# A Variable Structure Path Tracking Control Strategy for Robotics Applications

Song Yongduan, Gao Weibing

(The Seventh Research Division, Beijing University of Aeronautics and Astronautcs)

Abstract: This paper studies the path tracking control problem of a robot manipulator subject to parameter uncertainty and external disturbance. By using the theory of VSS, and taking advantage of the important property of the robot dynamics, a new control strategy is proposed in which both parameter uncertainty and external disturbance can be easily taken into account. It is shown that with this control strategy, the accurate path tracking is guaranteed. Moreover, the on—line parameter estimation is not required, which makes the strategy easy to implement.

Key words: path tracking; variable structure control; robot; uncertainty

#### 1 Introducation

It is well known that path tracking is one of the important issues in the context of robot control and much work has been done in this area during the past decade. However, the effectiveness of these methods is contingent on the assumptions that the parameters of the robot controlled are completely known and there is no external disturbance during operation.

In order to solve the path tracking problem when the above assumptions do not hold, as usually the case in practice<sup>[1]</sup>, many advanced control schemes such as learning control <sup>[2]</sup> and adaptive control<sup>[3-5]</sup> are developed. However, there is a need for repeating trial operation in learning control scheme. As for adaptive control, the complicated on — line parameter estimation is required, which generally results in an expensive control system.

In the present paper we intend to study a simple but effective control strategy for a robot manipulator in the face of parameter uncertainty and external disturbance. Using the theory of variable structure systems and the important properties concerning the robot dynamics, a new control strategy is proposed in which we can easily deal with the parameter uncertainty and external disturbance. It is shown that with this control strategy, the accurate path tracking is guaranteed and the on—line parameters estimation is not needed, which makes it easy to implement.

### 2 Some Properties of Robot Dynamics

The nonlinear dynamics of a robot manipulator in the face of unceratin parameters and disturbances is represented by

$$D(q, p_D)\ddot{q} + C(q, \dot{q}, p_C)\dot{q} + G(q, P_G) + f_i = \gamma.$$
 (1)

where  $q:n\times 1$  vector, shaft angular displacements;  $\gamma:n\times 1$  control torque vector;  $p_i$ ; equivalent pa-

rameter matrix (vector) with appropriate dimension,  $i = \{D, C, G\}$ ;  $f_d: n \times 1$  external disturbance vector (unknown but bounded);  $D(q, p_D): n \times n$  symmetric positive definite inertia matrix;  $C(q, q, p_D): n \times n$  matrix reflects Coriolis and centrifugal forces;  $G(q, p_D): n \times 1$  gravitational forces.

**Property** 1 For all q and  $p_D$ , the matrix  $D(q, p_D)$  is always symmetric positive definite, i. e.,

$$\forall q \in \mathbb{R}^*, p_D \in \mathbb{R}^1, \exists D(q, p_D)^T = D(q, p_D) \text{ and } \forall z \in \mathbb{R}^* - \{0\}, \exists z^T D(q, p_D) z > 0.$$

**Property** 2 For all q,  $p_D$  and  $p_C$ , the inertia matrix  $D(q, p_D)$  and matrix  $C(q, q, p_C)$  have the following relation

$$\frac{d}{dt} [D(q, p_D)] - 2[C(q, \dot{q}, p_C)] = J_A.$$

where  $J_A$  is a skew—symmetric matrix.

**Property** 3 For the robot dynamic structure given in (1), there exists a vector  $peR^n$ , such that

$$D(q,p_D)\dot{r} + C(q,\dot{q},p_C)r + G(q,p_G) = \varphi(q,\dot{q},r,\dot{r})p.$$

where  $\varphi(\cdot) \varepsilon R^{n \times s}$ , a known matrix, p is the proper combination of  $p_D$ ,  $p_C$  and  $p_G$ ,  $p_E R^n$ .

Remark It is interesting to note that if the three properties are used properly, powerful controllers can be obtained.

In this paper, we assume that the exact value of the equivalent parameter vector  $p_i$  is unknown, the only information about it is:  $p_i = p_{i0} + \delta p_i$ , and  $\delta p_i \in [p_i^-, p_i^+]$ , where  $p_i^+, p_i^-$  are known bounds,  $p_{i0}$  is the normal parameter vector,  $i = \{D, C, G\}$ .

#### 3 Path Tracking Controller Design

It is our objective to design a control torque such that, for any given desired path  $q^* \in \mathbb{R}^n$ ,  $q^* \in \mathbb{R}^n$ ,  $q^* \in \mathbb{R}^n$ ,  $q^* \in \mathbb{R}^n$ ,

$$\lim_{i \to \infty} (q_i - q_i^*) = 0$$
 and  $\lim_{i \to \infty} (\dot{q}_i - \dot{q}_i^*) = 0$ ,  $i = 1, 2, \dots, n$ . (2)

Motivated by the work of Yeung and Chen [6] [7], we use the theory of variable structure systems (for details, see[8][9]) to solve this problem. As a first step, we choose the sliding plane as

$$s = (\dot{q} - \dot{q}^*) + \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_n \end{bmatrix} (q - q^*)$$
(3a)

i. e.,

$$s_i = \dot{e}i + \sigma_i e_i, \quad i = 1, 2, \cdots, n. \tag{3b}$$

where  $\sigma_i > 0$ ,  $i = 1, 2, \dots, n$ , design parameters determining the rate of response of the system.  $e = q - q^*$  is the tracking error. Define

$$\delta D(q, \delta p_D) = D(q, p_D) - D(q, p_{D0}), 
\delta C(q, \dot{q}, \delta p_C) = C(q, \dot{q}, p_C) - C(q, \dot{q}, p_{C0}), 
\delta G(q, \delta p_G) = G(q, p_G) - G(q, p_{G0}).$$
(4)

For simplicity, we sometimes supress the variables  $q, \dot{q}$ , and  $p_i$  in  $D(\cdot), \delta D(\cdot)$  etc. hereafter, whenever in so doing no confusion is arised.

## 3.1 Controller Based on the Bounds of $\delta_D$ , $\delta_C$ , and $\delta_C$

Assume that the bounds of  $\delta_D, \delta_C, \delta_G$ , and  $f_d$  are given respectively as

$$d_{ij}^{-} \leq (\delta D)_{ij} \leq d_{ij}^{+},$$

$$c_{ij}^{-} \leq (\delta C)_{ij} \leq c_{ij}^{+},$$

$$g_{i}^{-} \leq (\delta G)_{i} \leq g_{i}^{+},$$

$$f_{ii}^{-} \leq f_{ii} \leq f_{ii}^{+},$$

$$i, j = 1, 2, \dots, n.$$
(5)

Theorem 1 For a robot manipulator with uncertain parameters and external disturbance as assumed above, the asymptotic path tracking is ensured if the following control torque is implemented,

$$y = D(q, p_{00})\dot{\beta} + C(q, \dot{q}, p_{00})\beta + G(q, p_{00}) - R(\dot{e} + \sigma e) + u_{\text{aux}}. \tag{6a}$$

$$u_{\text{auxi}} = \begin{cases} \sum_{j=1}^{n} \Gamma_{1ij} - \dot{\beta}_{j} + \sum_{j=1}^{n} \Gamma_{2ij} - \beta_{j} + g_{i}^{-} + \gamma_{di}^{-}, & \text{if } s_{i} > 0, \\ \sum_{j=1}^{n} \Gamma_{1ij} + \dot{\beta}_{j} + \sum_{j=1}^{n} \Gamma_{2ij} + \beta_{j} + g_{i}^{+} + \gamma_{di}^{+}, & \text{if } s_{i} < 0. \end{cases}$$
(6b)

where  $R \in \mathbb{R}^{n \times n}$ , a symmetric positive definite matrix chosen arbitrarily.

$$\beta_{j} = \dot{q}_{j}^{*} - \sigma_{j}e_{j}, 
\Gamma_{1ij}^{-} = \begin{cases} d_{ij}^{-} & \text{if } \dot{\beta}_{j} > 0, \\ d_{ij}^{+} & \text{if } \dot{\beta}_{j} < 0, \end{cases} 
\Gamma_{1ij}^{+} = \begin{cases} d_{ij}^{+} & \text{if } \dot{\beta}_{j} > 0, \\ d_{ij}^{-} & \text{if } \dot{\beta}_{j} < 0, \end{cases} 
\Gamma_{2ij}^{-} = \begin{cases} c_{ij}^{-} & \text{if } \beta_{j} > 0, \\ c_{ij}^{+} & \text{if } \beta_{j} < 0, \end{cases} 
\Gamma_{2ij}^{+} = \begin{cases} c_{ij}^{+} & \text{if } \beta_{j} > 0, \\ c_{ij}^{-} & \text{if } \beta_{j} < 0, \end{cases} 
i, j = 1, 2, \dots, n.$$
(6c)

Proof By (1) and (6a), we obtain

$$D(q, p_D)\dot{s} + C(q, \dot{q}, p_C)s$$

$$= u_{\text{aux}} - \left[\delta D(q, \delta p_D)\dot{\beta} + \delta C(q, \dot{q}, \delta p_C)\beta\right]$$

$$+ \delta G(q, \delta p_G) + f_d - Rs. \tag{7}$$

where the relation (4) has been used. Now choose a Lyapunov candidate

$$V = \frac{1}{2} s^T [D(q, p_D)] s. \tag{8}$$

Differentiating (8) with respect to time along the solutions of (7), gives

$$V = \frac{1}{2} s^{T} \left[ \frac{\mathrm{d}}{\mathrm{d}t} D(q, p_{D}) - 2C(q, \dot{q}, p_{C}) s - s^{T} R s + s^{T} \left\{ u_{\mathrm{aux}} - \left[ \delta D(q, \delta p_{D}) \dot{\beta} + \delta C(q, \dot{q}, \delta p_{D}) \beta + \delta G(q, \delta p_{G}) + f_{d} \right] \right\}.$$

$$(9)$$

Since  $J_A$  in [Property 2] is a skew—symmetric matrix, therefore  $s^T J_A s = 0$ , as a result the first term of (9) vanishes. Also noting that  $D(q, p_D)$  is symmetric positive definite, by the Lyapunov stability theorem, the sliding plane s = 0 is asymptotic stable if

$$s^{T}\left\{u_{\text{aux}}-\left[\delta D(q,p_{D})\beta+\delta C(q,q,\delta p_{C})\beta+\delta G(q,\delta p_{C})+f_{d}\right]\right\}<0. \tag{10}$$

Obviously (10) is fulfilled if  $u_{\text{aux}}$  is chosen as follows,

$$u_{\text{auxi}} < \sum_{j=1}^{n} \delta D_{ij} \dot{\beta}_{j} + \sum_{j=1}^{n} \delta C_{ij} \beta_{j} + \delta G_{i} + f_{di}, \quad \text{if } s_{i} > 0,$$

$$\tag{11}$$

$$u_{\text{auxi}} > \sum_{j=1}^{n} \delta D_{ij} \dot{\beta}_{j} + \sum_{j=1}^{n} \delta C_{ij} \beta_{j} + \delta G_{i} + f_{4i}, \quad \text{if } s_{i} < 0,$$

$$i = 1, 2, \dots n.$$

$$(12)$$

From the above conditions, (6b) is derived, which completes the proof.

**Remark** We can see that the controllers derived are related to the bounds of  $\delta D$ ,  $\delta C$ ,  $\delta G$ , which are generally available for a practical robot manipulator.

#### 3. 2 Controller Based on the Bounds of $\delta_P$

In this subsection, we present another controller which merely depends on the bounds of the equivalent parameters, rather than those of  $\delta D$ ,  $\delta C$ ,  $\delta G$ , as a result, the controller design procedure is much simpler.

Lemma 2 Let  $\delta p = p - p_0$ , then we can obtain the following relation.

$$\delta D(q, \delta p_D) \dot{\beta} + \delta C(q, \dot{q}, \delta p_C) \beta + \delta G(q, \delta p_G) 
= \varphi(q, \dot{q}, \beta, \dot{\beta}) \delta p.$$
(13)

where  $\varphi(q, \dot{q}, \beta, \dot{\beta}) \varepsilon R^{n \times n}$ , a known matrix.

Proof By using of [Property 3], we have here

$$D(q, p_D)\dot{\beta} + C(q, \dot{q}, p_C)\beta + G(q, p_G) = \varphi(q, \dot{q}, \beta, \dot{\beta})p,$$
  

$$D(q, p_{D0})\dot{\beta} + C(q, q, p_{G0})\beta + G(q, q_{G0}) = \varphi(q, \dot{q}, \beta, \dot{\beta})p_0.$$

In view of the definitions of  $\delta D$ ,  $\delta C$ ,  $\delta G$ , the following relation is readily established

$$\begin{split} \delta D(q,\delta p_D)\dot{\beta} + \delta C(q,\dot{q},\delta p_C)\beta + \delta G(q,\delta p_G) \\ &= \left[D(q,p_D) - D(q,p_{D0})\right]\dot{\beta} + \left[C(q,\dot{q},p_C) - C(q,\dot{q},P_{C0})\right]\beta + \left[G(q,p_G) - G(q,p_{D0})\right] \\ &= \left[D(q,p_D)\dot{\beta} + C(q,\dot{q},p_C)\beta + G(q,p_G)\right] - \left[D(q,p_{D0})\dot{\beta} + C(q,\dot{q},p_{D0})\beta + G(q,p_{D0})\right] \\ &= \varphi(q,\dot{q},\beta,\dot{\beta})p - \varphi(q,\dot{q},\beta,\dot{\beta})p_0 \\ &= \varphi(q,\dot{q},\beta,\dot{\beta})\delta p \end{split}$$

the result is obtained. With this lemma, we have the following theorem.

Theorem 3 For a robot manipulator in the face of uncertainty and disturbance, if control torque y is designed as

$$\gamma = D(q, p_{D0})\dot{\beta} + C(q, \dot{q}, p_{C0})\beta + G(q, p_{G0}) + u_{\text{aux}}, \qquad (14a)$$

$$u_{\text{aux}} = -Rs + \varphi(q, \dot{q}, \beta, \dot{\beta})d - \text{ksgn}(s), \qquad (14b)$$

$$d_{j} = \begin{cases} p_{j}^{-} - \varepsilon_{j}, & \text{if } s^{T} \varphi(q, \dot{q}, \beta, \dot{\beta}) ]_{j} > 0, \\ p_{j}^{+} - \varepsilon_{j}, & \text{if } [s^{T} \varphi(q, \dot{q}, \beta, \dot{\beta})]_{j} < 0, \end{cases}$$
(14c)

$$k_i = \sup_{t \in [0,\infty]} \{ |f_{di}| \} + \varepsilon_0, \tag{14d}$$

$$i = 1, 2, \dots, n, \quad j = 1, 2, \dots, s,$$

where  $\varepsilon_0$  and  $\varepsilon_j$  are small positive constants, then asymptotic path tracking is guaranteed.

Proof Invoking (13) into the asymptotic stability condition (10) gives

$$s^{T}\{u_{\text{aux}} - \left[\varphi(q, \dot{q}, \beta, \dot{\beta})\delta p + f_{d}\right]\} < 0, \tag{15}$$

$$u_{\text{aux}} = -Rs + \varphi(q, \dot{q}, \beta, \dot{\beta})d - \text{ksgn}(s). \tag{16}$$

let from (15) and (16), the following equivalent stability conditions are obtained,

$$s^{T}\{\varphi(q,\dot{q},\beta,\dot{\beta})(d-\delta p)\}<0, \tag{17}$$

$$s^{T}[f_{i}-ksgn(s)]<0. (18)$$

It is easily verified that (17) and (18) are satisfied if d and k are chosen as in (14c-14d), which completes the proof.

It is worth noting that the on—line parameter estimation is not required both in controller (6) and in (14), which makes them easy to implement.

#### Simulation Experiment

The model chosen for simulation is a two link planar "manipulator" with an unknown mass payload as shown in Fig. 1. The links are of lenth  $l_1$  and  $l_2$ , and mass  $m_1$  and  $m_2$  respectively, The mass is assumed to be concentrated in a point at the end of each link. The position state variables are the angles  $q_1$  and  $q_2$ , and the control input  $q_1$  and  $q_2$  are the torque applied at the joints. The terms in the dynamic equation (1) corresponding to simulation model are

$$D = \begin{bmatrix} (m_1 + m_{2'}) l_1^2 + m_2 l_2^2 + 2m_{2'} l_1 l_2 \cos q_2 \\ m_{2'} l_2^2 + m_{2'} l_1 l_2 \cos q_2, & m_{2'} l_2^2 \end{bmatrix},$$

$$C_q^* = \begin{bmatrix} -m_{2'} l_1 l_2 \sin q_2 (2\dot{q}_1 \dot{q}_2) - m_{2'} l_1 l_2 \sin q_2 \dot{q}_2^2 \\ m_{2'} l_1 l_2 \sin \dot{q}_2 q_1^2 \end{bmatrix}$$

$$= \begin{bmatrix} -m_{2'} l_1 l_2 \sin \dot{q}_2 \dot{q}_1 & m_{2'} l_1 l_2 \sin \dot{q}_2 \dot{q}_1 - m_{2'} l_1 l_2 \sin \dot{q}_2 \dot{q}_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$G = \begin{bmatrix} (m_1 + m_{2'}) l_1 g \cos q_1 + m_{2'} l_2 g \cos (q_1 + q_2), \\ m_{2'} l_2 g \cos (q_1 + q_2) \end{bmatrix}.$$

where  $g = 9.81 \text{m/s}^2$ , the acceleration due to gravity.

Fig. 1 Two-link manipulator

 $m'_2 = m_2 + m_L$ ,  $m_L$  is the mass of the payload. For the purpose of simulation, we set the bounds of the parameters as

 $l_1 = 2 + [-0.2, 0.3]m; l_2 = 1 + [-0.3, 0.4]m; m_1 = 1 \text{kg}; m_2 = 0.5 \text{kg}; m_L = [0.3] \text{kg}.$  Let the equivalent parameter vector p be

$$\begin{aligned} p_1 &= (m_1 + m_2) \, l_1^2, p_2 = m_2' \, l_2^2, p_3 = m_2' \, l_1 l_2, p_4 = (m_1 + m_2' \,) \, l_1, p_5 = m_2' \, l_2, \\ \text{then we have} \quad & \varphi_{11} &= \dot{\beta}_1, \varphi_{12} = \dot{\beta}_1 + \dot{\beta}_2, \varphi_{13} = \cos q_2 (2 \, \dot{\beta}_1 + \dot{\beta}_2) - \sin q_2 (\dot{q}_2 \beta_1 + \dot{q}_2 \beta_2 + \dot{q}_2 \beta_2), \\ \varphi_{14} &= g \cos q_1, \varphi_{15} = g \cos (q_1 + q_2), \varphi_{21} = 0, \varphi_{22} = \dot{\beta}_1 + \dot{\beta}_2, \varphi_{23} = \cos q_2 \dot{\beta}_1 + \sin q_2 \dot{q}_1 \beta_1, \varphi_{24} = 0, \\ \varphi_{25} &= g \cos (q_1 + q_2). \end{aligned}$$

and the bounds of the equivalent parameters be

$$p_1 = [4.86, 23.805] = 9.47 + [-4.60, 14.33],$$
  
 $p_2 = [0.245, 3.56] = 3.30 + [-3.06, 3.56],$   
 $p_3 = [0.63, 11.27] = 5.32 + [-4.69, 5.95],$ 

$$p_4 = [2.71, 10.35] = 3.83 + [-1.13, 6.53],$$
  
 $p_5 = [0.90, 8.05] = 3.58 + [-2.68, 4.48].$ 

In the simulation,  $p_i(i=1,2,\dots,n)$  is chosen arbitrarily between the bound, and the external disturbance vector is

$$f_{d1} = 2(Nm), f_{d2} = 1.5\sin(2t)(Nm).$$

The desired path is given as

$$q_1^* = \exp(-t), \quad \dot{q}_1^* = -\exp(-t), \ddot{q}_1^* = \exp(-t),$$
  
 $q_2^* = 2\exp(-2t), \quad \dot{q}_2^* = -4\exp(-2t), \quad \ddot{q}_2^* = 8\exp(-2t).$ 

The sliding plane is chosen as

$$(\dot{q}_1 - \dot{q}_1^*) + 12(q_1 - q_1^*) = s_1, \quad (\dot{q}_2 - \dot{q}_2^*) + 19.5(q_2 - q_2^*) = s_2$$

and matrix R is chosen as, R = diag(24, 24). Finally the initial values are set as  $q_1(0) = 0.2$  (rad),  $\dot{q}_1(0) = 0 (\text{rad/s})$ ,  $q_2(0) = 0.02 (\text{rad})$ ,  $\dot{q}_2(0) = 0 (\text{rad/s})$ . The simulation under the control of (16) was performed by using of the DQY program and the results are shown in Fig. 2-4. It should be noted that the chattering effects have been taken into account in this program.

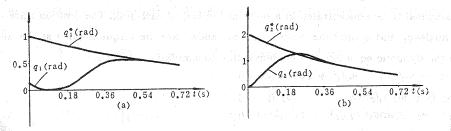


Fig. 2 Desired and actual positions of joint 1 (a) and joint 2 (b)

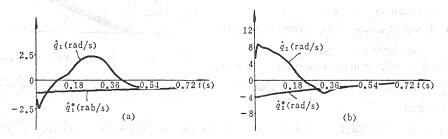


Fig. 3 Desired and actual velocities of joint 1 (a) and joint 2(b)

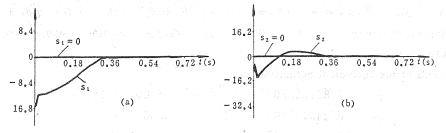


Fig. 4 Curves of sliding mode  $s_1(a)$  and  $s_2(b)$ 

# 5 Extention to Task Space

In general, for an n-link robot manipulator working in an m-DOF space, the mathematical statement of the problem of determining a joint angle vector  $q \in R^n$ , corresponding to a given position,  $x \in R^m$ , is

$$x = E(q) \quad \text{or} \quad q = Q(x). \tag{19}$$

Defferentiating (19) with respect to time one obtains

$$\dot{x}=rac{\delta E}{\delta q}\dot{q}=J(q)\dot{q}.$$
 The second constant  $\dot{x}$  , where  $\dot{x}$ 

where J(q) is known as the Jacobian matrix,  $J = \frac{\delta E}{\delta q} \in R^{m \times n}$ , in the case of non-redundent manipulator where m = n, equation (20) will have a unique solution when the Jacobian J is nonsingular. In the following, we assume  $J^{-1}$  exists.

$$\ddot{q} = J^{-1}\ddot{x} + \dot{J}^{-1}\dot{x}. \tag{21}$$

with this relation, Eq. (1) is rewritten as

$$\overline{D}(x,p_D)\ddot{x} + \overline{C}(x,\dot{x},p_C)\dot{x} + \overline{G}(x,p_G) + \overline{f}_d = F.$$
 (22)

where

$$\overline{D}(x, p_D) = J^{-T}D(Q(x), p_D)J^{-1}, 
\overline{C}(x, x, p_C) = J^{-T}[D(Q(x), p_D)J^{-1} + C(Q(x), J^{-1}x, p_C)J^{-1}], 
\overline{G}(x, p_G) = J^{-T}G(Q(x), p_G), 
T = J^TF, 
\overline{f}_4 = J^{-T}f_4.$$
(23)

It should be noted that the same properties mentioned in the section 2 still hold for the task space case. Thus we can easily obtain the following result.

Theorem 4 Suppose the desired path is given as  $\{x^*, \dot{x}^*, \dot{x}^*\}$ , if the following control toruge is used,

$$F = D(x, p_{D0})\dot{eta} + C(x, \dot{x}, p_{c0})\dot{eta} + G(x, \dot{x}, p_{C0}) + u_{
m aux}, \ u_{
m aux} = -Rs + arphi(x, \dot{x}, eta, \dot{eta})\dot{d} - {
m ksgn}(s), \ d_1 = egin{cases} p_i^- - arepsilon_i, & {
m if} & [s^T arphi(x, \dot{x}, eta, \dot{eta})]_i > 0 \ p_i^+ - arepsilon_i, & {
m if} & [s^T arphi(x, \dot{x}, eta, \dot{eta})]_i < 0, \end{cases} \ k_j = \sup_{i \in \epsilon[0, \infty]} \{|ar{f}_{dj}|\} + arepsilon_0, \ i = 1, 2, \cdots, s, \quad j = 1, 2, \cdots, n, \ s = (\dot{x} - \dot{x}^*) + \sigma(x - x^*), \ eta = \dot{x}^* - \sigma(x - x^*). \end{cases}$$

where  $\varepsilon_0$  and  $\varepsilon_i$  are small positive constants, then asymptotic path tracking is guaranteed.

#### 6 Conclusion

A variable structure control strategy is proposed to deal with the path tracking control problem for a robot manipulator in the presence of parameter uncertainty and external disturbance. By making use of the robot dynamics structure properties, control algorithms are derived

which guarantees the asymptotic path tracking. Simulation results are presented and the task space case is also addressed.

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- [1] Ahmad, S. . Analysis of Robot Drive Train Errors, Their Static Effects, and Their Compensations. IEEE J. Robotics and Automation, 1988, 4(2); 117-128
- [2] Arimoto S., Kawamura, S.. Learning Control Theory for Dynamical Systems. Proc. 24th IEEE CDC, 1985, 1375-1380
- [3] Bayard D. S., Wen. J. T.. New Class of Control Laws for Robotic Manipulators, Part 2. Adaptive Case. Int. J. Control, 1988, 41(5):1387—1397
- [4] Slotine J., Li W. On the Adaptive Control of Robot Manipulators, Int. J. Robotics Res, 1987, 6(3), 49-59
- [5] Craig, J., Hsu, P., Sastry, S.. Adaptive Control of Mechanical Manipulators. Int. J. Robotics Reserach, 1987, 6(2):16—28
- [6] Yeung K., Chen Y. P.. A New Controller Design for Manipulators Using Theory of Variable Structure Systems. IEEE Trans. Auto. Control, 1988, 33(2):200-206
- [7] Baily E., Arapostathis A. Simple Sliding Mode Control Scheme Applied to Robot Manipulators. Int. J. Control, 1987, 45 (4):1197-1209
- [8] Utkin V. I. . Sliding Modes and Their Applications, Moscow, USSR, 1978
- [9] Young K. D. A Variable Structure Model—Following Control for Robotic Applications. IEEE J. Robotics and Automation, 1988,4(5):556-561

## 用于机器人轨迹跟踪的结构控制

宋永端 高为炳(北京航空航天大学第七研究室)

摘要:实际机器人控制过程中,不确定参数及外部干扰几乎总是存在的,研究考虑了这些因素的机器人控制问题,在理论及应用中都有重要意义.本文考虑了存在外扰及不确定参数时,机器人的轨迹跟踪问题.利用变结构控制理论,结合机器人自身的结构特点,研制了两组控制规律,它一方面对外扰不敏感,另方面不要求精确的机器人参数,也勿需对参数辨识,因而运算简单,易于实现.理论分析及仿真试验都证明了所给控制能保证渐近轨迹跟踪.

rarah ira mara bar pagis na ake sa angar. Ta a aberdi patan sa washerah pagaba sa washerah

关键词: 轨迹跟踪;变结构控制;机器人;不确定性