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Self–tuning Robust Controller with Generalised Output Error Feedforward

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Abstract: The paper presents a new implicit self–tuning robust controller which is constructed via introducing a generalised output errer feedforward control (GOEFC) signal to an optimal pole / zero–placement self–tuning controller. The design criterion of the GOEFC and the simulations for the new adaptive control algorithm are given.

Key words: Plant uncertainties; self–tuning robust control; feedforward control; pole–placement

1. Introduction

Recently peopole have paid attention to the robust problems of the STC and gaven many research results [1, 2, 3]. Especially, Goodwin et.al. put forward the concept of the adaptive robust control [4].

When some factors make the STC systems unstable, the deviation between the expected output and the practical one, the generalised output error (GOE), will unceasingly increase in the absolutely. If GOE is used for the feedforwand control to ensure GOE to be decreased, the STC systems will asymptotically tend to a stable state. Based on the idea, we design a new implicit self–tuning robust controller (STRC). The analysis and simulations show the GOEFC can ensure the persistent excitation and equivalent disturbance acting on the process.

2. Process Model and Control Objective

Consider a liner SISO system

$$A(q^{-1})y(t)=q^{-d}B(q^{-1})u(t)+\sum(t), \tag{1}$$

where, $A(q^{-1})=1+a_1q^{-1}+\cdots+a_{n_a}q^{-n_a}$, $B(q^{-1})=b_0+b_1q^{-1}+\cdots+b_{n_b}q^{n_b}$, $b_0\neq 0$. $\{u(t)\}$ *and* $\{y(t)\}$ are the input and output. q^{-d} represents a time dely. $\sum(t)$ denots the equivalent disturbance of the process and we make the following Assumption:

A 2.1 The stachastic disturbances of $\{\sum(t)\}$ will be taken as a real stochastic process defined on a probability space (Ω, F, P) and the following is satisfied

$$\lim_{N\to\infty}\sup\frac{1}{N}\sum_{t=1}^{N}\sum(t)^2<\infty \qquad \text{a.s} \tag{2}$$

Consider the cost function

$$I(t+d)=E\left\{\left[P(q^{-1})y(t+d)-R(q^{-1})y_r(t)\right]^2+[Q'(q^{-1})u(t)]^2\right\}, \tag{3}$$

where $P(q^{-1})$ and $R(q^{-1})$ are weighting polynomial. $\{y_r(t)\}$ is the system input sequence and is bounded. The control objective is to achive

$$\lim_{N\to\infty}\sup\frac{1}{N}\sum_{t=1}^{N}y(t)^2<\infty \qquad \text{a.s.} \tag{4}$$

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$$\lim_{N\to\infty} \sup \frac{1}{N}\sum_{t=1}^{N} u(t)^2 < \infty \quad \text{a.s.} \tag{5}$$

and to make (3) minimum, Moreover, by tuning $P(q^{-1})$, $Q'(q^{-1})$ and $R(q^{-1})$ on – line to make the closing – loop system have the following transfer relation

$$y(t) = \frac{B_m(q^{-1})}{T(q^{-1})} y_r(t-d) + \frac{H(q^{-1})}{T(q^{-1})}\sum(t), \tag{6}$$

where $T(q^{-1})$ and $B_m(q^{-1})$ denote the expected pole and zero polynomials of the closing – loop system and satisfy (1) $B_m(q^{-1})$ and $T(q^{-1})$ are coprime (2) $B_m(1)/T(1)=1$.

3. Basic Self – tuning Controller

The STC algorithm which satisfies the above control objective is in the folloing

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{P(t-2)X(t-d)}{\lambda + X(t-d)^T P(t-2)X(t-d)}[\Phi(t) - X(t-d)^T\hat{\theta}(t-1)], \tag{7}$$

$$P(t-1) = \left[P(t-2) - \frac{P(t-2)X(t-d)X(t-d)^T P(t-2)}{\lambda + X(t-d)^T P(t-2)X(t-d)}\right]/\lambda, \tag{8}$$

$$\Phi(t) = P(q^{-1})y(t) = X(t-d)^T\theta + e(t),$$

$$X(t) = [y(t)y(t-1)\cdots y(t-n_g);\ u(t)\ \ u(t-1)\cdots u(t-n_{h'})],$$

$$\theta = \left[g_0 g_1 \cdots g_{n_g};\ h_0' h_1' \cdots h'_{n_{h'}}\right]^T,$$

$$H(q^{-1}) = H'(q^{-1}) + Q(q^{-1}), \tag{9}$$

where, $Q(q^{-1}) = q_0' Q'(q^{-1})/b_0$, $H'(q^{-1})B(q^{-1})F(q^{-1}) = h_0' + h_1' q^{-1} + \cdots + h'_{n_{h'}} q^{-n_{h'}}$,

$G(q^{-1}) = g_0 + g_1 q^{-1} + \cdots + g_{n_g} q^{-n_g}$. $G(q^{-1})$ and $F(q^{-1})$ satisfy $P(q^{-1}) = A(q^{-1})F(q^{-1}) + q^{-d}G(q^{-1})$. $e(t) = F(q^{-1})\sum(t)$ is prediction error. Then we can obtain the following STC equation

$$\hat{G}(q^{-1})y(t) + \hat{H}(q^{-1})u(t) = \hat{R}(q^{-1})y_r(t). \tag{10}$$

The pole / zero – placement equations are [5]

$$P(q^{-1})\hat{H}(q^{-1}) - q^{-d}Q(q^{-1})\hat{G}(q^{-1}) = \hat{F}(q^{-1})T(q^{-1}), \tag{11}$$

$$B(q^{-1})\hat{F}(q^{-1}) = \hat{H}'(q^{-1}), \tag{12}$$

$$R(q^{-1})\hat{B}(q^{-1}) = B_m(q^{-1}). \tag{13}$$

To sum up, the equivalent structure of the STC with the optimal pole / zero placement is shown in Fig.1.

Define $r(t-1) = r(t-2) + X(t-d)^T X(t-d)$. Remember $\Omega_1 = \{\omega : r(t) \to \infty|_{t\to\infty} \text{a.s.}\}$, $\Omega_1 \subset \Omega$. So we have the following robust. stability result.

Proposition 3.1 Consider the process (1), subject to assumption A 2.1, Provided the equations (11) – (13) is soluble for $\forall t \geqslant 0$, and

1) $\lim\limits_{N\to\infty}\sup\frac{\lambda_{\max}P(N)}{\lambda_{min}P(N)}<\infty$ a.s.

2) $\sum(t)^2 = O[\|X(t-d)\|^2]$ a.s. $\forall\omega\in\Omega_1$,

then the STC system advanced here has the robust stability in the following sense:

1° $\lim\limits_{N\to\infty}\sup\frac{1}{N}\sum\limits_{t=1}^{N}y(t)^2<\infty$ a.s. $\forall\omega\in\Omega_1$,

2° $\lim\limits_{N\to\infty}\sup\frac{1}{N}\sum\limits_{t=1}^{N}u(t)^2<\infty$ a.s. $\forall\omega\in\Omega_1$,

3° $\lim\limits_{N\to\infty}\sup\frac{1}{N}\sum\limits_{t=1}^{N}\varepsilon_g(t)^2<\infty$ a.s. $\forall\omega\in\Omega_1$,

4. Self-tuning Robust Controller

The proposition 3.1 shows, if the conditions 1) and 2) are satisfied, $\{u(t)\}$, $\{y(t)\}$ and $\{\varepsilon_y(t)\}$ are mean squarly bounded. Analysing the system in the Fig.1, we can know if $\varepsilon_y(t)$ is used for the feedforward control and combinde with STC, the conditions (1) and (2) can be ensure. Based on this idea, we structure a new STRC using GOEFC and it is shown in Fig.2. Where $\{\varepsilon_y(t)\}$ denotes the GOE, that is $\varepsilon_y(t)\triangleq y_m(t)-y(t)$, and $y_m(t)$ is the reference model output. In Fig.2, the STC equation is still (10), but now $u(t)$ is substituted by $u_1(t)$, that is

$$u_1(t)=\frac{\hat{R}(q^{-1})}{\hat{H}(q^{-1})}y_r(t)-\frac{\hat{G}(q^{-1})}{\hat{H}(q^{-1})}y(t), \tag{14}$$

$\{w(t)\}$ is the output of GOEFC. Let GOEFC equation be

$$M(q^{-1})w(t)=N(q^{-1})\varepsilon_y(t), \tag{15}$$

where $N(q^{-1})$ and $M(q^{-1})$ are coprime and $M(q^{-1})$ is a Hurwitz polynomial.

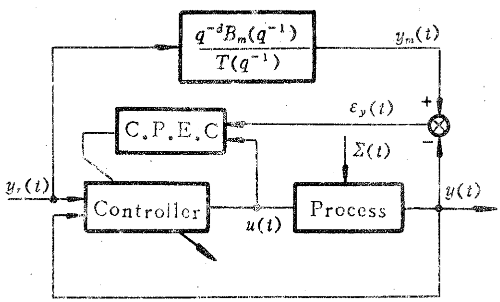

Fig.1 The equivalent structure of STC

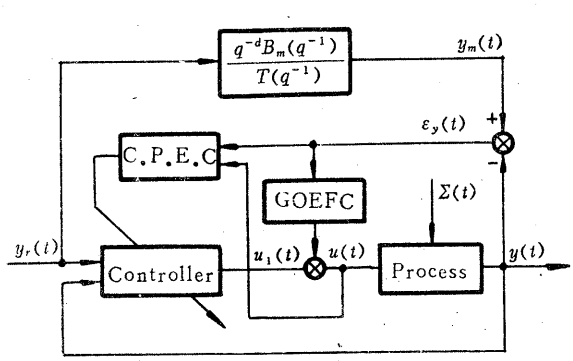

Fig.2 The STRC with GOEFC

In order to guarantee the asymptotical stability of the system, $\frac{N(q^{-1})}{M(q^{-1})}$ must be strictly positive real, so that the GOEFC sigral always ensures $\{u(t)\}$ changes in the direction of making $\{\varepsilon_y(t)\}$ absolutly decrease.

From Fig.2 we can know $\{u(t)\}$ includes $\{\varepsilon_y(t)\}$, that means, the effect of the equivalent dis turbance acting on the process is direct refleted in the $\{u(t)\}$. Therefore the persistent excitation can be strengthened. This shows the STRC with GOEFC has the robust stability in the structure,

5. Simulation Studies

Consider the process

$$(1-0.7q^{-1})\left(1-a_2q^{-1}\right)y(t)=q^{-2}(1-1.2q^{-1})u(t)+(1-0.5q^{-1})\omega(t)+\zeta(t),$$

where, $\{\omega(t)\}$ is a white noise sequence, $\zeta(t)$ is a additional equivalent disturbance, and $\zeta(t)\in\{0,\ y_r(t)\times 5\%\}$, $a_2\in[0,\ 0.5]$. To test the performance of the STRC with GOEFC on a system having unmodelled dynamics, a one order model will be assumped. Let $\frac{q^{-d}B_m(q^{-1})}{T(q^{-1})}=\frac{0.5q^{-2}}{1-0.5q^{-1}}$, and design the controller according to $n_a=n_b=1$, $d=2$. The equation of GOEFC is $(1-0.5q^{-1})w(t)=2\varepsilon_y(t)$.When order n_a, paremeter a_2 and $\zeta(t)$ change, the simula tion results are shown in the Fig.3.

6. Conclusions

The STRC with GOEFC has the robust stability in the structure. Under the action of the bounded equivalent disturbance, the STRC ensures the system to have the asymptotically tracking and requlating properties. The adaptive control algorithm assigns the pole and zero of the system to expected positions,

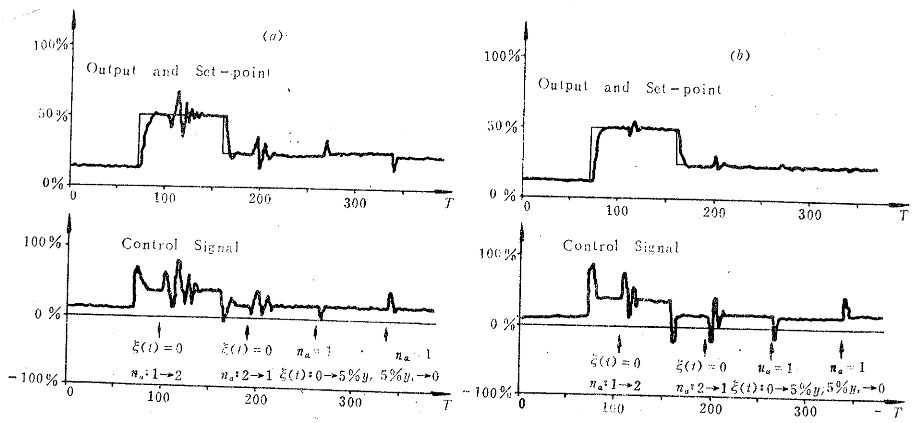

Fig. 3 Optimal pole / zero–placement STC without (a) and with (b) GOEFC

simultaneously, makes the cost function(3) and GOE minimum. The analysis and simulations show that the STRC with GOEFC can be applied to many real–time process controls.

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广义输出误差前馈自校正鲁棒控制器

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摘要 本文提出了一种新型的隐式自校正鲁棒控制器，即在最优零极点配置自校正控制器的基础上，引入了一个广义输出误差前馈控制信号．文中给出了广义输出误差前馈控制器的设计准则及仿真研究结果．

关键词： 对象不确定性；自校正鲁棒控制；前馈控制；极点配置