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An Extended Popov Criterion for a Class of Uncertain Systems

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Abstract: An extended Popov criterion will be presented for a class of nonlinear uncertain systems in which linear parts are the convex combination of vertex models, and the nonlinear functions lie within a sector. The criterion says that the uncertain system is absolutely stable in some sector if there exists a common Popov line such that the modified Nyquist loci of all vertex systems lie within the right of the line.

Key words: nonlinear systems; absolute stability

1 Introduction

In recent years, considerable attention has been focussed on the stability of uncertain systems. Since, in practical systems, uncertainty can not be avoided, and the property that the system remains stable must be preserved. Uncertainty is, in general, a result of modelling errors, changes in operating conditions, etc. A so-called uncertain system is, actually, a family of systems. The absolute stability or instability of the whole family of systems will be determined by the absolute stability or instability of all its member systems.

This paper will extend the notable Popov criterion to suite such an uncertain system that its linear part is a convex combination of finite vertex models, and the nonlinear part is as the one in the absolute stability problem. An extended Popov criterion will be presented which says that the whole is absolutely stable if all its vertex systems satisfy a common frequency-domain condition. Since this criterion deals only with the vertex systems, a significant reduction in computational complexity results. The motivation for this paper stems from the paper written by Zhao and Barmish[1] and the works of Willems[2].

2 Problem Formulation and Notations

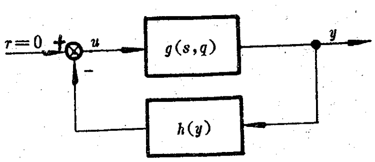

Fig. 1 Nonlinear Systems

Consider the nonlinear closed-loop system shown in Fig. 1, where $g(s,q)$, the transfer function of the linear part, contains an uncertain parameter vector $q\in Q$, and $h(\cdot)$ is a nonlinear feedback function. If $g(s,q)$ depends affine linearly on q, and Q is a polytope in parameter space, then $g(s,q)$ can be written as the following convex combination

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$$G: = \{g(s,q) = q_1g_1(s) + \cdots + q_rg_r(s): 0 \leqslant q_i \leqslant 1, q_1 + \cdots + q_r = 1\} \tag{1}$$

where $g_i(s)$, $i=1,\cdots,r$ are given and called vertex transfer functions of G. Let $h(\cdot)$ be any nonlinear function belonging to the sector

$$H[0,k): = \{h(\cdot): h(0) = 0, \quad 0 \leqslant h(y)y < ky^2\} \tag{2a}$$

or

$$H(0,k): = \{h(\cdot): h(0) = 0, \quad 0 < h(y)y < ky^2\} \tag{2b}$$

where k is a real constant or positive infinity. For simplicity, a pair $(q(s), h(y))$ denotes the nonlinear system shown in Fig. 1. Define

$$(g(s), H[0,k)): = \{(g(s), h(y)): h(\cdot) \in H[0,k)\}; \tag{3}$$

$$(G, H[0,k)): = \{(g(s), H[0,k)): g(\cdot) \in G\}. \tag{4}$$

Similarly, $(g(s), H(0,k))$ and $(G, H(0,k))$.

Definition 1 The system $(g(s), h(y))$ is called stable if its equilibrium point 0 is asymptotically stable in the large; $(g(s), H[0,k))$ is absolutely stable in the sector $[0,k)$ if $(g(s), h(y))$ is stable for any $h(y) \in H[0,k)$; The system family $(G, H[0,k)$ is absolutely stable in the sector $[0,k)$ if for any $g(s) \in G$, $(g(s), H[0,k))$ is absolutely stable in the sector $[0,k)$.

By using the nonlinear separate method, a large number of uncertain nonlinear systems can be represented by (4). How to test its absolute stablility? Does it suffice to apply Popov's criterion to all vertex systems $(g_i(s), H[0,k))$, $i=1,2,\cdots,r$? These questions will be answered in this paper.

An important class of $g_i(s)$ in practical systems is defined below.

Definition 2 A transfer function is said to belong to TRF_1 if 1) it is properly rational with no pole-zero cancellations in $\{s: \mathrm{Re}(s) \geqslant 0\}$, 2) all its poles lies within $\{s: s=0 \text{ or } \mathrm{Re}(s) < 0\}$, 3) the possible pole $s=0$ must be simple and have real positive residue. A transfer function is said to belong to TRF_0 if 1) it belongs to TRF_1, and has all its poles within $\{s: \mathrm{Re}(s) < 0\}$.

3 The Main Results

Before giving our main result, here is an useful lemma.

Lemma 1 The vertex models $g_i(s), i=1,\cdots,r$ of G belong to TRF_k, $k=0,1$, so does every member of G.

Proof Obviously for the case $k=0$. We need to prove it for the case $k=1$. By the set-theory, any $g(s) \in G$, apart from $\{g_i(s)\}$, can be represented as

$$g(s) = 0.5\{(g^*(s) + g^{**}(s)\}$$

for some $g^*(s)$, $g^{**}(s) \in G$. Without loss of generality, it suffices to prove that $g(s) \in TRF_1$ provided $g^*(s)$, $g^{**}(s) \in TRF_1$. There are three cases we should deal with:

i) $g^*(s)$, $g^{**}(s) \in TRF_0$. Clearly, $g(s) \in TRF_0$, futhermore, $g(s) \in TRF_1$.

ii) $g^*(s) \in TRF_0$, $g^{**}(s) \in TRF_1 \backslash TRF_0$. Obviously, $g(s)$ has the simple pole 0, too, and $\mathrm{res}\{g(s)\} = 0.5\,\mathrm{res}\{g^{**}(s)\} > 0$. No pole-zero cancellation arises at $s=0$ for $g(s)$, hence $g(s) \in TRF_1$.

iii) $g^*(s)$, $g^{**}(s) \in TRF_1 \backslash TRF_0$. $g(s)$ has the simple pole 0, too, and $\mathrm{res}\{g(s)\} = 0.5 \cdot (\mathrm{res}\{g^*(s)\} + \mathrm{res}\{g^{**}(s)\}) > 0$. No pole-zero cancellation arises at $s=0$ for $g(s)$, hence

$g(s) \in TRF_1$.

In a word, every member of G must belong to TRF_1. Q. E. D.

Theorem 1 Given vertex transfer functions $g_i(s) \in TRF_1$, $i=1,\cdots,r$, then the system family $(G, H(0,k))$ defined by (4) is absolutely stable in the sector $(0,k)$ if there exists a real number b such that

$$\mathrm{Re}\{(1+jb\mu)g_i(j\mu)+1/k\}>0 \tag{5}$$

for all $\mu>0$ and $i=1,\cdots,r$.

Proof Any $g(s) \in G$ can be written as

$$g(s)=q_1g_1(s)+\cdots+q_rg_r(s), \quad 0\leqslant q_i \leqslant 1, \quad q_1+\cdots+q_r=1.$$

Firstly, by lemma 1, $g(s)$ belongs to TRF_1; secondly,

$$\mathrm{Re}\{(1+jb\mu)g(j\mu)+1/k\}=\mathrm{Re}\{(1+jb\mu)\sum_{i=1}^{r}q_ig_i(j\mu)+1/k\}$$

$$=\sum_{i=1}^{r}q_i\mathrm{Re}\{(1+jb\mu)g_i(j\mu)+1/k\}>0$$

for all $\mu \geqslant 0$. By well-known Popov criterion (See [2]), the last inequility implies that $(g(s), H(0,k))$ is absolutely stable in the sector $(0,k)$. Because of arbitrariness of $g(s)$ hence the result. Q. E. D.

Theorem 2 Given vertex transfer functions $g_i(s) \in TRF_0$, $i=1,\cdots,r$, then the system family $(G, H[0,k))$ is absolutely stable in the sector $[0,k)$ if there exists a real number b such that the inequility (5) is true for all $\mu \geqslant 0$ and $i=1,\cdots,r$.

Proof By lemma 1, any $g(s)$ in G belongs to TRF_0. The remainder is similar to the proof of theorem 1, and also based on the classical popov criterion[2]. Q. E. D.

Remark i) We define a curve in the complex plane, called the modified Nyquist locus of $g_i(s)$, by means of

$$X_i(\mu):=\mathrm{Re}\{g_i(j\mu)\}, \tag{6a}$$

$$Y_i(\mu):=\mu\mathrm{Im}\{g_i(j\mu)\}. \tag{6b}$$

Substitution of (6) into (5) produces

$$X_i(\mu)-bY_i(\mu)+1/k \geqslant 0. \tag{7}$$

It follows that for (7) to be satisfied it must be possible to draw a straight line through the point $-1/k+j0$ with slope $1/b$, and for all loci of $X_i(\mu)+jY_i(\mu)$, $\mu \geqslant 0$, $i=1,\cdots,r$ has no point to the left of the line.

ii) Even if for every $g_i(s)$ there exists a real number b_i such that

$$\mathrm{Re}\{(1+jb_i\mu)g_i(j\mu)+1/k\} \geqslant 0 \quad \text{for all} \quad \mu \geqslant 0,$$

but a common real number satisfying condition (5) may not exist.

iii) Even if all $(g_i(s), H(0,k))$ are absolutely stable in the sector $(0,k)$, we can not say $(G, H(0,k))$ is absolutely stable in the sector $(0,k)$. This is because of the nonnecessity of Popov's criterion.

Example Given the vertex transfer functions

$$g_1(s) = (0.5s + 1)/(s^2 + s + 1),$$
$$g_2(s) = (s - 1)/(s^2 + s + 1),$$
$$g_3(s) = 1/(s^2 + 0.6s + 1).$$

Derive a $k>0$ as large as possible such that $(G, H[0,k))$ is absolutely stable in the sector $[0, k)$.

Solution

Step 1 Draw the modified Nyquist loci of $g_i(s)$, $i=1,2,3$. Here the locus of $g_1(s)$ is dashed; $g_2(s)$ solid; $g_3(s)$ dash-dot. (Fig. 2).

Step 2 Draw a straight line through the point $-1/k+j0$ with slope $1/b$ where $k>0$ is as large as possible, and all the modified Nyquist loci of $g_i(s)$, $i=1,2,3$ have no point to the left of the line.

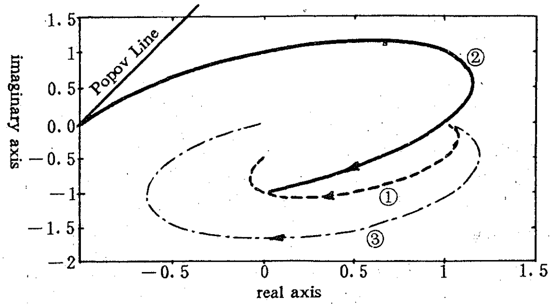

Fig. 2 The modified Nyquist loci

Step 3 Clearly $k_{max}=1$, and $(G, H[0,1))$ is absolutely stable in the sector $[0,1)$.

4 Conclusions

In this paper we considered the absolute stability for a class of nonlinear systems which linear parts are the convex combinations of vertex models, and the nonlinear feedback function lie in a sector. An extended Popov criterion has been proved. Since the criterion is only applied to the vertex systems, hence a significant reduction in computational complexity results. Another important class of systems is the one which contains interval transfer functions, and the further study about this problem will appear in another paper.

Reference

[1] Zhao, K. Y. and Barmish, B. R.. Stability Robustness of Plant-Controller Families. In Proc of 11th IFAC World Congress, Tallinn'90, 1990, 5:190—193

[2] Willems, J. L.. Stability Theory of Dynamical Systems. London:Nelson, 1970, 152

对一类不确定系统的推广波波夫判据

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摘要: 本文将对一类不确定系统提出推广的波波夫判据,这类系统的线性部分乃诸顶点模型的凸组合,而非线性函数是处于某扇区内.判据说:不确定系统是某扇区绝对稳定的如果存在一条公共的波波夫直线使所有顶点系统的修正奈氏曲线位于这直线右侧.

关键词: 非线性系统;绝对稳定性

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赵克友 1945年生. 1968年毕业于山东大学数学系. 后从事电气产品的研制，电站及电网的设计，维护与管理等技术工作. 1978年来先后任教于山东大学控制论专业，青岛大学应用数学专业及电气技术专业. 现为青岛大学电气工程系副教授，期间曾赴美国有关大学访问进修. 有兴趣于系统控制，微机应用，电气技术及应用数学. 近期研究领域为不确定线性及非线性系统的鲁棒控制.

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第三篇讨论了随机大系统在非李雅普诺夫意义下的稳定性问题，包括第七章和第八章。其中，第七章对由泛函方程描述的连续时间和离散时间随机大系统给出了随机输入输出稳定性和不稳定性的判据；第八章对随机大系统提出了实用稳定性的概念并建立了相应的稳定性判据。

第四篇论述了随机大系统的分散镇定，由第九章和第十章组成。第九章对Ito型线性随机大系统提出了分散指数镇定的一种方法，并给出了具体算法和仿真实例；第十章利用李雅普诺夫泛函，对线性和非线性多滞后随机大系统建立了利用局部状态反馈的分散镇定判据。

该书虽然是《大型动力系统的理论与应用》系列著作的第4卷，但前3卷均是关于确定性大系统的，而该书则是专门论述随机大系统的。它的主要特点是：

1. 书中注意收入有代表性的研究成果，特别是以较大篇幅(共6章)介绍了作者近年来发表在国际刊物和重要会议上的具有创造性的系列研究成果，主要表现在：在国际上，对具有多层递阶结构的随机大系统利用递阶李雅普诺夫函数给出了稳定性判据；利用李雅普诺夫泛函建立了具有结构化和非结构化不确定性的多滞后随机大系统的鲁棒稳定性判据；对随机大系统提出了在非李雅普诺夫意义下的均方实用稳定性概念并给出了相应的判据；讨论了滞后随机大系统的分散镇定问题。

2. 书中的研究方法在已有方法的基础上推陈出新，使之适合随机大系统的特点。作者注意把在长期从事确定性大系统稳定性与镇定研究中得到的方法和技巧，如专著《带有时滞的动力系统的运动稳定性》、《大型动力系统的理论与应用》(卷1～卷3)、"Stability, Stabilization and Control of Large Scale Systems"中的结论、方法和技巧，根据随机大系统的特点灵活地运用和推广至随机大系统的稳定性分析与镇定综合，如，第四章和第五章分别利用李雅普诺夫函数和李雅普诺夫泛函给出了多滞后随机大系统的滞后无关均方渐近稳定性(或称无条件稳定性)判据和鲁棒稳定性判据，第八章对随机大系统首次提出了各种均方渐近稳定性概念并给出了相应的判据。

3. 内容全面、结构完整。全书既论述了连续时间参数随机大系统的稳定性，又论述了离散时间参数随机大系统的稳定性；既讨论了随机大系统在李雅普诺夫意义下的稳定性，又讨论了随机大系统在非李雅普诺夫意义下的稳定性(如均方实用稳定性、二阶矩随机输入输出稳定性)；既对随机大系统给出了稳定性判据，又对随机大系统(包括滞后随机大系统)建立了分散镇定的判据，并给出了设计分散化控制器的具体算法及仿真实例；在论述随机大系统的稳定性与镇定时，既有一般性的定理，又有具体的例子以解释所得到的结果。该书全面地、系统地论述了随机大系统的稳定性与镇定，层次分明、结构清晰，每章均有引言、小结和参考文献，并注意以具体例子说明所得的结论，内容全面丰富，结构组织合理。

该书可作为理工科大学生和研究生(自动控制、系统工程、工业自动化、应用数学、运筹学与控制论等专业)的教学参考书或教材，也可供有关科研工作者、工程技术人员和高等院校有关专业的师生参考。

(司徒荣)