Decentralized Robust Stabilizing Control Design for Interconnected Time-Varying Uncertain Systems

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Abstract: For a class of interconnected uncertain systems, which are time-varying and assumed to satisfy the matching conditions, a sufficient condition for decentralized stabilization feedback control laws is presented. This condition is offered as the solvability problem of linear matrix inequalities (LMIs). Based on that, a convex optimization problem with linear matrix inequality (LMI) constraint is formulated to design a decentralized state feedback control with smaller gain parameters which enables the closed-loop system to be asymptotically stable.

Key words: uncertainty; interconnected systems; LMI; decentralized control; robustness

时变不确定性关联系统的分散鲁棒稳定控制器设计

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摘要: 对一类满足匹配条件的时变不确定性关联大系统,给出了其可分散状态反馈镇定的充分条件即一组线性矩阵不等式(LMI)有解. 在此基础上,通过求解一凸优化问题,指出了具有较小反馈增益的分散稳定化状态反馈控制律的设计方法. 文中的仿真示例说明了该方法的有效性和优越性.

关键词: 不确定性; 关联系统; LMI; 分散控制; 鲁棒性

1 Introduction

In recent years, decentralized control for interconnected large systems has been attracted much attention[1]. Since the models often contain uncertainties, expected performance can not be obtained if the controller designed is only based on nominal model. So the robust stabilization for uncertain interconnected large systems has given rise to a lot of attention[2~7]. However, in the above papers, the sufficient conditions are given in Riccati equalities or inequalities, which require the preselection of many parameters. There exist complex computation and inconvenient implement.

LMI method has been paid great attention to for its high solvability and becomes an effective method for robust analysis and synthesis[8,9]. In this paper LMI method is used to study the decentralized stabilization for time-varying uncertain interconnected large systems, which overcomes the drawbacks of Riccati approach. There are few reports about drawbacks in recent papers. For a class of time-varying uncertain interconnected large systems, which satisfy the so-called matching condition, the sufficient condition for decentralized state-feedback stabilizability is given and a convex optimization method for designing controller with smaller gain is proposed too.

2 Problem description and lemmas

Consider a class of time-varying uncertain interconnected large systems L with N subsystems L_i. The subsystems can be described as follows

$$\begin{aligned} L_i: \dot{x}_i(t) = & [A_i + B_i\Delta A_i(r_i(t))]x_i(t) + \\ & [B_i + B_i\Delta B_i(s_i(t))]u_i(t) + \\ & \sum_{j=1, j\neq i}^{N} B_iH_{ij}(x_j(t), t), \\ & i = 1, 2, \cdots, N, \end{aligned} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ is the state vector, $u_i(t) \in \mathbb{R}^{m_i}$ is the control vector, A_i, B_i are nominal matrices with suitable dimension, $\Delta A_i(r_i(t)), \Delta B_i(s_i(t))$ are continuous uncertainties on $r_i(t), s_i(t)$ with the compatible dimensions as A_i and B_i, the form of uncertainties implies that they satisfy the matching conditions. H_{ij} is the interconnected matrix of the jth subsystem to the ith subsystem, the interconnection can be nonlinear.

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connected system which contains two subsystems where

$$L_1:\dot{x}_1(t)=\begin{bmatrix}-1 & 2\\ -1+r_1(t) & r_1(t)\end{bmatrix}x_1+\begin{bmatrix}0\\ 1+s_1(t)\end{bmatrix}u_1+$$

$$\begin{bmatrix}0\\1\end{bmatrix}[\sin x_{21} \quad \cos x_{21}][x_2],$$

$$L_2:\dot{x}_2(t)=\begin{bmatrix}-3 & 0\\ 2r_2(t) & -1+r_2(t)\end{bmatrix}x_2+\begin{bmatrix}0\\ 1+s_2(t)\end{bmatrix}u_2+$$

$$\begin{bmatrix}0\\1\end{bmatrix}[\sin x_{12} \quad \cos x_{12}][2x_1],$$

where

$$|r_1(t)|\leqslant 0.5, \quad |r_2(t)|\leqslant 1,$$

$$|s_1(t)|\leqslant 0.4, \quad |s_2(t)|\leqslant 0.3.$$

Using the method introduced above, solve the convex optimization problem in LMITOOL, the decentralized stabilizing control law obtained is

$$u_1(t)=[-0.6829 \quad -5.3526]x_1(t),$$

$$u_2(t)=[-2.0779 \quad -3.2677]x_2(t),$$

Remark 2 Although the LMIs have many parameters, it can be solved at one time in LMITOOL. It is very convenient and does not necessitate the adjustment of parameters. Thus it overcomes the drawbacks of Riccati equation approach.

5 Conclusion

According to the Lyapunov stability theorem, for the time-varying uncertain interconnected large system, its decentralized state-feedback stabilizing control can be solved by LMI. By solving the convex optimization problem with LMI constraint, the design of smaller feedback gain is derived. The example shows that this method needs no adjustment of parameters and is very convenient to use.

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