第6卷 增刊1 控制理论与应用 Vol.6 Suppl.I. 1
1989年1月 CONTROL THEORY AND APPLICATIONS Jan., 1989

A New Algorithm for Constructing Internal Skew Prime Polynomial Matrices

Wang Qingguo, Sun Youxian, Zhou Chunhui

(Department of Chemical Engineering, Zhejiang University, Hangzhou)

Abstract

A new algorithm is presented for constructing an internally skew prime pair of polynomial matrices from a given pair of polynomial matrices and its effectiveness is illustrated by an example.

1. Introduction

Consider any $(p\times p)$ polynomial matrix $D(s)$ and $(p\times m)$ polynomial matrix $N(s)$. If $D(s)$ is nonsingular, then $D(s)$ and $N(s)$ are called externally skew prime [1] if and only if there exsists a pair of polynomial matrices, $\overline{N}(s)$ and $\overline{D}(s)$, of dimensions $p\times m$ and $m\times m$, such that

$$D(s)N(s)=\overline{N}(s)\overline{D}(s) \tag{1}$$

with $D(s)$ and $\overline{N}(s)$ relatively left prime and $N(s)$ and $\overline{D}(s)$ relatively right prime. In that cace, $\overline{D}(s)$ and $\overline{N}(s)$ are called internally skew prime [1]. The primary objective of this note is to present a new algorithm for constructing such an internal skew prime $\overline{D}(s)$ and $\overline{N}(s)$. It is now well known [2]-[4] that the problem considered is crucial for regulator synthesis of a linear multivariable system. The main result is developed in Section 2 and an example is also given to illustrate the result.

2. The Result

Assume that a $(p\times p)$ polynomial matrix D and a $(p\times m)$ polynomial matrix N are given with D nonsingular and N of full row rank.

Manuscript received May 19, 1986, revised Aug. 1, 1987.

Further assume that $\sigma(D)$ and $\sigma(N)$ are distinct, i.e. $\sigma(D)\cap\sigma(N)=\phi$. where $\sigma(N)$ denotes the set of all c such that $N(c)$ is of rank defect.

Lemma 1 If D and N satisfy assumptions described above, then D and N are externally skew prime.

Proof: This lemma follows from Lemma 6 in [3]. Its proof is strainghtforward and, is omitted here.

It is well known that a polynomial matrix is equivalent to its Smith form by elementary operations [5]. For D and N we have

$$D=L_1S_1R_1 \tag{2}$$

$$N=L_2S_2R_2 \tag{3}$$

$$DN=L_3S_3R_3 \tag{4}$$

where S_i, $i=1,2,3$, are in the Smith forms, and L_i and R_i, $i=1.2.3$, are unimodular. we now are ready to state and estabilish the following.

Theorem 1 If D is nonsingular, N of full row rank, and $\sigma(D)\cap\sigma(N)=\phi$, then D and N are skew-prime, and $\bar{D}$ and $\bar{N}$, defined by

$$\bar{D}=\text{Block diag}\{S_1,I\}R_3 \tag{5}$$

$$\bar{N}=L_3S_2 \tag{6}$$

are internally skew prime, where I is dentity matrix of order$(m-p)$.

In view of Lemma 1, we only need to prove that $\bar{D}$ and $\bar{N}$ are internally skew prime. In order to do this, we need the following first.

Lemma 2 If D is nonsingular, N of full row rank, and $\sigma(D)\cap\sigma(N)=\phi$, then the Smith form of DN is the product of those of D and N.

Proof: From (2) and(3), $DN=L_1(S_1R_1L_2S_2)R_2$, the Smith form of DN is the same as $S_1R_1L_2S_2$ because L_1 and R_2 are unimodular. It is therefore sufficient to prove that the Smith form of S_1MS_2 is S_1S_2, where M is unimodular. Let $Q=S_1MS_2$, and denote by d_{1k}, d_{2k}, and d_{3k} the determinatal divisors of order k of S_1, S_2 and Q, respectively. A determinantal divisor of order k of a polynomial matrix is the greatest common divisor of all its minors of order k. we know that the Smith form S of Q is

$$S=\left(\begin{array}{cccc:c} \lambda_{31} & & & & \\ & \lambda_{32} & & 0 & \\ & & \ddots & & 0 \\ 0 & & & \lambda_{3p} & \end{array}\right)$$

where $\lambda_{3i}=d_{3i}d_{3,i-1}^{-1}$, $i=1,2,\cdots,p$, and $d_{30}=1$. Similar statements can be applied to S_1 and S_2. Set $Q=S_1Q_1$, where $Q_1=MS_2$, applying the Binet-Cauchy formula [5], we can express a minor of Q of order k as a linear combination of minors of S_1 of the same order. It therefore follows that

$$d_{3k}d_{1k}^{-1}=p_{1k} \qquad k=1,2,\cdots,p, \tag{7}$$

where p_{1k} is polynomial. This also means that d_{1k} is a divisor of d_{3k}. In a similar way, by setting $Q=Q_2S_2$ where $Q_2=S_1M$, one can show

$$d_{3k}d_{2k}^{-1}=p_{2k}, \qquad k=1,2,\cdots,p \tag{8}$$

where p_{2k} is a polynomial. For a fixed k, partition M into $[M_k \vdots M_{p-k}]$, where M_k is the first k columns of M. It can be noted that $g_km_kd_{2k}$ is a minor of order k of Q, where m_k is any minor of order k of M_k and g_k is the product of the corresponding diagonal elements of S_1 to m_k. The greatest common divisor of all m_k must be a nonzero real constant, otherwise Laplance expansion of the determinant would show that M were not unimodular. Therefore the greatest common divisor of all $g_km_kd_{2k}$ can be expressed as $G_{1k}d_{2k}$ where G_{1k} is a polynomial and $\sigma(G_{1k})\subset\sigma(S_1)=\sigma(D)$. By the definition of d_{3k}, we have

$$G_{1k}d_{2k}d_{3k}^{-1}=\overline{p}_{2k}, \qquad k=1,2,\cdots,p \tag{9}$$

where $\overline{p}_{2k}$ is a polynomial, similarly one may show

$$G_{2k}d_{1k}d_{3k}^{-1}=\overline{p}_{1k}, \qquad k=1,2,\cdots,p \tag{10}$$

where G_{2k} and $\overline{p}_{1k}$ are polynomials and $\sigma(G_{2k})\subset\sigma(S_2)=\sigma(\mathrm{N})$. Farctorize d_{3k} into $d_{3k}=d_{3k}^{+}\ d_{3k}^{-}$ where $\sigma(d_{3k}^{+})\subset\sigma(D)$ and $\sigma(d_{3k}^{-})\subset\sigma(N)$, then we see that (7) and (10) are satisfied simultaneously if and only if both $d_{3k}^{+}\ d_{1k}^{-1}$ and $d_{1k}\ (d_{3k}^{+})^{-1}$ are polynomials. we thus have $d_{3k}^{+}=d_{1k}$. Similarly, we also have $d_{3k}^{-}=d_{2k}$. In view of these relations, we obtain

$$\begin{aligned}\lambda_{3k}&=d_{3k}(d_{3,k-1})^{-1}=d_{3k}^{+}(d_{3,k-1}^{+})^{-1}d_{3k}^{-}(d_{3,k-1}^{-})^{-1}\\ &=(d_{1k}d_{1,k-1}^{-1})(d_{2k}d_{2,k-1}^{-1})\\ &=\lambda_{1k}\lambda_{2k}\end{aligned}$$

The proof is thus completed.

Now we can prove Theorem 1.

From Lemma 2, $DN = L_3S_3R_3 = L_3S_1S_2R_3$ for some unimodular matrices L_3 and R_3. Set $\overline{D} = \text{Block diag}\{S_1, I\}R_3$, then $\det(\overline{D}) = a_1 \det(S_1) = a_2 \det(D)$ with a_1 and a_2 being nonzero real numbers, Since $DN\overline{D}^{-1} = L_3S_2 = \overline{N}$, a polynomial matrix, then $DN = \overline{N}\overline{D}$. Furthermore, D and $\overline{N}$ must be left coprime because otherwise there is a nonunimodular common divisor Q such that $\sigma(Q) \subset \sigma(D)$ and $\sigma(Q) \subset \sigma(\overline{N}) = \sigma(S_2) = = \sigma(N)$, which contradicts the fact that $\sigma(D) \cap \sigma(N) = \phi$. In a similar way, one can shows that $\overline{D}$ and N are right prime. The proof is thus completed.

Remark The above contruction of $\overline{D}$ and $\overline{N}$ is essentially different from those given by wolovich [1] and Chen and Pearson [3] who considered the problem of determining the solution X and Y of the equation

$$N(s)X(s) + Y(s)D(s) = I$$

for skew prime N and D.

We next consider an example[1]. Suppose that

$$D(s) = \begin{bmatrix} 1 & 0 \\ -1 & s+1 \end{bmatrix}, \quad N(s) = \begin{bmatrix} 1 & 1 \\ 0 & s \end{bmatrix}$$

$\sigma(D) \cap \sigma(N)$ is clearly empty and by Lemma 1, D and N are externallo skew prime. Furthermore, it follows from some simple calculations that

$$D(s) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & s+1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} := L_1S_1R_1$$

$$N(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} := L_2S_2R_2$$

and

$$D(s)N(s) = \begin{bmatrix} 1 & 0 \\ -1 & s+1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & s \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & s^2+s-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & s^2+s \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} := L_3S_3R_3$$

As shown in Lemma 2, we see that

$$S_3 = S_1S_2$$

By the Theorem, we obain

$$\overline{D}(s)=S_1R_3=\begin{pmatrix}1 & 0\\ 0 & s+1\end{pmatrix}\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}=\begin{pmatrix}1 & 1\\ 0 & s+1\end{pmatrix}$$

$$\overline{N}(s)=L_3S_2=\begin{pmatrix}1 & 0\\ -1 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & s\end{pmatrix}=\begin{pmatrix}1 & 0\\ -1 & s\end{pmatrix}.$$

which are internally skew prime.

3. Conclusion

A new algorithm has been presented for constructing an internally skew prime pair of polynomial matrices.

References

〔1〕 W.A.Wolovich, Skew prime polynomial matrices , IEEE Trans. Automat. Contr., Vol.AC-23, (1978), 880-887.

〔2〕 W.A.Wolovich and P.Ferreira, Output regulation and traching in linear multivariable system, IEEE Trans. Automat.Contr., Vol. AC-24, (1979), 460-465.

〔3〕 L.Chen and J.B.Pearson, Freguency domain synthesis of multivari- able linear regulators , IEEE Trans.Automat. Contr., Vol.AC-23, (1978), 3-15.

〔4〕 Q.G.Wang, Modelling and Control of Multivariable Systems:New Results and Applications.Ph.D.Thesis, Zhejiang University, (1987).

〔5〕 F.R.Gantmacher, Theory of Matrices, Vol.I, New York: Chelsea, (1959).

一个构造ISP多项式矩阵的新算法

王庆国 孙优贤 周春晖

（浙江大学化工系，杭州）

摘 要

本文给出了一个构造ISP（Internally Skew Prime）多项式矩阵的新算法，例子说明了算法的应用。