An Approach to Hybrid Position/force Control of Robot Manipulators

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Abstract: In this paper, the problem of tracking along a constraint surface is studied. Towards this goal, the main contribution in this study is the introduction of two task specification projective operators. with these operators, the dynamic equation of a constrained robot manipulator can be decoupled into two subsystems, one of which describes the motion and the other describes the constraint force. A feedforward and feedback control law is designed to guarantee the tracking of the desired motion and force.

Keywords: robot control; nonlinear system; target tracking; control system synthesis; hybrid control

1 Introduction

Recent research in dynamics and control of robot manipulators has gained much attention on developing the task specification for motion and contact force of the end—effector. This problem is known as the compliance control of robot manipulators. Two main categories of methods have been developed: impedance control and hybrid position/force control.

Impedance control is aimed at developing a relationship between interaction force (torque) and position (orientation) of manipulator[1,2]. Hybrid position/force control is designed to control the force (torque) and position (orientation) in a nonconflicting way. In this method, force is generated along those directions constrained by the environment, while position is restricted along those directions where the manipulator is unconstrained and free to move.

The hybrid control method has been first proposed by Raibert and Craig (1981)[3], in which, however, the dynamics of manipulator is not taken into account rigorously. Then, Yoshikawa (1986)[4] developed a dynamical hybrid control method, but the motion of the robot manipulator can track only the desired acceleration. Khatib (1987)[5] established the generalized task specification matrices matching with the specifications of the motion and the contact forces. Although the explicit formulation was not given therein, it is pointed out that the control of motion and force can be achieved with respect to different directions in the task space. McClamoroch

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(1988)[6] suggested a control law for a constrained robot to track the desired motion and contact force. This control law is comparatively perfect, but the assumption of global solvability for the constraint equation seems too strict.

In the present paper, the concept of a couple of nonlinear projective operators, called task specification projective operators, is defined and analyzed. Based on these operators, the dynamic equation of a constrained robot manipulator can be decoupled into two subsystems, one of which represents the motion and the other describes the constraint force. With this decoupled dynamics, a feedforward and feedback control law is established such that the desired velocity and constraint force can be tracked satisfactorily.

This paper is organized as follows: 1) The dynamic model equation and the constraint equation; 2) Task specification projective operators; 3) The decoupled dynamic equations; 4) A feedforward and feedback control law.

2 The dynamic model and the constraint equation

We consider a constrained robot, its dynamic equation is[6]

$$D(q)\ddot{q} + c(q,\dot{q}) = u + f. \tag{1}$$

where $q \in R^n$ denotes the generalized coordinate vector of a robot manipulator. In general, the components of q are the joint angles of robot in the joint space. Besides, q may also represent the generalized coordinate in other space instead, for example, the task variable in the task space. $\dot{q}, \ddot{q}$ are the generalized velocity and acceleration, respectively. $D(q): R^n \rightarrow R^{n\times n}$, is a positive definite matrix containing the inertia of the system. u, f denote the generalized control force and the constraint force. $c(q,\dot{q}): R^n \times R^n \rightarrow R^n$, is a nonlinear term containing gravity, gyroscopic and centrifugal inertial forces.

We assume that $p \in R^n$ is also a coordinate vector of the robot manipulator, which is suitable to describe the constraint of its end—effector. Consequently, the constraint equation can be expressed as

$$\widetilde{F}(p) = 0. \tag{2}$$

where $\widetilde{F}: R^n \rightarrow R^m$, is assumed to be analytic in $U \subset R^n$, an open set in R^n. In addition, the relation between p and q can be given as follows

$$p = H(q). \tag{3}$$

At any nonsingular position of the nonredundant manipulator considered here, p and q in Eq. (3) can be transformed to each other one to one. Substitution of Eq. (3) into Eq. (2) yields:

$$F(q) = \widetilde{F}(H(q)) = 0. \tag{4}$$

By differentiationg Eq. (4) with respect to time, we obtain

$$J(q)\dot{q} = 0, \quad J(q) =: \delta F(q)/\delta q. \tag{5}$$

where $J(q) \in R^{m\times n}$ is assumed to be of full rank, that is,

$$\mathrm{Rank}[J(q)] = m. \tag{6}$$

The generalized constraint force, in the ideal case, can be represented by

$$f = J^T(q)\lambda. \tag{7}$$

where $\lambda \in R^m$ is a so—called Lagrange multiplier vector. It is revealed that the n dimensional generalized constraint force only depends on an m dimensional vector. According to Eq. (4), we define a set in R^n as

$$s = \{q \in U \subset R^n | F(q) = 0\}. \tag{8}$$

This set, being fundamental in our development, is generally a manifold in R^n, on which the motion of the manipulator is constrained. Therefore, the dynamic model of a constrained robot is singular. This is the essential difference between the constrained robot and the free one in their motions.

3 Task specification projective operators

Let us define a pair of projective operators for each $q \in U \subset R^n$ as follows

$$\begin{aligned} \tilde{P}(q) &=: D^{-1}(q) J^T(q) [J(q) D^{-1}(q) J^T(q)]^{-1} J(q) \\ &= D^{-1}(q)(\delta F/\delta q)^T [(\delta F/\delta q) D^{-1}(q)(\delta F/\delta q)^T]^{-1}(\delta F/\delta q), \end{aligned} \tag{9}$$

$$P(q) =: I - \tilde{P}(q). \tag{10}$$

Note that these definitions are well posed since $D(q)$ is a positive definite matrix. Some properties of the aforementioned definitions are given in the following Lemma.

Lemma 1 $P(q)$ and $\tilde{P}(q)$ are all projective operators, and the

1) $P^2(q) = P(q), \tilde{P}^2(q) = \tilde{P}(q)$;

2) $P(q) + \tilde{P}(q) = I$;

3) $P(q)\tilde{P}(q) = \tilde{P}(q)p(q) = 0$;

4) $\tilde{P}(q)[D^{-1}(q)f] = D^{-1}(q)f$;

5) $P(q)\dot{q} = \dot{q}$.

The proof of Lemma 1 is straightforward and is, therefore, omitted.

In order to illustrate the geometric concept of task specification projective operators, for each point $q \in U \subset R^n$, we define two subspaces in the tangent space $T_q(\Omega)$

$$s_1(q) =: \{x_1 \in T_q(U) | J(q)x_1, q \in U\}, \tag{11}$$

$$s_2(q) =: \{x_2 \in T_q(U) | x_2 = D^{-1}(q)J(q)\lambda, \quad q \in U, \lambda \in R^m\}. \tag{12}$$

It is clear that

$$s_1(q) + s_2(q) = T_q(U). \tag{13}$$

From Eqs. (9—13) and Lemma 1, we also see that s_1 is along the tangent surface, but s_2 does not coincide with the normal surface of the constraint manifold(5). Besides, $\tilde{P}(q)$ defined in $T_q(U)$ is a projective mapping along s_1 to s_2, while $P(q)$ is along s_2 to s_1.

4 The decoupled dynamic equations

By using the task specification operators, the dynamic equations of a constrained robot manipulator can be decoupled into

$$P(q)\ddot{q} = P(q)\tilde{u}, \tag{14}$$

$$\tilde{P}(q)\ddot{q} = \tilde{P}(q)\tilde{u} + D^{-1}(q)f, \tag{15}$$

$$\tilde{u} =: D^{-1}(q)[u - c(q, \dot{q})].$$

where $P(q)\ddot{q}$ denotes the projective mapping of generalized acceleration $\ddot{q}$ along s_2 to s_1, and

$\tilde{P}(q)\ddot{q}$ maps $\ddot{q}$ along s_1 to s_2, In the case that robot manipulator were a particle, the two vectors aforementioned would be the tangent and the normal accelerations, respectively. In general, the component of the generalized acceleration, used to calculate the generalized constraint force, is not the normal acceleration of the constraint manifold. It is also interesting to know that from

$$\ddot{q} = P(q)\ddot{q} + \tilde{P}(q)\ddot{q}, \quad \text{and} \quad \ddot{q} = \frac{\mathrm{d}}{\mathrm{d}t}[P(q)\dot{q}] = P(q)\ddot{q} + \dot{P}(q)\dot{q},$$

we can obtain

$$\tilde{P}(q)\ddot{q} = \dot{P}(q)\dot{q}. \tag{16}$$

By summing up the equations mentioned above, the following theorem is obtained.

Theorem 1 The dynamic equations of constrained robot manipulator can be decoupled into Eqs. (14) and (15). The generalized constraint force is independent of the generalized acceleration $\ddot{q}$.

5 A feedforward and feedback control law

In order to derive the control law, two nonsingular matrices $Q(q)$ and $Q^{-1}(q)$ are defined such that

$$Q^{-1}(q)P(q)Q(q) = \begin{bmatrix} I_{n-m} & 0 \\ 0 & 0 \end{bmatrix}, \tag{17}$$

$$Q^{-1}(q)\tilde{P}(q)Q(q) = \begin{bmatrix} 0 & 0 \\ 0 & I_m \end{bmatrix}. \tag{18}$$

The transformation $Q^{-1}(q)$ means that $n-m$ of the independent columns of $P(q)$ and m columns of $\tilde{P}(q)$ are taken as a group of new base in the tangent space for each point $q \in U \subset R^n$. Hence $Q(q)$ can be calculated by the independent column searching algorithm.

Theorem 2 Given a desired trajectory q^d satisfying the constraint equation (5) and a desired generalized constraint force $f^d = J^T(q)\lambda^d$, the following feedforward and feedback control law,

$$\begin{aligned} \tilde{u} &= P(q)\tilde{u} + \tilde{P}(q)\tilde{u} \\ &= P(q)[Q(q)\dot{Q}^{-1}(q,\dot{q})(\dot{q}^d - \dot{q}) + \ddot{q}^d + \hat{K}(q)(\ddot{q}^d - \ddot{q})] \\ &\quad + \tilde{P}(q)[-D(q)^{-1}J^T(q)\lambda^d + \dot{P}(q)\dot{q}^d]. \end{aligned} \tag{19}$$

will guarantee

$$\lim(\dot{q}^d - \dot{q}) = 0, \quad \text{and} \quad \lim(f^d - f) = 0$$

where

$$\dot{Q}^{-1}(q,\dot{q}) =: \frac{\mathrm{d}}{\mathrm{d}t}Q^{-1}(q,\dot{q}),$$

$$\hat{K}(q) =: Q(q)\tilde{K}Q^{-1}(q), \quad \tilde{K} =: \mathrm{Diag}(K_1, 0). \tag{20}$$

and K_1 is an $n-m$ dimensional stable matrix.

Proof Let us define a nonlinear transformation for the generalized velocity as follows:

$$\dot{z} = Q^{-1}(q)\dot{q},$$

$$z =: \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad z_1 \in R^{n-m}. \tag{21}$$

and suppose that

$$\dot{z}^d = Q^{-1}(q)\dot{q}^d, \quad \ddot{z}^d = \frac{d}{dt}\dot{z}^d. \tag{22}$$

then, we have

$$\begin{aligned} \ddot{z}^d &= Q^{-1}(q)\ddot{q}^d + \dot{Q}^{-1}(q,\dot{q})\dot{q}^d, \\ \ddot{z} &= Q^{-1}(q)\ddot{q} + \dot{Q}^{-1}(q,\dot{q})\dot{q}. \end{aligned} \tag{23}$$

Substitution of Eqs. (21) and (22) into Eq. (19) yields

$$\begin{aligned} P(q)\tilde{u} &= P(q)\{Q(q)\dot{Q}^{-1}(q,\dot{q})[Q(q)\dot{z}^d - Q(q)\dot{z}] \\ &\quad + Q(q)\ddot{z}^d - Q(q)\dot{Q}^{-1}(q,\dot{q})Q(q)\dot{z}^d + Q(q)\widetilde{K}(\dot{z}^d - \dot{z})\} \\ &= P(q)[-Q(q)\dot{Q}^{-1}(q,\dot{q})Q(q)\dot{z} + Q(q)\ddot{z}^d + Q(q)\widetilde{K}(\dot{z}^d - \dot{z})]. \end{aligned}$$

$$\dot{Q}^{-1}(q,\dot{q}) = \frac{d}{dt}Q^{-1}(q,\dot{q})$$

Substituting Eq. (23)into Eq. (14) and premultiplying the two sides by $Q^{-1}(q)$, respectively, we can obtain

$$Q^{-1}PQ\ddot{z} - Q^{-1}PQ\dot{Q}^{-1}Q\dot{z} = - Q^{-1}PQ\dot{Q}^{-1}Q\dot{z} + Q^{-1}PQ\ddot{z}^d + Q^{-1}PQ\widetilde{K}(\dot{z}^d - \dot{z}).$$

that is

$$(\ddot{z}_1^d - \ddot{z}_1) + K_1(\dot{z}_1^d - \dot{z}_1) = 0.$$

Thus, we have

$$\lim(\dot{z}_1^d - \dot{z}_1) = 0.$$

Also, because

$$\dot{z}_2^d - \dot{z}_2 = Q^{-1}\widetilde{P}Q(\dot{z}^d - \dot{z}) = Q^{-1}\widetilde{P}(\dot{q}^d - \dot{q}) = 0.$$

the following limit is guaranteed

$$\lim_{t\to\infty}(\dot{z}^d - \dot{z}) = 0.$$

Moreover, substituting Eq. (19) into Eq. (15), we obtain

$$\tilde{P}\dot{q} = \tilde{P}\dot{q}^d - D^{-1}(q)J^T(\lambda^d - \lambda).$$

Consequently, the limit of the difference between the generalized constraint force and its desired value is guaranteed to be zero. Thus, the proof is completed.

In addition, the closed loop of the system can be improved for the following cases. In most of the practical problems about compliance control of a robot manipulator, in order to satisfy the quality requirements of the closed—loop system, it is enough to tracking only the desired velocity, $\dot{q}^d(t)$ However, in the case where tracking of the desired trajectory $\dot{q}^d(t)$ is also a requirement, an additional outer loop is needed to guarantee the difference between the practical generalized coordinate and the desired one to approach zero. This difference, multiplied by an appropriate gain matrix, can be used as the input of this additional loop. The structure of this system is shown in Fig. 1.

In the case that the end—effector of robot manipulator is free to move, we suppose that

$$\delta F / \delta x = J = 0.$$

in Eq. (9), thus,

$$P(q) = I, \quad \tilde{P}(q) = 0.$$

In order to achieve the compliance control of a robot manipulator for the tasks in a variety of environments, for example, where the constraint is variable, the complete control system may be hierarchically organized. A high—level frame is designed to plan the motion and to generate a set of commands to the lower level (real—time), which gives the task specification projective operators associated with a variable constraint in real time. This arrangement is plotted in Fig. 2.

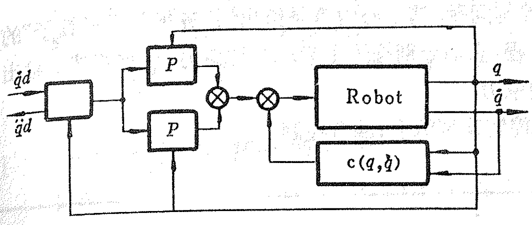

Fig. 1 The Structure of Control System

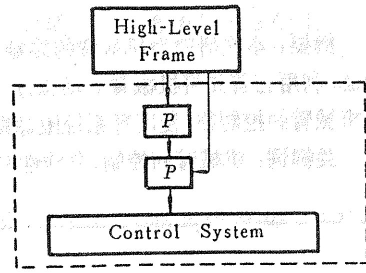

Fig. 2 Hierarchical Control System

6 Conclusions

A methodology for the description of constrained motion tasks of the end—effector based on the construction of task specification projective operators has been proposed. By using these operators, the dynamic equation of constrained robot manipulator can then be decoupled into two subsystems, representing the motion and the constraint forces, respectively. Moreover, an approach of designing the controller for the dynamic hybrid position/force control of a robot manipulator has been presented. Finally, some suggestions for improving the designed closed loop system have been stated for some cases.

Recently some methods have been developed (for example, Slotine, J. J. E, 1986) to enhance the robustness of the closed—loop systems of robot manipulators, though it is open in this paper. In addition, the VSC law may be used to improve the robustness of the compliance control of robot manipulators.

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机械臂的动态混合控制

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摘要：本文研究当机械臂的终端受有约束时的控制问题. 其中心内容是给出“任务规范投影算子”的概念. 利用它首先将机械臂的动态方程解耦为两组方程，它们分别描述了运动与约束力. 在此基础上给出了机械臂的控制律，使闭环系统跟踪期望的速度与约束力.

关键词：机械臂的控制；非线性系统；目标跟踪；控制系统综合；混合控制

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