Pseudo Separate-Bias Estimation of Nonlinear Systems with Colored Noise *

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Abstract: This paper presents a separate-bias estimation algorithm for a class of nonlinear time-varying stochastic systems with colored noise, the bias may be nonlinear, random and time-varying with some unknown changing law. Compared with the state augmentation technique for the state and parameter estimation of a class of nonlinear systems, the proposed algorithm is greatly improved in the real-time tracking ability.

Key words: nonlinear system; time-varying; state estimation; separate-bias estimation; colored-noise

有色噪声干扰下的非线性系统的伪偏差分离估计

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摘要: 本文给出了有色噪声干扰下的一类非线性时变随机系统的伪偏差分离估计方法. 偏差允许是非线性的, 随机的和时变的, 并且时变规律是未知的. 与基于扩展状态变量的一类非线性系统的状态估计方法相比, 本文方法在实时跟踪性方面得到了很大提高.

关键词: 非线性系统; 时变; 状态估计; 偏差分离估计; 有色噪声

1 Introduction

State augmentation technique is usually adopted to estimate parameters and states of nonlinear systems by the use of the extended Kalman filter (EKF). Unfortunately, the EKF has been proved to be only valid for time-invariant parameters, and generally the parameter estimates have constant bias. In a recent work[1], a strong tracking filter (STF) is proposed and it can provide state and consistent parameter estimation of a class of nonlinear systems with colored noise, wherein state augmentation technique is also adopted, and the parameters can be randomly time-varying with unknown changing law.

Since Friedland[2] proposed the famous separate-bias estimation algorithm for linear systems with time-invariant bias, dozens of papers have been published on the subject for the extension of this technique to other classes of linear or nonlinear systems[3~5]. More recently, an extension of the Friedland's separate-bias algorithm to randomly time-varying bias of a class of nonlinear systems is reported in [6], where the nonlinear systems are time-varying with zero-mean, Gaussian white noise.

The aim of this paper is to give an equivalent separate-bias estimation algorithm of the STF algorithm proposed in [1] so as to make the STF algorithm more practical and easy to be implemented. Since the colored process noise can be transformed into white noise by adopting the state augmentation method, only colored measurement noise is considered in this paper.

The present work is based on the former work in [6]. The difference between [6] and the present paper is that two further extensions have been carried out in the latter. First, the system considered in this paper is a nonlinear time-varying stochastic system with none zero-mean colored noise, while in paper [6] only zero-mean, Gaussian white noise was considered. Second, the bias in this paper is a real nonlinearity entering the process and the measurement equation.

2 Problem formulation

A class of nonlinear time-varying stochastic systems Ⅰ are described by the following model:

* Supported by National Natural Science Foundation (69774022), Education Ministry and 863 plan of China.
Manuscript received May 4, 1998, revised May 31, 1999.

$$x(k+1)=f(x(k),b(k),u(k),k)+\Gamma(x(k),k)v(k), \tag{1}$$

$$y(k+1)=h(x(k+1),b(k+1),k+1)+e(k+1), \tag{2}$$

$$e(k+1)=F(k+1,k)e(k)+G(k+1,k)\xi(k), \tag{3}$$

where state $x \in \mathbb{R}^n$; bias $b \in \mathbb{R}^p$; input $u \in \mathbb{R}^q$; output $y \in \mathbb{R}^m$; f, h are nonlinear functions, and are assumed to have continuous derivative with respect to the state x and the bias b. The process noise $v \in \mathbb{R}^r$ is a Gaussian white noise; $e \in \mathbb{R}^m$ is a colored measurement noise; $\xi \in \mathbb{R}^r$ is a Gaussian white noise. The following statistics are assumed to be known:

$$\begin{cases} Ev(k) = m_v(k); & \mathrm{var}\{v(k)\} = Q(k), \\ E\xi(k) = m_\xi(k); & \mathrm{var}\{\xi(k)\} = S(k). \end{cases} \tag{4}$$

$x(0)$ is a Gaussian white noise with the statistics:

$$Ex(0) = x_0; \quad \mathrm{var}\{x(0)\} = P_0. \tag{5}$$

$x(0), v(k)$ and $\xi(k)$ are mutually statistically independent. The matrices $\Gamma(\cdot)$, $F(\cdot)$ and $G(\cdot)$ have proper dimensions. We assume that the bias $b(k)$ is unknown, it may be nonlinear (with respect to time), random and time-varying and also system Ⅰ have some model uncertainties.

The present objective is to obtain the pseudo separate-bias estimation algorithm of system Ⅰ [2,6]. In the sequel, we introduce a lemma, which will be used in the next section.

Consider the following dynamic system Ⅱ:

$$x(k+1) = A_1(k)x(k) + B_1(k)b(k) + t_1(k) + \Gamma_1(k)v_1(k), \tag{6}$$

$$y(k+1)=H_1(k+1)x(k+1) + D_1(k+1)b(k+1) + z_1(k+1) + e_1(k+1), \tag{7}$$

where x, b, y are defined the same as in system Ⅰ; A_1, B_1, Γ_1, H_1, D_1 are matrices with proper dimensions; t_1 and z_1 are known vectors. The process noise $v_1 \in \mathbb{R}^r$ is a Gaussian white noise; $e_1 \in \mathbb{R}^m$ is the measurement Gaussian white noise; $v_1(k)$ and $e_1(k)$ are correlated, and have the following statistics:

$$\begin{cases} Ev_1(k)=m_{v_1}(k), \ \mathrm{cov}\{v_1(k),v_1(j)\}=Q_1(k)\delta_{k,j}, \\ Ee_1(k)=m_{e_1}(k), \ \mathrm{cov}\{e_1(k),e_1(j)\}=R_1(k)\delta_{k,j}, \\ \mathrm{cov}\{v_1(k),e_1(j)\} = S_1(k)\delta_{k,j}. \end{cases} \tag{8}$$

Lemma 2.1[7] With a matrix defined by

$$J_1(k) = \Gamma_1(k)S_1(k)R_1^{-1}(k),$$

system Ⅱ can be equivalently transformed to be:

$$x(k+1)=A_1^*x(k)+B_1^*(k)b(k)+t_1^*(k)+v_1^*(k), \tag{9}$$

$$y(k+1)=H_1(k+1)x(k+1)+D_1(k+1)b(k+1) + z_1^*(k+1) + e_1^*(k+1), \tag{10}$$

where

$$A_1^*(k) = A_1(k) - J_1(k)H_1(k), \tag{11}$$

$$B_1^*(k) = B_1(k) - J_1(k)D_1(k), \tag{12}$$

$$t_1^*(k) = t_1(k) + \Gamma_1(k)m_{v_1}(k) + J_1(k)\cdot [y(k) - z_1(k) - m_{e_1}(k)], \tag{13}$$

$$v_1^*(k) = \Gamma_1(k)[v_1(k) - m_{v_1}(k)] - J_1(k)[e_1(k) - m_{e_1}(k)], \tag{14}$$

$$z_1^*(k+1) = z_1(k+1) + m_{e_1}(k+1), \tag{15}$$

$$e_1^*(k+1) = e_1(k+1) - m_{e_1}(k+1). \tag{16}$$

Now, v_1^* and e_1^* are uncorrelated, zero-mean, Gaussian white noise, i.e.

$$Ev_1^*(k) = 0, \tag{17}$$

$$Ee_1^*(k) = 0, \tag{18}$$

$$\mathrm{cov}\{v_1^*(k),v_1^*(j)\} = [\Gamma_1(k)Q_1(k)\Gamma_1^{\mathrm{T}}(k) - J_1(k)R_1(k)J_1^{\mathrm{T}}(k)]\delta_{k,j} \overset{\text{def}}{=} Q_1^*(k)\delta_{k,j}, \tag{19}$$

$$\mathrm{cov}\{e_1^*(k),e_1^*(j)\} = R_1(k)\delta_{k,j}, \tag{20}$$

$$\mathrm{cov}\{v_1^*(k),e_1^*(j)\} = 0. \tag{21}$$

3 Main results

Expanding $f(\cdot)$ into Taylor series at the state and bias filtering estimates $\hat{x}(k \mid k)$ and $\hat{b}(k \mid k)$, and only retaining the linear terms, (1) is transformed into:

$$x(k+1)=A(k)x(k)+B(k)b(k)+t(k)+\Gamma(k)v(k), \tag{22}$$

with

$$A(k) = \left.\frac{\partial f(x(k),b(k),u(k),k)}{\partial x}\right|_{\hat{x}(k|k),\hat{b}(k|k)}, \tag{23}$$

$$B(k) = \left.\frac{\partial f(x(k),b(k),u(k),k)}{\partial b}\right|_{\hat{x}(k|k),\hat{b}(k|k)}, \tag{24}$$

$$t(k) = f(\hat{x}(k \mid k),\hat{b}(k \mid k),u(k),k) - A(k)\hat{x}(k \mid k) - B(k)\hat{b}(k \mid k), \tag{25}$$

$$\Gamma(k) = \Gamma(\hat{x}(k \mid k),k). \tag{26}$$

Similarly (2) is transformed into:

$$y(k+1) = H(k+1)x(k+1) + D(k+1)\cdot b(k+1) + d(k+1) + e(k+1), \tag{27}$$

with

$$H(k+1) = \left.\frac{\partial h(x(k+1),b(k+1),u(k+1),k+1)}{\partial x}\right|_{\hat{x}(k|k),\hat{b}(k|k)}, \tag{28}$$

$$D(k+1) = \left.\frac{\partial h(x(k+1),b(k+1),u(k+1),k+1)}{\partial b}\right|_{\hat{x}(k|k),\hat{b}(k|k)}, \tag{29}$$

$$d(k+1) = h(\hat{x}(k\mid k),\hat{b}(k\mid k),k+1) - H(k+1)\hat{x}(k\mid k) - D(k+1)\hat{b}(k\mid k). \tag{30}$$

Define an auxiliary output as:

$$z(k) \overset{\text{def}}{=} y(k+1) - F(k+1,k)y(k), \tag{31}$$

and set up a rough bias equation as:

$$b(k+1) = b(k). \tag{32}$$

Substitute (27) into (31), and with the aid of (3), (22) and (32), it yields:

$$z(k) = h'(x(k),b(k),k) + e^*(k), \tag{33}$$

with

$$h'(x(k),b(k),k) \overset{\text{def}}{=} H^*(k)x(k) + D^*(k)b(k) + d^*(k), \tag{34}$$

$$H^*(k) = H(k+1)A(k) - F(k+1,k)H(k), \tag{35}$$

$$D^*(k) = H(k+1)B(k) - F(k+1,k)D(k) + D(k+1), \tag{36}$$

$$d^*(k) = H(k+1)t(k) + d(k+1) - F(k+1,k)d(k), \tag{37}$$

$$e^*(k) = H(k+1)\Gamma(k)v(k) + G(k+1,k)\xi(k), \tag{38}$$

From (38) and (4) we have:

$$Ee^*(k) = H(k+1)\Gamma(k)m_v(k) + G(k+1,k)m_\xi(k) \overset{\text{def}}{=} m_{e^*}(k), \tag{39}$$

$$\operatorname{var}\{e^*(k)\} = H(k+1)\Gamma(k)Q(k)\Gamma^{\mathrm{T}}(k)\cdot H^{\mathrm{T}}(k+1) + G(k+1,k)S(k)\cdot G^{\mathrm{T}}(k+1,k) \overset{\text{def}}{=} R(k), \tag{40}$$

$$\operatorname{cov}\{v(k),e^*(j)\} = [Q(k)\Gamma^{\mathrm{T}}(k)H^{\mathrm{T}}(k+1)]\delta_{k,j} \overset{\text{def}}{=} S^*(k)\delta_{k,j}. \tag{41}$$

Equation (22) and (33) and their accompanying equations constitute an approximate description of system Ⅰ, and only correlated noises exist in the new description.

Define:

$$\Omega(k) \overset{\text{def}}{=} \{z(1),z(2),\cdots,z(k)\}. \tag{42}$$

Let $P_0(k\mid k)$ and $P_b(k\mid k)$ be the covariance matrices of the bias-free state estimate and the bias estimate of system Ⅰ respectively[2]. Let $\hat{x}(k\mid k)$ be the state estimate of system Ⅰ in the presence of bias, $\hat{b}(k\mid k)$ be the bias estimate. Obviously we have:

$$\begin{cases} \hat{x}(k\mid \Omega(k-1)) = \hat{x}(k\mid k); \\ \hat{x}(k\mid \Omega(k)) = \hat{x}(k\mid k+1), \\ P_0(k\mid \Omega(k-1)) = P_0(k\mid k); \\ P_0(k\mid \Omega(k)) = P_0(k\mid k+1), \\ P_b(k\mid \Omega(k-1)) = P_b(k\mid k); \\ P_b(k\mid \Omega(k)) = P_b(k\mid k+1). \end{cases} \tag{43}$$

From (32) it leads to:

$$P_b(k\mid k+1) = P_b(k+1\mid k+1). \tag{44}$$

Based on the approximate model (22), (32) and (33) of system Ⅰ, and with the aid of Lemma 2.1, the results in [6] can be directly applied. Through a tedious deduction and by using (43) and (44), we finally obtain a pseudo separate-bias estimation algorithm as follows:

Algorithm 3.1 (Pseudo separate-bias estimation algorithm)

$$\begin{aligned} \hat{x}(k\mid k+1) = {} & f(\hat{x}(k-1\mid k),\hat{b}(k-1\mid k),u(k-1),k-1) + \\ & \Gamma(k-1)m_v(k-1) + \\ & J(k-1)[y(k) - F(k,k-1)y(k-1) - \\ & h'(\hat{x}(k-1\mid k),\hat{b}(k-1\mid k),k-1) - \\ & m_{e^*}(k-1)] + [K_0(k) + V(k)K_b(k)]\gamma(k), \end{aligned} \tag{45}$$

$$\hat{b}(k\mid k+1) = \hat{b}(k\mid k) + K_b(k)\gamma(k), \tag{46}$$

with

$$K_0(k) = P_0(k\mid k)(H^*(k))^{\mathrm{T}}\cdot [H^*(k)P_0(k\mid k)(H^*(k))^{\mathrm{T}} + R(k)]^{-1}, \tag{47}$$

$$V(k) = [I - K_0(k)H^*(k)]U(k-1) - K_0(k)D^*(k), \tag{48}$$

$$K_b(k) = P_b(k+1\mid k+1)[H^*(k)V(k) + D^*(k)]^{\mathrm{T}}R^{-1}(k), \tag{49}$$

$$P_b^{-1}(k+1 \mid k+1) = [\lambda(k)P_b(k \mid k)]^{-1} + C^{\mathrm{T}}(k) \cdot [H^*(k)P_0(k \mid k)(H^*(k))^{\mathrm{T}} + R(k)]^{-1}C(k), \tag{50}$$

$$\lambda(k) = \begin{cases} 1, & \text{if} \quad k = 0, \\ 1, & \text{if} \quad \lambda_0 < 1, \quad k \geqslant 1, \\ \lambda_0, & \text{if} \quad \lambda_0 \geqslant 1, \quad k \geqslant 1, \end{cases} \tag{51}$$

$$\lambda_0 = \frac{\mathrm{tr}[N(k)]}{\mathrm{tr}[M(k)]}, \tag{52}$$

$$N(k) = V_0(k) - R(k), \tag{53}$$

$$M(k) = D^*(k)P_b(k \mid k)(D^*(k))^{\mathrm{T}}, \tag{54}$$

$$V_0(k) = \begin{cases} \gamma(1)\gamma^{\mathrm{T}}(1), & \text{when} \quad k = 1, \\ \dfrac{[\rho_0 V_0(k-1) + \gamma(k)\gamma^{\mathrm{T}}(k)]}{1+\rho}, & \text{when} \quad k \geqslant 2, \end{cases} \tag{55}$$

$$C(k) = H^*(k)U(k-1) + D^*(k), \tag{56}$$

$$\gamma(k) = y(k+1) - F(k+1,k)y(k) - h'(\hat{x}(k \mid k),\hat{b}(k \mid k),k) - m_{e^*}(k), \tag{57}$$

$$Q^*(k) = \Gamma(k)Q(k)\Gamma^{\mathrm{T}}(k) - J(k)R(k)J^{\mathrm{T}}(k), \tag{58}$$

$$J(k) = \Gamma(k)S^*(k)R^{-1}(k), \tag{59}$$

$$A^*(k) = A(k) - J(k)H^*(k), \tag{60}$$

$$B^*(k) = B(k) - J(k)D^*(k), \tag{61}$$

$$\hat{x}(k+1 \mid k+1) = A^*(k)\hat{x}(k \mid k+1) + B^*(k)\hat{b}(k \mid k+1) + t^*(k). \tag{62}$$

$$\hat{b}(k+1 \mid k+1) = \hat{b}(k \mid k+1), \tag{63}$$

$$t^*(k) = t(k) + \Gamma(k)m_v(k) + J(k)[y(k+1) - F(k+1,k)y(k) - d^*(k) - m_{e^*}(k)], \tag{64}$$

$$U(k) = A^*(k)V(k) + B^*(k), \tag{65}$$

$$P_0(k \mid k+1) = [I - K_0(k)H^*(k)]P_0(k \mid k), \tag{66}$$

$$P_0(k+1 \mid k+1) = A^*(k)P_0(k \mid k+1)(A^*(k))^{\mathrm{T}} + Q^*(k). \tag{67}$$

The initial value

$$V(0) = 0. \tag{68}$$

In (50) $\lambda(k) \geqslant 1$ is a fading factor, its value is on-line adaptively determined by (51) ~ (55), which make it possible to estimate unknown time-varying bias[6]. In (55) $0 < \rho \leqslant 1$ is a forgetting factor, usually we select $\rho = 0.95$. It has been shown that ρ has minor influence on the whole algorithm because of the further adaptive regulation of the fading factor $\lambda(k)$ (see [6]).

Proposition 3.1

On the basis of Algorithm 3.1, we have the following equality:

$$f(\hat{x}(k-1 \mid k),\hat{b}(k-1 \mid k),u(k-1),k-1) + \Gamma(k-1)m_v(k-1) + J(k-1)[y(k) - F(k,k-1)y(k-1) - h'(\hat{x}(k-1 \mid k), \hat{b}(k-1 \mid k),k-1) - m_{e^*}(k-1)] \equiv \hat{x}(k \mid k). \tag{69}$$

The proof of this proposition is straightforward after substituting (45), (46) and (64) into (62) and through a simple deduction.

With the aid of (45), (69) is simplified into

$$\hat{x}(k \mid k+1) = \hat{x}(k \mid k) + [K_0(k) + V(k)K_b(k)]\gamma(k). \tag{70}$$

4 Conclusion

An efficient pseudo separate-bias estimation algorithm for a class of nonlinear systems with colored noise has been proposed in this paper. One of the main applications of the proposed algorithm may be in parameter adaptive control of stochastic systems with colored noise, where the system parameters can be represented in the form of "bias" and estimated with the states simultaneously. The proposed algorithm can also be applied to the field of parameter estimation based fault detection and diagnostics of closed-loop nonlinear systems[8,9].

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