# A New Approach to Obtaining the Hedging Points in Multi-Part Production System by Genetic Algorithms

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**Abstract**: In this paper, we suggest a method to obtain the values of hedging points for a multi-part-type production system by using the genetic algorithms. We establish a genetic algorithm model for hedging point policy and show that the results of the genetic algorithms can reflect the properties of hedging point policy accurately.

**Key words**: hedging point policy; hedging point; genetic algorithms

# 安全点策略中安全点求取的一种新的方法——遗传算法的应用

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**摘要**: 给出一种利用遗传算法求取单机器、多工件生产系统的安全点策略中安全点近似值的方法.建立了安全点策略的遗传算法运算模型,编制了遗传算法程序.通过举例运行说明了遗传算法的运算结果能够正确地体现安全点策略的特性.

**关键词**: 安全点策略; 安全点; 遗传算法

## 1 Introduction

Consider the problem of a stochastic manufacturing system containing one machine that produces $N$ commodity types must meet the demand at a minimum cost. The stochastic nature is due to the machine failure.

The problem belongs to the field of flowshop with failure-prone machines. Kimema and Gershwin[1] have showed that the optimal control for such a system has a specific structure called the hedging point policy. In such a policy, a nonnegative surplus of products called hedging point must be guided as quickly as possible and is maintained after reached to hedge against future capacity shortages caused by machine failures. The focus of the policy is to solve the value of the hedging points. Unfortunately, no analytic solution for it can be obtained except for the one-part-type one-machine system[2] because of mathematics difficulties. Although there are a few papers[3] which have considered multiple-part systems and proposed some ways to solve the problem, it is difficult to use those solution when you deal with a more than two part-type system.

In this paper, we first apply the genetic algorithms to this field. The most advantage of the method is that the complexity will not increase as the dimensions (the number of part-type) of a system increase.

The remainder of this paper is organized as follows. Section 2 describes the problem we aim to solve. Section 3 studies the application of GAs, runs the GAs program and analyzes the results. Section 4 deduces conclusion from the previous section.

## 2 The problem

### 2.1 A multi-part-type one-machine system and hedging point policy

#### 2.1.1 The model of dynamic programming

a) State variables, control variables and parameters.

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The continuous state variables:

$X(t)$: $(x_1(t), x_2(t), \cdots, x_n(t))$—part surplus.

The discrete state variables:

$\alpha(t)$: The state of machine—a finite-state homogeneous Markov process, the state set: $\{0, 1\}$, "1"—functional, "0"—repaired.

The control variables:

$U(t)$: $(u_1(t), u_2(t), \cdots, u_n(t))$ —the production rates.

The parameters:

$D$: $(d_1, d_2, \cdots, d_n)$ —the demand rate of part.

b) The state equation and the constraint condition.

The state equation:

$$\frac{\mathrm{d}(X(t))}{\mathrm{d}t} = U(t) - D. \tag{1}$$

The constraint condition:

$$\sum_{i=1}^{n} \tau_i u_i \leqslant \alpha(t), \tag{2}$$

$$u_i(t) \geqslant 0, \quad i = 1, \cdots, n. \tag{3}$$

c) The objective function.

$$J(X, \alpha, t) = \min_{v(t)} E\left\{\int_t^T g(X(s))\mathrm{d}s \mid X(t) = X, \alpha(t) = \alpha\right\}, \tag{4}$$

where, $g(X(s))$ is a penalizing function.

#### 2.1.2 The structure of optimal production rate

Gershwin[4] proposes the structure of an optimal control policy. Note that the constraint of $U$ is a polyhedron. The optimal policy divides the state space into a set of areas and in each area the production rate is constant. The boundaries of each area intersect at one point, called hedging point, and the running trend of system should be toward the point. Finally the system will maintain at the point after reaching it. The activity of the system can be expressed as Fig. 1.

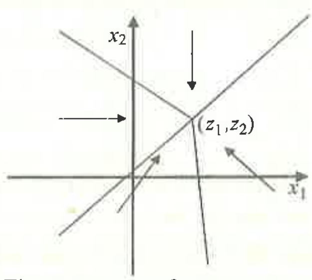


Fig. 1 The state space of a two-part-type system

Because we usually suppose that production rates are far higher than machine state change rates, the system will be at the hedging point in most period of functional state. The division of the state space determines the way to reach the hedging point. But in practice, we do not implement the policy in the way shown in Fig. 1 because of complexity. We just operate the system to run toward the hedging point by making it work at its full capacity. Therefore, the most important problem we are concerned with is the value of the hedging point.

## 3 Application of GAs

### 3.1 The idea

In the above discussion, we show that the structure of hedging point policy is very distinct and the focus is the solution of hedging points. The optimal policy will be almost achieved if the hedging point is obtained.

After the hedging point is obtained, we implement the optimal policy in practice as follows: We choose the part type, whose difference between its surplus and hedging point is the biggest, to produce when the machine is idle. Stimulated by this way and the distinct structure of hedging point policy, we propose to apply the GAs to this field. If the regions in which the values of the hedging points is inside can be estimated, we could choose the initial values of the hedging points in the regions randomly. By simulating the way implemented in practice, the value function according to the hedging point chosen can be calculated. Then the evaluating value, called "fitness" in GAs, can be gotten by the transition of value function. Therefore, we can seek the optimal value of the hedging point by GAs. The focus of this method is how to determine the fitness so as to reflect the properties of the value function.

### 3.2 The model of GAs

#### 3.2.1 The parameters of production system

In order to obtain the optimal policy for the production system, the following parameters must be given:

a) Failure rate and repair rate of the machine

$$p, r.$$

b) Demand rate for each part type

$$d_i, \ i = 1, 2, \cdots, n.$$

c) Processing time for each part type (unit time)

$$\tau_i, \ i = 1, 2, \cdots, n.$$

d) Penalizing coefficients for each part type.

$C_i^+$: the surplus is positive, $C_i^-$: the surplus is negative (backlog).

**Remark** In this paper, we only consider the situation that the functional state of the machine is feasible. So the parameters of the model must satisfy:

$$\sum_{i=1}^{n} d_i \leqslant \frac{r}{r+p}. \tag{5}$$

The inequality (5) implies that the capacity of the machine is powerful enough to meet the demand. If the capacity of machine can not satisfy the inequality (5), we called it infeasible state in which the machine must always produces in its maximal capacity and we usually think that the hedging point is infinity.

### 3.2.2 Representation

We denote the coordinates of the hedging point as $Z_i$, $i = 1, 2, \cdots, n$. Each coordinate is represented by a float point number and called a gene in GA's literature. The hedging point is a vector $Z_i$ and represents as:

$$(z_1, z_2, \cdots, z_n). \tag{6}$$

The expression (6) is called chromosome in GAs. Our goal is to seek the optimal value of the chromosome.

### 3.2.3 The expression of the fitness value

How to determine the expression of the fitness value is the focus of the application of GAs. In the model established in 2.1.1, we suppose that the time region of the machine up and down belongs to exponential distributions. Then, the transition of machine state is homogeneous Markov process. Since the objective value function is the expectation of an integral expression, we can not use the value function as the fitness value directly. To calculate the fitness value, we use several approximation ways as follows:

i) Indicate the time region of the up and down of the machine as average number. Suppose that the failure rate is $p$ and the repair rate is $\gamma$. Thus, the expectation of those time region are $1/p$ and $1/\gamma$.

ii) Suppose the sum of a range of up time and a range of down time, which is $1/p + 1/\gamma$, as a running cycle of the machine and as a time scale calculating the fitness value. The time scale is divided into $M$ intervals, as shown in the Fig. 2.

iii) Suppose the capacity of the machine can make the surplus of each part type reach its hedging point during the up region. By denoting the moment when the machine breakdown as $t_0$ and the surplus of part type $i$ in

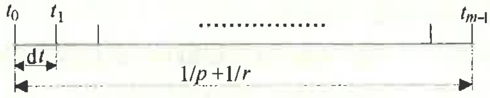


Fig. 2 The time scale for calculation of fitness

$t_j$ moment as $x_{ij}$, we have

$$x_{ij} = x_{i(j-1)} + \mathrm{d}t \times (u_{i(j-1)} - d_i), \quad x_{i0} = Z_i, \tag{7}$$

where the $u_{ij}$ denotes the production rate of part type $i$ in $t_j$ moment, which must satisfy:

$$\sum_{i=1}^{n} \tau_i u_{ij} \leqslant 1. \tag{8}$$

As the up state comes up, we must decide the part type whose difference between its surplus and hedging point is the biggest, and then produce it as maximal production rate. For example, supposing the part type $m$ is chosen, we have $u_{mj} = 1/\tau_m$ and meanwhile the production rates of other part types are zero. Then, we must calculate the difference at every point $t_j$. Whenever the difference of another part type, for example, part type $n$, is bigger than the part type $m$, the production rate of $m$ will reduce to $d_m$ and the remainder of machine capacity will be given to the part type $n$, $u_{nj} = (1 - \tau_m d_m)/\tau_n$. Analogize as above until the surplus of each part type rise to the hedging point, then their production rates will be equal to the demand rates. The idea of the method is based on the basic rule of hedging point policy, which points out that the system should operate toward the hedging point as quickly as possible, then keep at the point until the down state comes up. This method can be illuslrated by Fig. 3 for a two part type system.

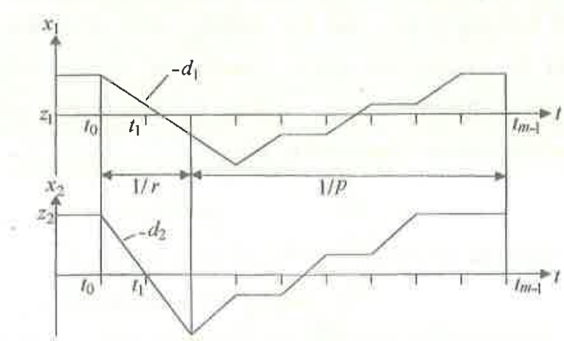


Fig. 3 The description of the method to determine the production rate and the corresponding track of state

iv) Calculate the expression below with the results obtained in iii)

$$\sum_{j=0}^{m-1}\sum_{i=1}^{n}(C_i^+x_{ij}^+ + C_i^-x_{ij}^-), \tag{9}$$

where $x_{ij}^+ = \max\{0, x_{ij}\}, x_{ij}^- = \max\{0, -x_{ij}\}$. The goal of optimization is to make expression (9) minimum. But, it can not be used as fitness value function since the GAs need the fitness value as the maximum form. Therefore, we make the transition

$$M - \sum_{j=0}^{m-1}\sum_{i=1}^{n}(C_i^+x_{ji}^+ + C_i^-x_{ji}^-), \tag{10}$$

where $M$ represents the maximal value of expression (9) obtained so far.

#### 3.2.4 Determine the region of the hedging points

The region where the hedging points will be sought must be decided before the GAs can be implemented. According to the idea of hedging point policy, we can see that the lower boundary is zero. To decide the upper boundary, we think that the surplus level will reduce at the speed of $d_i$ in the down state. So the minimum part storage to prevent the surplus level from dropping below the zero during down state is $d_i\frac{1}{\gamma}$. Therefore the region of the hedging points is

$$[0,\ d_i\frac{1}{\gamma}]. \tag{11}$$

#### 3.2.5 Program of the GAs

We program the GAs software by modifying the basic program frame offered in the appendix in [5]. The fundamental rules we used are described as follows:

i) Standard proportional selection.

ii) Arithmetical crossover[6].

iii) Non-uniform mutation[6].

iv) Elitist rule.

### 3.3 The experiment, results and analysis

#### 3.3.1 Parameters setting and stop rule

Parameters:

Population scope: $N = 100$;

Number of generation: $M = 5000$;

Probability of selection: $p_r = 0.5$;

Probability of crossover: $p_c = 0.8$;

Probability of mutation: $p_m = 0.2$.

Stop rule:

Stop when the $m$th reproduction have finished.

#### 3.3.2 The results and explanation

We apply the GAs to a three-part-type one-machine system so as to show the efficiency of the method. We will see that the results of GAs can reflect the fundamental properties of the hedging point policy accurately.

a) Zero-inventory control policy.

We operate the GAs for three examples with different parameters and get the results in Table 1.

We can see from the table that the Hedging Points in the three examples are zero or can be approximated by zero. In contrast to these results, we usually think that positive inventories are used as a buffer against uncertainties. Thus, zero-inventory level can only be optimal when there is no uncertainty at all, and that is never possible. But, in [7], a condition in which a zero-inventory policy is actually provably optimal even when there is uncertainty given for one-part-type, one-machine system. They show that the zero-inventory is optimal when the parameters of a system satisfy:

$$\frac{Kp(C^+ + C^-)}{C^+\,(K-d)(r+p)} \leqslant 1. \tag{12}$$

Table 1 The result 1 of the GAs

| | parameters of part | part type 1 | part type 2 | part type 3 | parameters of machine |
|---|---|---|---|---|---|
| | $d$ | 1 | 1 | 1 | $p=0.$ |
| | $\tau$ | 0.1 | 0.1 | 0.1 | 05 |
| Example 1 | $C^+$ | 2 | 2 | 2 | $r=0.$ |
| | $C^-$ | 8 | 8 | 8 | 5 |
| | HP $Z$ | **0.00** | **0.00** | **0.00** | |
| | $d$ | 3 | 7 | 5 | $p=0.$ |
| | $\tau$ | 0.01 | 0.03 | 0.07 | 1 |
| Example 2 | $C^+$ | 4 | 4 | 4 | $r=0.$ |
| | $C^-$ | 5 | 5 | 5 | 5 |
| | HP $Z$ | **0.03** | **0.05** | **0.01** | |
| | $d$ | 1 | 1 | 1 | $p=0.$ |
| | $\tau$ | 0.1 | 0.15 | 0.05 | 1 |
| Example 3 | $C^+$ | 2.5 | 2.5 | 2.5 | $r=0.$ |
| | $C^-$ | 7.5 | 7.5 | 7.5 | 5 |
| | HP $Z$ | **0.05** | **0.02** | **0.04** | |

In Equation (12), $K$ denote the maximum production rate. This inequality happens whenever r is adequately reduce or $p$ is increased to the full, i.e., whenever the system is made effcient. Obviously this situation should be suitable for the multi-part-type system. We deal with the parameters of the multiple-part-type system as follows:

$$d = \sum_{i=1}^{n} d_i, \quad K = \frac{1}{\left(\sum_{i=1}^{n} \tau_i / n\right)},$$

$$C^+ = \frac{\sum_{i=1}^{n} C_i^+}{n}, \quad C^- = \frac{\sum_{i=1}^{n} C_i^-}{n}.$$

Replacing parameters in left side of (12) by the four expressions, intuitively the inequality is modified by expressing the condition in which the zero-inventory policy is optimal for a multiple-part-type system. The left side of (12) for the specified parameters of the three examples in Table 1 is 0.65, 0.884, 0.95 respectively. So we can expect that the zero-inventory policy is optimal for those cases. The results of GAs have proved our hypothesis.

b) Effects of change of parameters.

In the following examples, we take Example 1 as a reference model and change the specified parameters in other examples in order to show that the GAs can reflect the properties of hedging point policy when some parameters are changed. We make 5 experiments and the results are showed in Table 2.

Table 2 The result 2 of GAs

| | parameters of part | part type 1 | part type 2 | part type 3 | parameters of machine |
|---|---|---|---|---|---|
| | $d$ | 2.5 | 2.5 | 2.5 | $p = 0.15$ |
| | $\tau$ | 0.1 | 0.1 | 0.1 | $r = 0.8$ |
| Example 1 | $C^+$ | 2 | 2 | 2 | |
| | $C^-$ | 8 | 8 | 8 | |
| | HP $Z$ | **1.93** | **2.02** | **1.93** | |
| | $d$ | 2.5 | 2.5 | 2.5 | $p = \mathbf{0.1}$ |
| | $\tau$ | 0.1 | 0.1 | 0.1 | $r = 0.8$ |
| Example 2 | $C^+$ | 2 | 2 | 2 | |
| | $C^-$ | 8 | 8 | 8 | |
| | HP $Z$ | **0.28** | **0.24** | **0.28** | |
| | $d$ | 2.5 | 2.5 | 2.5 | $p = 0.15$ |
| | $\tau$ | 0.1 | 0.1 | 0.1 | $r = 0.8$ |
| Example 3 | $C^+$ | **4** | 2 | 2 | |
| | $C^-$ | **6** | 8 | 8 | |
| | HP $Z$ | **0.905** | **1.86** | **1.91** | |
| | $d$ | 2.5 | 2.5 | 1 | $p = 0.15$ |
| | $\tau$ | 0.1 | 0.1 | 0.1 | $r = 0.8$ |
| Example 4 | $C^+$ | 2 | 2 | 2 | |
| | $C^-$ | 8 | 8 | 8 | |
| | HP $Z$ | **0.98** | **1.10** | **0.58** | |
| | $d$ | 2.5 | 2.5 | 2.5 | $p = 0.15$ |
| | $\tau$ | 0.1 | **0.3** | 0.1 | $r = 0.8$ |
| Example 5 | $C^+$ | 2 | 2 | 2 | |
| | $C^-$ | 8 | 8 | 8 | |
| | HP $Z$ | **3.01** | **3.10** | **3.05** | |

The boldfaces in the table as well as the hedging points mean the parameters changed with respect to Example 1. We explain the consequences as follows:

i) Influences of the parameters of the machine.

In Example 2, we reduce the failure rate to 0.1, thus, the hedging points of three part types decrease obviously. The reason is that inventory level should drop as long as the reliability of the machine is enhanced.

ii) Influences of the penalizing coefficients.

In Example 3, we change the penalizing coefficient of the first part type by increasing $C^+$ and decreasing $C^-$. The results show that hedging point of first part is smaller than the others. The result is coincident with our hypothesis.

iii) Influences of demand rate.

In Example 4, we reduce the demand rate of the third part type and the results show the corresponding hedging point also decreases.

iv) Influences of the part processing time.

In Example 5, we increase the processing time of the second part type. In contradiction with our hypothesis that the corresponding hedging point should increase because the capacity of the machine for this part is low, the hedging points of the second part types are not different from other types obviously. We can explain the results as below: as mentioned in Section 3.1, when we decide which part type should be produced we think the difference of the part type between its surplus and hedging point is the biggest. The longer process time the higher chosen probability. So the actual production rates of the part rypes with longer process region are not lower than others. Therefore the corresponding hedging points is not higher.

## 4 Conclusion

In this paper, we apply GAs to get the approximation solution of hedging points for a multi-part-type one-machine system since the analytic results can not be achieved. The advantage of the method is that the complexity will not increase when the dimension of a system increase except the longer time of calculation. The disadvantage is that the way can only be used in one-machine system. How to expand it to multi-machine system is one of our future concerns with the field under discussion.

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