z)$ is nonzero polynomial matrix. This is a contrary to the degree of $F_N(z)$.

3. Extensive Structural and Parametric Identification Algorithm

In this section, it is always assumed that we can give the estimative values of $\bar{n}\geqslant\max(v_i)$ and $\bar{v}\geqslant v$ before identification and the $\deg F_N(z)$ of input date $\{u(k),\ i=k,k+1,\cdots,k+N-1\}$ is larger than $2\bar{n}+\bar{v}$. Therefore, by Theorem 2.2, this implies that we can identify the structural indices and numerical parameters for the following subsystem(3.1) instead of for the i-th subsystem of (1.5a) and must lead to $r_{il,j}$ being zero for all $l>v_i+v+1$.

$$y_i(k+v_i)=\sum_{j=1}^{p}\sum_{l=1}^{v_{ij}}p_{ij,l}y_j(k+l-1)+\sum_{j=1}^{m}\sum_{l=1}^{\bar{n}+\bar{v}+1}r_{il,j}u_j(k+l-1) \quad (3.1)$$

Let the input-output data $\{y(i),u(i),i=1,2,\cdots\}$ be arranged as

$$\bar{y}_i^T(k+j)=(y_i^T(k+j),\ y_i^T(k+j+1),\cdots,y_i^T(k+j+N-1)),\quad 1\leqslant i\leqslant p,0\leqslant j$$

$$\bar{u}_i^T(k+j)=(u_i^T(k+j),\ u_i^T(k+j+1),\cdots,u_i^T(k+j+N-1)),\quad 1\leqslant i\leqslant p,0\leqslant j \quad (3.2)$$

Equation (3.1) shows that the vector $\bar{y}_i(k+v_i)$ is a linear combination of vectors $\bar{y}_j(k+l-1)(1\leqslant j\leqslant p,1\leqslant l\leqslant v_{ij})$ and $\bar{u}_j(k+l-1)$ $(1\leqslant j\leqslant m,\ 1\leqslant l\leqslant\bar{n}+\bar{v}+1)$.

Denote with $L_j(\bar{y}_i)$ and $L_j(\bar{u}_i)$ the vectors of (3.3) defined as follows

$$L_j(\bar{y}_i)=(\bar{y}_i(k),\ \bar{y}_i(k+1),\ \cdots,\ \bar{y}_i(k+j)) \quad (3.3a)$$

$$L_j(\bar{u}_i)=(\bar{u}_i(k),\ \bar{u}_i(k+1),\ \cdots,\ \bar{u}_i(k+j)) \quad (3.3b)$$

Also denote with $R(\delta_1,\delta_2,\cdots,\delta_{p+m})$ the matrix defined by

$$R(\delta_1,\cdots,\delta_{p+m})=L_{\delta_1}(\bar{y}_1),\cdots,\ L_{\delta_p}(\bar{y}_p),\ L_{\delta_{p+1}}(\bar{u}_1),\cdots,\ L_{\delta_{p+m}}(\bar{u}_m)) \quad (3.4)$$

and with $S(\delta_1,\cdots,\delta_{p+m})$ the matrix defined by

$$S(\delta_1,\delta_2,\cdots,\delta_{p+m})=R^T(\delta_1,\delta_2,\cdots,\delta_{p+m})R(\delta_1,\delta_2,\cdots,\delta_{p+m}) \quad (3.5)$$

Construct now the sequence of symmtrical increasing-dimension matrices given by

$$S(\underbrace{1,0,\cdots,0}_{p},\underbrace{\bar{n}+\bar{v},\cdots,\bar{n}+\bar{v}}_{m}),\ S(\underbrace{1,1,\cdots,0}_{p},\underbrace{\bar{n}+\bar{v},\cdots,\bar{n}+\bar{v}}_{m}),\cdots,$$

$$S(\underbrace{1,1,\cdots,1}_{p},\underbrace{\bar{n}+\bar{v},\cdots,\bar{n}+\bar{v}}_{m}),S(\underbrace{2,1,\cdots,1}_{p},\underbrace{\bar{n}+\bar{v},\cdots,\bar{n}+\bar{v}}_{m}),\cdots \quad (3.6)$$

and select, in the sequence (3.6), the singular matrix in the considered sequence and let μ_i be the index increased by one with respect to the previous (nonsingular) matrix, then if follows that $v_i=\mu_i$, $v_{ij}=\mu_j-1(j\neq i,1\leqslant j\leqslant p)$. Therefore, when a singular matrix is found, one of the indices is determined; the procedure ends, all structural indices are determined exvept v.

Define the vector of parameters for all $i=1,2\cdots,p$

$$\theta_{Gi}=(p_{i1,1},\cdots,p_{i_1,v_1}\,\vdots\cdots\vdots\,p_{ip,1},\cdots,p_{ip,v_p}\,\vdots\,r_{i1,1},\cdots,$$
$$r_{i\bar{n}+\bar{v}+1,1}\,\vdots\cdots\vdots\,r_{i1,m},\cdots,r_{i\bar{n}+\bar{v}+1,1,m}) \tag{3.7}$$

and, for simplicity of nontation, let

$$R_{Gi}=R(v_{i1},\cdots,v_{ii}-1,\cdots,v_{ip},\bar{n}+\bar{v},\cdots,\bar{n}+\bar{v}) \tag{3.8a}$$
$$S_{Gi}=S(v_{i1},\cdots,v_{ii}-1,\cdots,v_{ip},\bar{n}+\bar{v},\cdots,\bar{n}+\bar{v}) \tag{3.8b}$$

then the parameters estimation $\hat{\theta}_{Gi}$ of the i-th subsystem are

$$\hat{\theta}_{Gi}=S_{Gi}^{-1}\,R_{Gi}^T\,\bar{y}_i(k+v_i) \tag{3.8c}$$

Notice that, by Theorem 2. 2, the numerical parameters $r_{il,j}$ $(i>v_i+v+1)$ of $\hat{\theta}_{Gi}$ must be zero for all i,j $(1\leqslant i\leqslant p,1\leqslant j\leqslant m)$. Then

$$v=\max\{\max(l-v_i|r_{ik,j}=0,k\geqslant l;$$
$$r_{ik,j}\neq 0,k<l,\text{for all } 1\leqslant j\leqslant m)-2,-1\} \tag{3.9}$$

4.Conclusion

The concept of the lowest degree polynomial matrix play an important role in this paper. By the concept, we give a sufficient condition which guarantees the linear independence of the extensive identification algorithm proposed in section 3.

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最低阶多项式阵和多变量奇异系统辨识*

陈 东 涂奉生

（南开大学计算机与系统科学系，天津）

摘 要

这篇论文的主要工作是引进了输入数据的最低阶多项式阵的概念和将多变量系统结构和参数辨识算法推广到奇异多变量系统。

关键词：奇异系统；最低阶多项式阵；等价；辨识；标准形

*国家自然科学基金资助的课题。

（上接封三）

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