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Application of Kalman Filter to Redundant Control Systems

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Abstract

This paper puts forward a principle of failure detection with Kalman filters as main elements in digital redundant control systems. It also presents and proves a method of failure location by the comparison of output error variances. The method was tested and verified by the satisfactory results of digital simulations, and it is of great importance in the improvement of the reliability of the system.

1. Introduction

As well known, when the number of channels in a digital control systems is two, the existing failure can only be discovered by comparison, but cannot be located. So some compensation means must be introduced. Analytical redundancy with kalman filters as main elements, which is discussed here, is just an effective method.

2. Principle of Failure Detection of Analytical Redundancy

Assume the redundant system has two channels and each channel has two kinds of signals, then a failure detection method is shown in Fig.1, where S_{12} is the second kind signal in the first channel; C is the comparator whose output is"1"when the difference of its input signals is greater than the threshold, otherwise, the output is "0" ; DF is a diagnostic filter, L_1 and L_2 are the state or output error variances of two channels respectively.

It can be seen from Fig.1 that this structure can be used for locating the failure. Let A_1, A_2, A_3, A_4 be the failure events of S_{11}, S_{21}, S_{12},

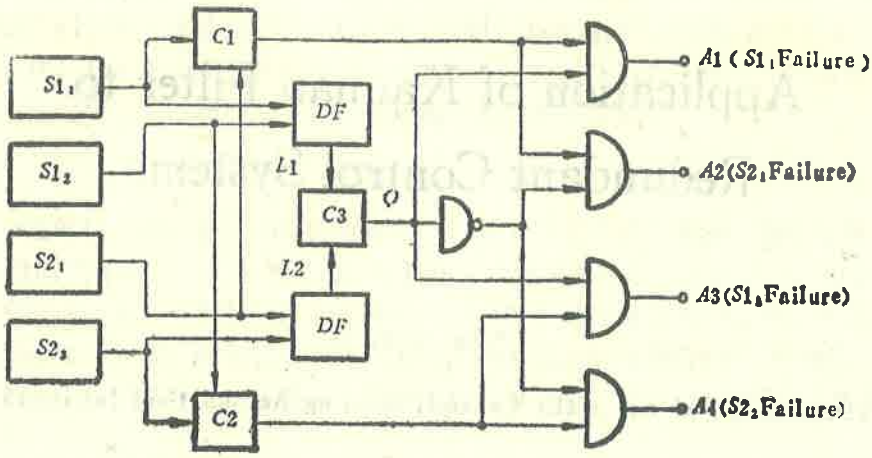

Fig. 1 $Q=$ "1" when $L_1>L_2$

S_{22} respectively. Their logical representations can be written as follows:

$$A_1=C_1\cdot Q=(|S_{11}-S_{21}|>\varepsilon)\,\mathrm{AND}(L_1>L_2) \tag{1}$$

Similarly we have $A_2=C_1\cdot\overline{Q},\ A_3=C_2\cdot Q,\ A=C_2\cdot\overline{Q}.$

Evidently DF is a key part for this method. Here we discuss the analytical redundancy with kalman filters as main elements because it will be seen that Kalman filter can deal with not only noisy problems bnt also deviation problems.

3. Failure Detection Princtple of Dual Kalman Filters

Let the system model be

$$x_{n+1}=F_nx_n+G_nw_n+P_nu_n \tag{2}$$

$$y_n=H_n^Tx_n+v_n \tag{3}$$

where $\{u_n\}$ is a known input sequence, $\{w_n\}$ and $\{v_n\}$ are white noises with zero means and covariances Q_n、R_n respectively, x_0 is a Gaussian random variable with mean $\overline{x_0}$ and covariance P_0. And x_0, w_n, v_n are independent from one another. $x_n\in R^n$, $u_n\in R^m$, $y_n\in R^r$, The Kalman filter algorithms are as follows,

$$\hat{x}_{n+1/n}=F_n\hat{x}_{n/n-1}+P_nu_n+K_n(y_n-H_n^T\hat{x}_{n/n-1}) \tag{4}$$

$$\Sigma_{n+1/n}=(F_n-K_nH_n^T)\Sigma_{n/n-1}(F_n-K_nH_n^T)^T+G_nQ_nG_n^T+K_nR_nK_n^T \tag{5}$$

$$K_n=F_n\Sigma_{n/n-1}H_n(H_n^T\Sigma_{n/n-1}H_n+R_n)^{-1} \tag{6}$$

with initial conditions $\hat{x}_{0/-1}=\bar{x}_0$, $\Sigma_{0/-1}=P_0$, where $\hat{x}_{n+1/n}$ is the predicted estimate of contaminated state vector, $\Sigma_{n+1/n}$ is the estimation error covariance, K_n is the Kalman filter gain.

Suppose $\{y_{1n}\}$, the measurements of first channel, are contaminated by noise more seriously than $\{y_{2n}\}$, the measurements of the second channel, that is, $R_{1n}\geqslant R_{2n}$ for every n, then their corresponding covariances are respectively

$$\Sigma^1_{n+1/n}=(F_n-K_{1n}H_n^T)\Sigma^1_{n/n-1}(F_n-K_{1n}H_n^T)^T+G_nQ_nG_n^T+K_{1n}R_{1n}K_{1n}^T \tag{7}$$

$$\Sigma^2_{n+1/n}=(F_n-K_{2n}H_n^T)\Sigma^2_{n/n-1}(F_n-K_{2n}H_n^T)^T+G_nQ_nG_n^T+K_{2n}R_{2n}K_{2n}^T \tag{8}$$

According to the inductive method, we can prove (the details can be seen in appendix)

$$\Sigma^1_{n+1/n}\geqslant\Sigma^2_{n+1/n} \qquad \forall n \tag{9}$$

It can be seen from the above that based on the error covariances, we can judge which channel is more seriously contaminated by noise, and therefore locate the failure.

If failure is deviation problem, or if failure includes both noise and deviation, we can draw the same conclusions. Consequently no matter what type a failure is, it can be located based on the above scheme of analytical redundancy.

But unfortunately it is difficult to calculate the equations of Kalman filter in real time. However, because the state estimate is only used for failure detection, we here propose a program of analytical redundancy with a constant gain (suboptimal) Kalman filter. The existance conditions of the suboptimal Kalman filter could be seen in [1]. Apparently there is no problem about the real time computation of the suboptimal Kalman filter with a constant gain. And we can also prove the above conclusions if K_n is substituted by a constant K.

However when a constant gain Kalman filter is used (gain K is calculated off—line), $\Sigma_{n+1/n}$ is never calculated and never known, so it can not be used for failure detection. In a practical system, what we know are only the measurements of outputs and their estimates (or the estimates of the states). We can also prove that the failure could be located based on the output error variance as follows,

Let

$$\hat{y}_{n/n-1}=H_n^T\hat{x}_{n/n-1} \tag{10}$$

be the estimated value of outputs, then

$$y_n - \hat{y}_{n/n-1} = H_n^T(x_n - \hat{x}_{n/n-1}) + v_n + \xi \tag{11}$$

hence the output error covariance (i.e. the error covariance between the measurements and estimates of the outputs) is

$$\Sigma^y_{n/n-1} = H_n^T \Sigma_{n/n-1} H_n + R_n + \xi \cdot \xi^T \tag{12}$$

If $R_{1n} \geqslant R_{2n}$ and/or $\xi_1\xi_1^T \geqslant \xi_2\xi_2^T$ for two channels, then their corresponding state error covariances satisfy

$$\Sigma^1_{n/n-1} \geqslant \Sigma^2_{n/n-1} \quad \forall n$$

Upon above we can easily prove that their corresponding output error covariances satisfy

$$\Sigma^{y_1}_{n/n-1} \geqslant \Sigma^{y_2}_{n/n-1} \qquad \forall n \tag{13}$$

hence

$$\mathrm{tr}(\Sigma^{y_1}_{n/n-1}) \geqslant \mathrm{tr}(\Sigma^{y_2}_{n/n-1}) \qquad \forall n \tag{14}$$

where tr(A) is the trace of matrix A, it is just the output error variance.

Up to now, we have proved the principle of failure detection of analytical redundancy with constant gain Kalman filters. It is worthwile to note that the above program may bring about notable errors to theestimates of the states, but it cannot influence the correctness of the failure detection, it also can be shown that the above conclusions could be extended to the continuous systems.

4. Digital Simulation of Analytical Redundancy for Sensors of An Airplane

In a redundant digital control system of an airplane, given the pitch short—period dynamic equation of the airplane(for Mach number $M = 0.8$, height $H = 0$),

$$\dot{x} = Ax + B\delta + \omega \tag{15}$$

$$y = Cx + v \tag{16}$$

where

$$A = \begin{pmatrix} -2.670444 & 15.032745 \\ 1 & -2.603823 \end{pmatrix} \qquad x = \begin{pmatrix} \dot{\theta} \\ \alpha \end{pmatrix}$$

$$B = \begin{pmatrix} -47.61327 \\ -0.26095 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

α (angle of attack), $\dot{\theta}$ (pitch rate) are the states of the system, and they can ba measured, so they can be regarded as outputs. And given the controller $\delta = f(x)$, whose details can be seen in[2]. In addition w, v are white noises, $E(w) = E(v) = 0$, $Q = \mathrm{cov}\,(w,\ w) = \begin{pmatrix} 0.001 & 0 \\ 0 & 0.001 \end{pmatrix}$

$R = \mathrm{cov}\,(v,\ v) = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.075 \end{pmatrix}$[2]

and x_0 is a Gaussian random variable $N(0,1)$, x_0, w, v are independent from one another.

When a Kalman filter is realized in a microcomputer, the state equations are calculated with the corresponding discrete-time equations, that is,

$$x_n = F x_{n-1} + G\delta_n + w_n \tag{17}$$

$$y_n = x_n + v_n \tag{18}$$

where $F = e^{AT}$, $G = \int_0^T e^{AT}\, B dt$, T is the sampling period.

Evidently, based on the above the Kalman filter will tend to be constant when $t \to \infty$. Therefore we can find the constant gain K in advance for the computation of the Kalman filter. Above analytical redundancy was simulated in a computer FACOM 230. The following is a table of the steady values of the state error variances and output error variances under some failure conditions.

Failure condition	Σ_x	Σ_y
$\hat{\dot{\theta}} = \dot{\theta} - 0.06\,\dot{\theta}$	1.1386	0.2021
$\hat{\dot{\theta}} = \dot{\theta} - 0.02\,\dot{\theta}$	0.2013	0.0508
$\hat{\alpha} = \alpha + 0.04\,\alpha$	0.1024	0.1477
$\hat{\alpha} = \alpha + 0.05\,\alpha$	0.1855	0.2467
$\hat{\alpha} = \alpha + 0.4$	0.0545	0.4338
$\hat{\alpha} = \alpha + 0.2$	0.0312	0.0886
No failure	0.0212	0.0212

where $\hat{\dot{\theta}}$ is the measurement of $\dot{\theta}$, $\hat{\alpha}$ is the measurement of α; Σ_x is the state error variance; Σ_y is the output error variance.

According to the above table, we can know that the more serious the failure, the greater the corresponding Σ_x or Σ_y. Hence the failure can be located based on the comparison of Σ_y of the two channels. This just verifies what we proposed.

5. Concluding Remarks

The method of analytical redundancy proposed here is both simple and practical. The use of suboptimal Kalman filter does not influence the correctness of the failure detection.

In addition, a real airplane model is time—variant. In order to express the change of the parameters of the airplane, we usually adopt the method of parameter—adjustment, that is, K varies with the change of the airplane state, from which the greater estimate error may arise, but the correctness of the failure detection cannot be influenced.

I would like to give my hearty thanks to professor Guo Suo—Feng, under whose supervision this work was done.

Appendix: Proof of the Failure Detection Principle of Dual Kalman Filters

Let the equations of a Kalman filter be

$$\hat{x}_{n+1/n} = (F_n - K_n H_n^T)\ \hat{x}_{n/n-1} + K_n y_n + P_n u_n$$

$$\Sigma_{n+1/n} = F_n[\Sigma_{n/n-1} - \Sigma_{n/n-1} H_n (H_n^T \Sigma_{n/n-1} H_n + R_n)^{-1} H_n^T \Sigma_{n/n-1}] F_n^T + G_n Q_n G_n^T$$

$$K_n = F_n \Sigma_{n/n-1} H_n (H_n^T \Sigma_{n/n-1} H_n + R_n)^{-1}$$

assume for two channels, $R_{1n} \geqslant R_{2n}$ $\forall n$, other conditions are the same. We prove that their corresponding state error covariances satisfy the following by the inductive method,

$$\Sigma^1_{n+1/n} \geqslant \Sigma^2_{n+1/n} \qquad \forall n$$

Evidently when $n=0$, $\Sigma^1_{0/-1} = \Sigma^2_{0/-1} = P_0$, $\Sigma^1_{0/-1} \geqslant \Sigma^2_{0/-1}$ holds, suppose

when $n=k$, we have $\Sigma^1_{k/k-1} \geqslant \Sigma^2_{k/k-1}$, then when $n=k+1$,

$$\begin{aligned}
\Sigma^1_{k+1/k} - \Sigma^2_{k+1/k} &= F_k[\Sigma^1_{k/k-1} - \Sigma^1_{k/k-1} H_k (H_k^T \Sigma^1_{k/k-1} H_k + R_{1k})^{-1} H_k^T \Sigma^1_{k/k-1}] F_k^T \\
&\quad - F_k[\Sigma^2_{k/k-1} - \Sigma^2_{k/k-1} H_k (H_k^T \Sigma^2_{k/k-1} H_k + R_{2k})^{-1} H_k^T \Sigma^2_{k/k-1}] F_k^T \\
&= F_k\{[(\Sigma^1_{k/k-1})^{-1} + H_k R_{1k}^{-1} H_k^T]^{-1} - [(\Sigma^2_{k/k-1})^{-1} + H_k R_{2k}^{-1} H_k^T]^{-1}\} F_k^T
\end{aligned}$$

$\because \quad \Sigma^{1}_{k/k-1} - \Sigma^{2}_{k/k-1} \geqslant 0, \; R_{1k} - R_{2k} \geqslant 0,$

$$\therefore \quad (\Sigma^{1}_{k/k-1})^{-1} - (\Sigma^{2}_{k/k-1})^{-1} \leqslant 0, \quad R^{-1}_{1k} - R^{-1}_{2k} \leqslant 0$$

$$(\Sigma^{1}_{k/k-1})^{-1} + H_k R^{-1}_{1k} H^T_k \leqslant (\Sigma^{2}_{k/k-1})^{-1} + H_k R^{-1}_{2k} H^T_k$$

Hence $\Sigma^{1}_{k+1/k} - \Sigma^{2}_{k+1/k} \geqslant 0$, i.e. $\Sigma^{1}_{k+1/k} \geqslant \Sigma^{2}_{k+1/k}$

we know the conclusion holds based on the inductive method, that is,

$$\Sigma^{1}_{n+1/n} \geqslant \Sigma^{2}_{n+1/n} \qquad \forall n$$

where the following equation is used,

$$(\Sigma^{-1} + HR^{-1}H^T)^{-1} = \Sigma \cdot \Sigma H (H^T \Sigma H + R)^{-1} H^T \Sigma$$

References

〔1〕 Anderson, B.D.O., J.B. Moore, Optimal Filtering, Prence—Hall, Inc., Englewood Cliffs, New Jersey, (1979), 77—82.

〔2〕 Feng Gang, Digital Redundant Flight Control System Study, Thesis of Master, Nanjing Aeronautical Institute, (1984), 69—77.

卡尔曼滤波器在余度控制系统中的应用

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摘 要

本文提出了一种余度控制系统中的以卡尔曼滤波器为主体的故障检测原理，提出并证明了一种根据输出误差方差定位故障的方法，并进行了数字仿真，取得了满意的结果。该方法对于提高系统可靠性具有重要意义。