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The Identification of Parameters in Heterometric Autoregulation of Cardiac Contractility

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Abstract

A biocybernetics system identification technique was applied to study the dynamic process of the heterometric autoregulation in ventricular contraction. The process can be described by a second order differential equation. Using Gauss-Newton method the parameters a, b, c and d were obtained. The meaning of the parameters were discussed. Experiment indicates that parameter c determines the stability of the system and assesses the contractility of the ventricular myocardium.

1. Introduction

The regulation of cardiac stroke volume is one of the important problems in cardiac physiology. Sarnoff and Mitchell [1] coined the term heterometric autoregulation to describe the original Starling phenomenon. Some former papers discussed this phenomenon from static point of view. We are trying to probe into this problem by using the system identification to analyse the heterometric autoregulation from the dynamic point of view.

2. Computational Methoes

A definite amount of normal saline (3ml/100gm) is quickly injected into the left ventricle through the jugular vein. It increases pre-load and so does the initial sarcomere length, so that the end diastolic pressure (EDP) in left ventricle is increased immediately. The transient state of the heterometric autoregulation ensues [2]. From our experiments, the transient state can be described by the

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following differential equation:

$$\dot{X}_1 = -aX_1 - bX_2, \qquad X_1(0) = C_1 \tag{1}$$
$$\dot{X}_2 = cX_1 - dX_2, \qquad X_2(0) = C_2$$

where X_1 is the deviation of the aortic pressure (PAO), mmHg. after the saline injection. X_2 is the deviation of the end diastolic pressure (EDP), mmHg after the saline injection. The observing equations are given by

$$Z_1(t_i) = X_1(t_i) + \text{zeta } 1(t_i), \tag{2}$$
$$Z_2(t_i) = X_2(t_i) + \text{zeta } 2(t_i), \quad i = 1, 2\cdots, L,$$

where zeta $1(t_i)$ and zeta $2(t_i)$ represent respectively the PAO and EDP measurement noise.

The performance criterion for least squares estimation is given by

$$J(\overline{P}) = \overline{E}^T \times \overline{E} = \sum_{j=1}^{L} [\overline{E}_i(\overline{P})]^T \times [\overline{E}_i(\overline{P})], \tag{3}$$

where for equations (1) E_i is the two dimensional column vector

$$\overline{E}_i(\overline{P}) = \begin{vmatrix} E_1(t_i, \overline{P}) \\ E_2(t_i, \overline{P}) \end{vmatrix} = \begin{vmatrix} Z_1(t_i) - X_1(t_i, \overline{P}) \\ Z_2(t_i) - X_2(t_i, \overline{P}) \end{vmatrix}$$
$$= \overline{Z}(t_i) - \overline{X}(t_i, \overline{P}), \quad i = 1, 2, \cdots, L. \tag{4}$$

Here L is the number of observations, $\overline{Z}(t_i)$ is a vector of observation at time t_i, $\overline{X}(t_i, \overline{P})$ is a state vector, $\overline{P}$ is a vector of unknown parameters, and

$$\overline{E} = \begin{vmatrix} \overline{Z}(t_1) - \overline{X}(t_1, \overline{P}) \\ Z(t_2) - X(t_2, P) \\ \vdots \\ \overline{Z}(t_L) - \overline{X}(t_L, \overline{P}) \end{vmatrix}, \quad 2L \text{ vector}, \tag{5}$$

The Gauss-Newton method [3] is based on the Taylor series expansion

$$\overline{X}(\overline{P} + \Delta\overline{P}) \doteq \overline{X}(\overline{P}) + \frac{d\overline{X}}{d\overline{P}} \Delta\overline{P} = \overline{X}(\overline{P}) + X_p \Delta\overline{P}, \tag{6}$$

where X_p is a $(2L \times 4)$ parameter influence matrix. The iteration equation for the least-squares estimate is

$$\overline{P}_j + 1 = \overline{P}_j + \Delta\overline{P}_j, \tag{7}$$

where

$$\Delta \overline{P}_j = (X_p^T X_p)^{-1} X_p^T \overline{E}. \tag{8}$$

and

$$\overline{P}_j = |a \quad b \quad c \quad d|^T \tag{9}$$

is the estimate of the unknown parameters at iteration j. The method requires the solution of the eight auxiliary equations obtained by taking the partial derivatives of equation (1) with respect to the unknown parameters.

Defining

$$x_{1a} = \partial x_1/\partial a,$$
$$x_{2a} = \partial x_2/\partial a,$$
$$x_{1b} = \partial x_1/\partial b,$$
$$\vdots$$
$$x_{2d} = \partial x_2/\partial d.$$

the auxiliary equations can be conveniently expressed in the form

$$\dot{Y} = FY + U. \tag{10}$$

with initial condition $Y(0) = 0,$ (11)

Where $\dot{Y} = \begin{vmatrix} \dot{x}_{1a} & \dot{x}_{1b} & \dot{x}_{1c} & \dot{x}_{1d} \\ \dot{x}_{2a} & \dot{x}_{2b} & \dot{x}_{2c} & \dot{x}_{2d} \end{vmatrix}$, $F = \begin{vmatrix} -a & -b \\ c & -d \end{vmatrix}$,

$Y = \begin{vmatrix} x_{1a} & x_{1b} & x_{1c} & x_{1d} \\ x_{2a} & x_{2b} & x_{2c} & x_{2d} \end{vmatrix}$, $U = \begin{vmatrix} -x_1 & -x_2 & 0 & 0 \\ 0 & 0 & x_1 & -x_2 \end{vmatrix}$

The sequence for obtaining the least squares estimate is as follows: using an initial approximation for the unknown parameters, integrate equations (1) and (10) from $t=0$ to $t=T$. Compute a new set of parameters using equations (7) and (8) and repeat the cycle, starting with the integration of equations (1) and (10).

3. Physiological Experiments

The experiments were carried out in two groups of adult wistar rats. One group was fed with normal food, i. e. 1# Rat, the other was fed with special cake made by sweet potato mill only i. e. 2# Rat. They were anesthetized intraperitoneally with a mixture of 10% urethane and 1% chloralose (2.5ml/kg). Chest was opened and artificially ventilated. The ventilation rate was 80/min and tidal volume was 10–15ml. Ventricular pressure tracing was obtained from a catheter inserted into the left ventricle and connected to a TP–

200T pressure transducer; aortic pressure was recorded from a catheter passed into the ascending aorta from the carotid artery and connected to another TP-200T pressure transducer.

The resultant waveforms were recorded on RM-6000 Polygraph (NIHON KOHDEN, Japan), and then digitalized at a rate of 200 samples/sec by an A/D converter and stored in PDP-11/24 Minicomputer.

The measured deviations from the control level are given in Table 1, for 1# Rat and Table 2 for 2# Rat. The first sampling was made 2.5 second after the normal saline injection. In table 1, the aortic systolic pressure of 1# rat before the saline injection was 90.5 mmHg and the EDP was 4 mmHg. In table 2, the aortic systolic pressure of 2# rat before the normal saline injection was 70.0 mmHg and the EDP was 5 mmHg.

4. System Identification and Numerical Results

The system identification was carried out by the program in PDP-11 BASIC PLUS-2 language, for its MAT statement is very suitable for the matrix arithmetic [4].

To obtain the least-squares estimation of the unknown parameters, the algorithms described above were utilized with a fourth-order Runge-Kutta method [5] for the integration of the differential equations.

Using the Gauss-Newton method for a time interval of 25 sec, a convergent set of parameter estimates was obtained. The results were given in Table 3 and Table 4. The computed data coincided very well with the experimental ones.

5. Discussion

In equation (1), parameter $a = -[(dPAO/dt)/PAO]$ is an index of cardiac perfermance [2]. Parameter $b = -[(dPAO/dt)/EDP]$ represents Staring Law of heart [1]. Parameter c represents the dynamic changes of EDP with respect to PAO which could be used as a direct criterion for ventricular contractility. Parameter d represents the unit change of EDP with respect to the derivation of EDP which has little physiological significance.

The injected normal saline acts as a disturbance to the contrac-

tion of the left ventricle. After disturbance, the 1# Rat(control) tended to recover rapidly (see Table 1) while the 2# Rat(experiment) did not (see Table 2).

Table 1 The Measurement Data for Rat #1

TIMES (S)	PAO (mmHg)	EDP (mmHg)
0	10	6.4
2.5	1.45	5.5
5	−4.55	4.8
7.5	−9.0	4.3
10	−6.2	3.9
12.5	−4.5	3.3
15	−2.4	2.6
17.5	−1.2	1.9
20	0.0	1.3
22.5	0.3	0.6
25	0.59	0.2

Table 2 The Measurement Data for Rat #2

TIMES (S)	PAO (mmHg)	EDP (mmHg)
0	19	12
2.5	−3.8	8.5
5	−15.7	6
7.5	−24	4.5
10	−30	3.7
12.5	−36	3.4
15	−39.4	3.2
17.5	−43	3
20	−46	2.9
22.5	−48	2.8
25	−50.1	2.75

Table 3 Estimated Coefficient for Rat #1

$a=0.29$ $b=43$ $c=0.074$ $d=0.021$

Computed Value for Rat #1

TIMES (S)	PAO (mmHg)	EDP (mmHg)
0	10	6.4
2.5	1.4	6.1
5	−4.9	5.2
7.5	−7.5	4.5
10	−6.0	3.13
12.5	−4.8	2.98
15	−3.5	1.12
17.5	−2.33	1.54
20	0.04	0.58
22.5	0.36	0.15
25	0.39	0.09

Table 4 Estimated Coefficient for Rat 2#

$a=0.03$ $b=0.83$ $c=-0.01$ $d=0.15$

Computed Value for Rat #2

TIMES (S)	PAO (mmHg)	EDP (mmHg)
0	19	12
2.5	−3.17	8.14
5	−15.7	5.8
7.5	−24	4.45
10	−31	3.66
12.5	−35.7	3.22
15	−39.3	3.01
17.5	−42.4	2.9
20	−45.3	2.86
22.5	−47.9	2.9
25	−50.6	3

The courses of the regulation are determined by parameters a,

b, c, d. Among them parameter c is the most important one which reflects the relationship between blood pressure deviation (PAO) and d(EDP)/dt. Because the characteristic equation of (1) is

$$\det\begin{vmatrix} \lambda+a, & b \\ -c, & \lambda+d \end{vmatrix} = \lambda^2+(a+d)\lambda+(ad+bc)=0.$$

Then the necessary and sufficient condition for the system to be stable are $a+d>0$, $ad+bc>0$. When $a>0$, $b>0$, $d>0$, and $c>-ad/b$, the system is stable. That is, after disturbance, the blood pressure tends to recover. If the value of parameter c is less than $-ad/b$, the system is unstable. The blood pressure tends to drop down indicating the contractility of the myocardium is being impaired. The regulation time of the system depends on parameter a. The greater value of the parameter a is, the shorter the regulation time of the system and the better myocardial function would be. If the value of a is small, the system could be stable but the regulatory time would be long and the pressure oscillation is large suggesting that the contractile function of the myocardium be impaired.

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心肌收缩性异长自身调节的参数辨识

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摘 要

应用系统辨识方法，研究了心肌收缩性异长自身调节的动态过程。这种动态过程可由二阶微分方程描述，并用 Gauss-Newton 法求出方程的参数 a、b、c与d，探讨了各参数的的意义，指出参数c对于判断系统的稳定性有十分重要的意义，从而估计心肌的收缩功能。