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Mixed Scalar l_1/H_2 Problem for Discrete Time Systems *

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Abstract: A discrete time l_1 control design problem involving a constraint on H_2 performance is formulated and an upper approximation method for the solution of this infinite dimensional optimization problem is introduced. Suboptimal solutions of the problem can be obtained by solving a sequence of truncated problems. The continuity property of the optimal value with respect to changes in the H_2 constraint is studied.

Key words: l_1 control; H_2 control; discrete time systems; continuity property

单变量离散混合 l_1/H_2 问题

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摘要: 本文描述了 H_2 性能约束下的离散控制系统 l_1 设计问题,并提出了该无穷维优化问题的一种上逼近解法,通过求解一系列的截断问题能够得到无穷维问题的次优解,混合 l_1/H_2 问题最优值对 H_2 约束连续依赖的性质也在本文得到研究.

关键词: l_1 控制; H_2 控制; 离散系统; 连续性

1 Introduction

For the Linear Shift Invariant (LSI) systems, the fundamental differences between H_2 control design and l_1 control design can be traced to the modeling and treatment of uncertain exogenous disturbances[1,2]. The object here is to consider the simultaneous treatment of both H_2 and l_1 performance criteria. The design of controllers to satisfy mixed performance criteria, such as mixed H_2/H_∞ control, mixed l_1/H_∞ control, mixed H_2/l_1 control has recently been the focus of researchers[3~7]. For SISO control systems, mixed l_1/H_2 problem of minimizing the l_1 norm of the closed loop map while maintaining its H_2 norm at a prescribed level was addressed and studied in [8]. This paper intends to consider the general mixed l_1/H_2 control problem, i.e. to minimize the l_1 norm of a closed loop transfer function, subject to an inequality constraint on the H_2 norm of another closed loop transfer function.

2 Problem formulation

Let $\mathbb{R}$ denote the field of real numbers, $\mathbb{R}^m$ denote the m-dimensional real vectors, C denote the field of complex numbers, Z_+ denote the nonnegative integers. A causal SISO LSI transfer function $\hat{G}$ can be described as $\hat{G} = G(0) + G(1)\lambda + G(2)\lambda^2 + \cdots, \quad G(k) \in \mathbb{R}$. As $\hat{G}$ can be represented uniquely by its impulse response sequence

$$[G(0), G(1), G(2), \cdots]^T,$$

$\hat{G}$ and its impulse response sequence are not differentiated in notation through this paper. Define

$$l_e = \{\hat{G} \mid \hat{G} = G(0) + G(1)\lambda + \cdots, G(k) \in \mathbb{R}\},$$

$$l_2 = \{\hat{G} \in l_e \mid \sum_{k=0}^{\infty}(G(k))^2 < \infty\},$$

$$l_1 = \{\hat{G} \in l_e \mid \sum_{k=0}^{\infty} |G(k)| < \infty\},$$

$$Rl_e = \{\hat{G} \in l_e \mid \hat{G} \text{ is a rational function of } \lambda\},$$

$$Rl_1 = Rl_e = \cap\, l_1.$$

For any $\hat{G} \in l_1$, the l_1-norm of $\hat{G}$ is given by

$$\|\hat{G}\|_1 = \sum_{k=0}^{\infty} |G(k)|.$$

For any $\hat{G}_1, \hat{G}_2 \in l_2$, the inner product of $\hat{G}_1$ and $\hat{G}_2$ is given by

$$\langle \hat{G}_1, \hat{G}_2 \rangle = \sum_{k=0}^{\infty} G_1(k) G_2(k).$$

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Then l_2 is Hilbert space, and $\forall\ \hat{G} \in l_2$, the l_2-norm of $\hat{G}$ is

$$\| \hat{G} \|_2 = \sqrt{\langle \hat{G}, \hat{G} \rangle} = \sqrt{\sum_{k=0}^{\infty} (G(k))^2}.$$

$\| \hat{G} \|_2$ is also the H_2-norm of $\hat{G}$.

The mixed l_1 /H_2 control problem can be stated as: Given $\hat{T}_1, \hat{T}_2, \hat{V}_1, \hat{V}_2 \in Rl_1$ and a constant γ, find $\hat{Q} \in Rl_1$ such that $\| \hat{T}_1 - \hat{Q}\hat{V}_1 \|_1$ is minimized and $\| \hat{T}_2 - \hat{Q}\hat{V}_2 \|_2 \leqslant \gamma$. Define

$$\xi(\gamma) = \{ \hat{Q} \in Rl_1 \mid \| \hat{T}_2 - \hat{Q}\hat{V}_2 \|_2 \leqslant \gamma \}$$

and

$$\gamma_0 = \inf_{\hat{Q} \in Rl_1} \| \hat{T}_2 - \hat{Q}\hat{V}_2 \|_2,$$

obviously, $\xi(\gamma)$ is nonempty when $\gamma > \gamma_0$. The mixed l_1/H_2 problem is described as

$$\mu(\gamma) = \inf_{\hat{Q} \in \xi(\gamma)} \| \hat{T}_1 - \hat{Q}\hat{V}_1 \|_1. \qquad \text{(OPT)}$$

It is easy to see that $\mu(\gamma)$ is a decreasing function. Throughout this paper, there are the following assumptions:

a) $\gamma \in (\gamma_0, \infty)$.

b) $\hat{V}_1 = \lambda^m + V_1(m-1)\lambda^{m-1} + \cdots + V_1(0) = \prod_{i=1}^{m} (\lambda - \lambda_i) \in \mathbb{R}^{m+1}$.

This assumption is not restrictive and is made to streamline the presentation of the paper. For

$$\hat{V}_1 = \frac{\lambda^m + a_{m-1}\lambda^{m-1} + \cdots + a_0}{b_q\lambda^q + b_{q-1}\lambda^{q-1} + \cdots + b_0} \in Rl_1,$$

we have

$$\hat{Q}\hat{V}_1 = \hat{Q}'\hat{V}_1' = \left(\frac{\hat{Q}}{b_q\lambda^q + \cdots + b_0}\right)(\lambda^m + a_{m-1}\lambda^{m-1} + \cdots + a_0).$$

Obviously, the denominator part of $\hat{V}_1$ can be absorbed into $\hat{Q}$.

c) $\Lambda = \{\lambda_1, \cdots, \lambda_m\} \subset D$, where D is the open unit disk in C. When there is one element of $\{\lambda_1, \cdots, \lambda_m\}$ on the unit circle, we say the problem is singular. The singular problem is difficult to be solved and still an open problem now. This paper only deals with the non-singular problems.

d) $\hat{V}_2 = \lambda^n + V_2(n-1)\lambda^{n-1} + \cdots + V_2(0) \in \mathbb{R}^{n+1}$. Similar to Assumption b), the denominator part of $\hat{V}_2$ can be absorbed into $\hat{Q}$.

e) There exists a $\hat{Q}_0 \in Rl_1$ such that

$$\| \hat{T}_2 - \hat{Q}_0\hat{V}_2 \|_2 = \inf_{\hat{Q} \in Rl_1} \| \hat{T}_2 - \hat{Q}\hat{V}_2 \|_2 = \gamma_0.$$

3 Continuity

Let $M = \| \hat{T}_1 - \hat{Q}_0\hat{V}_1 \|_1 + 1$, where $\hat{Q}_0$ is defined in Assumption e). Obviously, $\hat{Q}_0 \in \xi(\gamma)$ and

$$\inf_{\hat{Q} \in \xi(\gamma)} \| \hat{T}_1 - \hat{Q}\hat{V}_1 \|_1 < M.$$

Define

$$\xi_1 = \{ \hat{Q} \in Rl_1 \mid \| \hat{T}_1 - \hat{Q}\hat{V}_1 \|_1 \leqslant M \}.$$

Obviously,

$$\inf_{\hat{Q} \in \xi(\gamma)} \| \hat{T}_1 - \hat{Q}\hat{V}_1 \|_1 = \inf_{\hat{Q} \in \xi(\gamma) \cap \xi_1} \| \hat{T}_1 - \hat{Q}\hat{V}_1 \|_1.$$

Define

$$\xi_2 = \left\{ \hat{Q} \in Rl_1 \,\middle|\, \| \hat{Q} \|_1 \leqslant \frac{\| \hat{T}_1 \|_1 + M}{\prod_{i=1}^{m} (1 - |\lambda_i|)} \right\}.$$

Proposition 3.1 $\xi_1 \subset \xi_2$.

Proof Space C_1 is defined as

$$\left\{ \hat{G} \,\middle|\, \hat{G} = \begin{bmatrix} G(0) \\ G(1) \\ \vdots \end{bmatrix}, \quad \sum_{k=0}^{\infty} |G(k)| < \infty, \quad G(k) \in C \right\}.$$

For any $\hat{G} \in C_1$, the C_1-norm of $\hat{G}$ can be given by $\| \hat{G} \|_1 = \sum_{k=0}^{\infty} |G(k)|$. Obviously, l_1 is a subset of C_1 and C_1-norm in l_1 space is exactly l_1-norm. With C_1-norm, for any $\hat{Q} \in \xi_1$, it follows that

$$\| \hat{T}_1 \|_1 + M \geqslant \| \hat{T}_1 \|_1 + \| \hat{T}_1 - \hat{Q}\hat{V}_1 \|_1 \geqslant$$

$$\| \hat{Q}\hat{V}_1 \|_1 \geqslant \| \prod_{i=1}^{m} (\lambda - \lambda_i)\hat{Q} \|_1 \geqslant$$

$$\| \lambda \prod_{i=2}^{m} (\lambda - \lambda_i)\hat{Q} \|_1 - \| -\lambda_1 \prod_{i=2}^{m} (\lambda - \lambda_i)\hat{Q} \|_1 \geqslant$$

$$(1 - |\lambda_1|) \| \prod_{i=2}^{m} (\lambda - \lambda_i)\hat{Q} \|_1 \geqslant \cdots \geqslant$$

$$\prod_{i=1}^{m} (1 - |\lambda_i|) \| \hat{Q} \|_1.$$

The above means that

$$\| \hat{Q} \|_1 \leqslant \frac{\| \hat{T}_1 \|_1 + M}{\prod_{i=1}^{m} (1 - |\lambda_i|)}. \qquad \text{Q.E.D.}$$

Proposition 3.2 $\inf_{\hat{Q} \in \xi(\gamma)} \| \hat{T}_1 - \hat{Q}\hat{V}_1 \|_1 = \inf_{\hat{Q} \in \xi(\gamma) \cap \xi_2} \| \hat{T}_1 - \hat{Q}\hat{V}_1 \|_1 = \mu(\gamma)$.

Proof Since $\xi(\gamma) \cap \xi_2 \subset \xi(\gamma)$,

$$\inf_{\hat{Q} \in \xi(\gamma)} \| \hat{T}_1 - \hat{Q}\hat{V}_1 \|_1 \leqslant \inf_{\hat{Q} \in \xi(\gamma) \cap \xi_2} \| \hat{T}_1 - \hat{Q}\hat{V}_1 \|_1.$$

On the other hand, from Proposition 3.1, we have

$$\inf_{\hat{Q} \in \xi(\gamma) \cap \xi_2} \| \hat{T}_1 - \hat{Q}\hat{V}_1 \|_1 \leqslant \inf_{\hat{Q} \in \xi(\gamma) \cap \xi_1} \| \hat{T}_1 - \hat{Q}\hat{V}_1 \|_1.$$

Hence

$$\inf_{\hat Q\in\xi(\gamma)}\|\hat T_1-\hat Q\hat V_1\|_1\geqslant$$
$$\inf_{\hat Q\in\xi(\gamma)\cap\xi_1}\|\hat T_1-\hat Q\hat V_1\|_1\geqslant$$
$$\inf_{\hat Q\in\xi(\gamma)\cap\xi_2}\|\hat T_1-\hat Q\hat V_1\|_1.$$

Thus

$$\inf_{\hat Q\in\xi(\gamma)}\|\hat T_1-\hat Q\hat V_1\|_1=\inf_{\hat Q\in\xi(\gamma)\cap\xi_2}\|\hat T_1-\hat Q\hat V_1\|_1.$$

Q.E.D.

Proposition 3.3 Given $\gamma_0<\gamma_1<\gamma_2<\infty$, then for any $\hat Q_2\in\xi(\gamma_2)\cap\xi_2$, there exists a $\hat Q_1\in\xi(\gamma_1)\cap\xi_2$ such that

$$\|\hat Q_1-\hat Q_2\|_1\leqslant 2\frac{\gamma_2-\gamma_1}{\gamma_2-\gamma_0}\frac{\|\hat T_1\|_1+M}{\prod_{i=1}^{m}(1-|\lambda_i|)}.$$

Proof From Assumption e and Proposition 3.1, we can conclude that there exists

$$\hat Q_0\in\xi(\gamma_1)\cap\xi_2\subset\xi(\gamma_2)\cap\xi_2.$$

Let $\rho=\frac{\gamma_2-\gamma_1}{\gamma_2-\gamma_0}$ and $\hat Q_1=\rho\hat Q_0+(1-\rho)\hat Q_2$.

Obviously $\rho\in(0,1)$ and $\hat Q_1\in\xi(\gamma_1)\cap\xi_2$. Next,

$$\|\hat Q_1-\hat Q_2\|_1\leqslant\|(\rho\hat Q_0+(1-\rho)\hat Q_2)-\hat Q_2\|_1\leqslant$$
$$\rho(\|\hat Q_0\|_1+\|\hat Q_2\|_1)\leqslant$$
$$2\frac{\gamma_2-\gamma_1}{\gamma_2-\gamma_0}\frac{\|\hat T_1\|_1+M}{\prod_{i=1}^{m}(1-|\lambda_i|)}.$$

Q.E.D.

Proposition 3.4 Given $\gamma_0<\gamma_1<\gamma_2<\infty$ and

$$\mu(\gamma_1)=\inf_{\hat Q\in\xi(\gamma_1)\cap\xi_2}\|\hat T_1-\hat Q\hat V_1\|_1,$$
$$\mu(\gamma_2)=\inf_{\hat Q\in\xi(\gamma_2)\cap\xi_2}\|\hat T_1-\hat Q\hat V_1\|_1,$$

then

$$H\frac{\gamma_2-\gamma_1}{\gamma_2-\gamma_0}\geqslant\mu(\gamma_1)-\mu(\gamma_2)\geqslant 0,$$

where

$$H=2\frac{\|\hat V_1\|_1(\|\hat T_1\|_1+M)}{\prod_{i=1}^{m}(1-|\lambda_i|)}.$$

Proof $\mu(\gamma_1)-\mu(\gamma_2)\geqslant 0$ is the direct result of the fact

$$\xi(\gamma_2)\cap\xi_2\supset\xi(\gamma_1)\cap\xi_2.$$

From Proposition 3.3, $\forall\hat Q_2\in\xi(\gamma_2)\cap\xi_2$, there exists a $\hat Q_1\in\xi(\gamma_1)\cap\xi_2$ such that

$$\|\hat T_1-\hat Q_2\hat V_1\|_1\geqslant$$
$$\|\hat T_1-\hat Q_1\hat V_1\|_1-\|\hat Q_2-\hat Q_1\|_1\|\hat V_1\|_1\geqslant$$
$$\mu(\gamma_1)-H\frac{\gamma_2-\gamma_1}{\gamma_2-\gamma_0}.$$

So

$$\inf_{\hat Q_2\in\xi(\gamma_2)\cap\xi_2}\|\hat T_1-\hat Q_2\hat V_1\|_1\geqslant\mu(\gamma_1)-H\frac{\gamma_2-\gamma_1}{\gamma_2-\gamma_0},$$

which means

$$H\frac{\gamma_2-\gamma_1}{\gamma_2-\gamma_0}\geqslant\mu(\gamma_1)-\mu(\gamma_2).$$

Q.E.D.

Proposition 3.5 $\mu(\gamma)$ is a continuous function.

Proof $\forall\varepsilon>0$ and $\forall\gamma\in(\gamma_0,\infty)$, Since $\lim_{\delta\to 0}H\frac{\delta}{\gamma-\gamma_0}=0$ and since $\lim_{\delta\to 0}H\frac{\delta}{\gamma+\delta-\gamma_0}=0$, there must exist a $\delta^*>0$ which satisfies

$$0<H\frac{\delta^*}{\gamma-\gamma_0}<\varepsilon,$$
$$0<H\frac{\delta^*}{\gamma+\delta^*-\gamma_0}<\varepsilon,$$
$$\delta^*<\gamma-\gamma_0.$$

From Proposition 3.4, the above implies $\forall\gamma_1\in(\gamma-\delta^*,\gamma]$,

$$0\leqslant\mu(\gamma_1)-\mu(\gamma)\leqslant\mu(\gamma-\delta^*)-\mu(\gamma)<\varepsilon$$

and

$$\forall\gamma_1\in(\gamma,\gamma+\delta^*),$$
$$0\leqslant\mu(\gamma)-\mu(\gamma_1)\leqslant\mu(\gamma)-\mu(\gamma+\delta^*)<\varepsilon.$$

Hence

$$\forall\gamma_1\in(\gamma-\delta^*,\gamma+\delta^*),$$
$$|\mu(\gamma_1)-\mu(\gamma)|<\varepsilon.$$

Q.E.D.

4 Approximate analysis

$$\forall N\in Z_+\geqslant 0,$$

define

$$\xi_N(\gamma)=\{\hat Q\mid\|\hat T_2-\hat Q\hat V_2\|_2\leqslant\gamma,\hat Q\in\mathbb{R}^{N+1}\}.$$

Since $\xi(\gamma)$ is nonempty, it is easy to see the following:

Proposition 4.1 $\forall\gamma\in(\gamma_0,\infty)$, $\exists N_0\in Z_+$ such that for any $N\geqslant N_0$ in Z_+, $\xi_N(\gamma)$ is nonempty.

For $N\geqslant N_0$ in Z_+, the Nth truncated problem of (OPT) can be constructed as

$$\mu_N(\gamma)=\inf_{\hat Q\in\xi_N(\gamma)}\|\hat T_1-\hat Q\hat V_1\|_1. \quad \text{(OPTN)}$$

(OPTN) can be posed as

$$\mu_N(\gamma)=\inf[\sum_{k=0}^{m+N}(\Psi_+(k)+\Psi_-(k))+\beta],$$

subject to

$$\Phi(j)=T_2(j)-\sum_{i=0}^{N}Q(i)V_2(j-i),$$

$$\Psi_+(k) - \Psi_-(k) = T_1(k) - \sum_{i=0}^{N} Q(i)V_1(k-i)$$

$$\sum_{j=0}^{n+N}(\Phi(j))^2 \leqslant \gamma^2 - \alpha,$$

$$\Psi_+(k) \geqslant 0, \quad \Psi_-(k) \geqslant 0,$$

where

$$k \in \{0,\cdots,m+N\}, \quad j \in \{0,\cdots,n+N\},$$

$$\alpha = \sum_{j=n+N+1}^{\infty}(T_2(j))^2, \quad \beta = \sum_{k=m+N+1}^{\infty}|T_1(k)|.$$

The above is a finite dimensional optimization problem which can be solved with many numerical optimization techniques.

Proposition 4.2 $\mu_{N_0}(\gamma) \geqslant \mu_{N_0+1}(\gamma) \geqslant \mu_{N_0+2}(\gamma) \geqslant \cdots$ and

$$\lim_{N\to\infty}\mu_N(\gamma) = \mu(\gamma).$$

Proof $\mu_{N_0}(\gamma) \geqslant \mu_{N_0+1}(\gamma) \geqslant \mu_{N_0+2}(\gamma) \geqslant \cdots$ is the direct result of

$$\xi_{N_0}(\gamma) \subset \xi_{N_0+1}(\gamma) \subset \xi_{N_0+2}(\gamma) \subset \cdots.$$

Since $\forall N \geqslant N_0$ in Z_+, $\xi_N(\gamma) \subset \xi(\gamma)$, it follows that $\mu_N(\gamma) \geqslant \mu(\gamma)$. Applying Proposition 3.5, $\forall \varepsilon > 0$, there exists a $\delta > 0$ such that

$$\mu(\gamma) + \frac{\varepsilon}{3} > \mu(\gamma - \delta) \geqslant \mu(\gamma).$$

Since $\mu(\gamma-\delta) = \inf_{\hat{Q}\in\xi(\gamma-\delta)} \|\hat{T}_1 - \hat{Q}\hat{V}_1\|_1$, one can find $\hat{Q}' \in Rl_1$ such that $\|\hat{T}_2 - \hat{Q}'\hat{V}_2\|_2 \leqslant \gamma - \delta$ and $\mu(\gamma-\delta) \leqslant \|\hat{T}_1 - \hat{Q}'\hat{V}_1\|_1 < \mu(\gamma-\delta) + \frac{\varepsilon}{3}$.

Choose positive integer N such that

$$\|\hat{Q}''\|_1 < \min\left(\frac{\delta}{\|\hat{V}_2\|_1}, \frac{\varepsilon}{\|3\hat{V}_1\|_1}\right),$$

where

$$\hat{Q}'' = \hat{Q}'(N+1)\lambda^{N+1} + \hat{Q}'(N+2)\lambda^{N+2} + \cdots.$$

Then

$$\|\hat{T}_1 - (\hat{Q}' - \hat{Q}'')\hat{V}_1\|_1 \leqslant \|\hat{T}_1 - \hat{Q}'\hat{V}_1\|_1 + \|\hat{Q}''\|_1\|\hat{V}_1\|_1 < \mu(\gamma) + \varepsilon$$

and

$$\|\hat{T}_2 - (\hat{Q}' - \hat{Q}'')\hat{V}_2\|_2 \leqslant \|\hat{T}_2 - \hat{Q}'\hat{V}_2\|_2 + \|\hat{Q}''\hat{V}_2\|_1 \leqslant \gamma.$$

Notice that $\hat{Q}' - \hat{Q}'' \in \xi^{N+1}$. Hence

$$\mu(\gamma) + \varepsilon > \|\hat{T}_1 - (\hat{Q}' - \hat{Q}'')\hat{V}_1\|_1 \geqslant \mu_N(\gamma) \geqslant \mu_{N+1}(\gamma) \geqslant \cdots \geqslant \mu(\gamma),$$

which means $\lim_{N\to\infty}\mu_N(\gamma) = \mu(\gamma)$. Q.E.D.

5 An example

In this section we illustrate the theory developed in the previous sections with an example: Given $\hat{T}_1 = 10\lambda + 5\lambda^2$, $\hat{T}_2 = 10 + 5\lambda$, $\hat{V}_1 = 4 + 4\lambda + \lambda^2$, $\hat{V}_2 = 2 + \lambda$ and $\gamma = 8$, find $\hat{Q} \in Rl_1$ such that $\|\hat{T}_1 - \hat{Q}\hat{V}_1\|_1$ is minimized and $\|\hat{T}_2 - \hat{Q}\hat{V}_2\|_2 \leqslant \gamma$. Table 1 summarizes our calculation results. From Table 1, we can see $\mu_0 \geqslant \mu_1 \geqslant \mu_2 \geqslant \cdots$ and μ_N converges to 4.1834 with the increasing of N as Proposition 4.2 shows.

Table 1 Calculation results with different N

N	0	1	2	3	4
μ_N	12.5000	7.2255	4.9723	4.3407	4.3407
N	5	6	7	8	9
μ_N	4.3407	4.3407	4.2614	4.2384	4.2131
N	10	11	12	13	14
μ_N	4.2055	4.1933	4.1909	4.1865	4.1852
N	15	16	17	18	19
μ_N	4.1843	4.1840	4.1836	4.1835	4.1835
N	20	21	22	23	24
μ_N	4.1834	4.1834	4.1834	4.1834	4.1834

References

1 Shu H and Chen T. State space approach to discrete time H_2 optimal control with a causality constraint. System and Control Letter, 1995, 26(1): 69–73

2 Dahleh M A and Diaz-Bobillo I J. Control of uncertain systems: a linear programming approach. New Jersey: Prentice Hall, 1995

3 Kaminer I, Khargonekar P P and Rotea M A. Mixed H_2/H_∞ control for discrete time systems via convex optimization. Automatica, 1993, 29(1): 57–70

4 Sznaier M and Bu J. On the properties of the solutions to mixed l_1/H_∞ control problems. Proceedings of 13th IFAC Congress, San Francisco, USA, 1996, Vol. G: 249–254

5 Voulgaris P. Optimal H_2/l_1 control: the SISO case. IEEE Proc. Decision and Control, Orlando, USA, 1994, 4: 3181–3186

6 Voulgaris P. Optimal H_2/l_1 control via duality theory. IEEE Trans. Automat. Contr., 1995, 40(11): 1881–1888

7 Wu J and Chu J. Mixed H_2/l_1 control for discrete time systems. Proceedings of 13th IFAC Congress, San Francisco, USA, 1996, Vol. G: 453–457

8 Salapaka M V, Dahleh M and Voulgaris P. Mixed objective control synthesis: optimal l_1/H_2 control. Proc. Amer. Contr. Conf., Seattle, USA, 1995, 1438–1442

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