

# A New Decomposition Method and Partial Stability of Nonlinear Large Scale Time-Delay Systems

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**Abstract:** In this paper, we study decomposition techniques for nonlinear large scale systems, which have the feature that the interactions between the various subsystems are nonadditive. Using the technique of decomposing a graph into its strongly connected components, we first rewrite the nonlinear large scale systems with delays into a hierarchical form. Some simple criteria for partial stability are obtained.

**Key words:** decomposition; graph theory; time-delay systems; partial stability

## 1 Introduction

With fast development of science and technology, nonlinear large scale systems with complex structure have appeared in many subjects. Nowadays, we are still at the beginning of studying the theory of nonlinear large scale systems and are short of methods as the problems of the stability of nonlinear large scale systems with delays are difficult and complicated.

As you know, it is very important for us how to decompose the systems into a desirable form. In this paper, we study decomposition techniques for nonlinear large scale systems with delays. Using the technique (see [1]~[4]) of decomposing a graph into its strongly connected components, we first rewrite the nonlinear large scale systems with delays into a hierarchical form, by renumbering and aggregating the original state variables, if necessary. Once the system equations have been rearranged in this hierarchical form, we can easily obtain the partial stability properties of the nonlinear large scale time-delay systems.

## 2 Decomposition Method

Consider the nonlinear large scale time-delay system described by

$$\begin{aligned} \dot{Z}_i(t) = & \sum_{j=1}^n C_{ij}(t)Z_j(t) + \sum_{j=1}^n D_{ij}(t)Z_j(t - \tau_j(t)) \\ & + G_i(t, Z_1(t), \dots, Z_n(t), Z_1(t - \tau_1(t)), \dots, Z_n(t - \tau_n(t))), \end{aligned} \quad (2.1)$$

( $i=1, \dots, n$ ) where  $Z_i(t)$  is the state of the  $i$ th subsystem and  $n$  is the number of subsystems. The delays  $\tau_j(t)$  ( $j=1, \dots, n$ ) are nonnegative, bounded and continuous functions.

Given the system description (2.1), we associate with it a digraph (i. e., directed graph)  $G$ , constructed as follow: we label  $n$  vertices as  $v_1, \dots, v_n$ , and we introduce an edge from  $v_j$  to  $v_i$  if A):  $C_{ij}(t) \not\equiv 0$  or  $D_{ij}(t) \not\equiv 0$ , for  $j \neq i$ ; or B): the function  $G_i$  explicitly depends on the quan-

ity  $Z_j(t)$  or  $Z_j(t - \tau_j(t))$ , for  $j \neq i$ . Once this is done for all functions  $C_{ij}(t)$ ,  $D_{ij}(t)$ ,  $G_1$ , ...,  $G_n$ , the resulting digraph is  $G$ . Now, the digraph  $G$  is really representative of the interaction pattern of the system (2.1), because  $G$  contains an edge from  $v_j$  to  $v_i$  if and only if the dynamics of  $Z_i$  are influenced by  $Z_j(t)$  or  $Z_j(t - \tau_j(t))$ .

Once we construct the associated digraph  $G$ , we identify the strongly connected components of  $G$ . Recall [2~4] that a pair of vertices  $(v_i, v_j)$  in  $G$  is said to be strongly connected if there is a path from  $v_i$  to  $v_j$  and vice versa. Also, the notion of strong connections defines an equivalence relation on the vertex set  $V = \{v_1, \dots, v_n\}$  of  $G$ . Now, partition  $V$  into its equivalence classes  $V_1, \dots, V_m$  under this relation, and renumber these equivalence classes in such a way that, whenever  $v_a \in V_i$ ,  $v_b \in V_j$  and  $i < j$ , there is no edge from  $v_b$  to  $v_a$  in  $G$ , such a renumbering can always be done, although perhaps in more ways than one.

Once we identify the equivalence classes of vertices  $V_1, \dots, V_m$ , and if  $\tau_a(t) = \tau_b(t)$ , whenever  $v_a, v_b \in V_i$ , we define

$$x_i(t) = \{Z_j(t), v_j \in V_i\}, \quad x_i(t - \tau_i(t)) = \{Z_j(t - \tau_j(t)), v_j \in V_i\},$$

where  $\tau_i(t) \equiv \tau_j(t)$ , whenever  $v_j \in V_i$ , and

$$A_{ij}(t) = \{C_{\alpha\beta}(t), v_\alpha \in V_i, v_\beta \in V_j\}, \quad B_{ij}(t) = \{D_{\alpha\beta}(t), v_\alpha \in V_i, v_\beta \in V_j\},$$

$$F_i = \{G_j, v_j \in V_i\}$$

With these definitions, the system equations (2.1) assume the hierarchical form

$$\begin{aligned} \dot{x}_i(t) = & \sum_{j=1}^i A_{ij}(t)x_j(t) + \sum_{j=1}^i B_{ij}(t)x_j(t - \tau_j(t)) \\ & + F_i(t, x_1(t), \dots, x_i(t), x_1(t - \tau_1(t)), \dots, x_i(t - \tau_i(t))), \end{aligned} \quad (2.2)$$

( $i=1, \dots, m$ ). Note that the new quantities  $x_i$  are obtained by renumbering and aggregating the old quantities  $Z_1, \dots, Z_n$  as needed.

Once we have rearranged the system differential difference equations in the form (2.2), we can easily obtain some criteria for the partial stability.

### 3 Stability Theorems

In this section, we shall discuss the partial stability properties of the large scale system with time varying delays in hierarchical form:

$$\begin{aligned} \dot{x}_i(t) = & \sum_{j=1}^i A_{ij}(t)x_j(t) + \sum_{j=1}^i B_{ij}(t)x_j(t - \tau_j(t)) \\ & + F_i(t, x_1(t), \dots, x_i(t), x_1(t - \tau_1(t)), \dots, x_i(t - \tau_i(t))), \end{aligned} \quad (3.1)$$

( $i=1, \dots, m$ ), where  $x_i \in \mathbb{R}^{n_i}$ ,  $t \in J = [a, +\infty)$ ,  $\sum_{j=1}^m n_j = n$ ,  $x^T = (x_1^T, \dots, x_m^T)^T$  and  $0 \leq \tau_i(t) \leq \tau = \text{const}$ . The delay  $\tau_i(t)$  is continuous function.

We assume that  $F_i$  is continuous,

$$F_i(t, 0, \dots, 0) = 0, \quad \forall t \in J, \quad (i=1, \dots, m), \quad (3.2)$$

and the solution  $X(t) = X(t, t_0, \Phi)$  of system (3.1) exists and is unique corresponding to each initial value condition

$$x_i(t) = \varphi_i(t), \quad t_0 - \tau \leq t \leq t_0, \quad i=1, \dots, m, \quad (3.3)$$

( $t_0 - \tau \geq a$ ), where  $\varphi_i(t)$  is continuous and  $\|\Phi\| = \max_{1 \leq i \leq m} \sup_{t_0 - \tau \leq t \leq t_0} \|\varphi_i(t)\|$ .

Let partial variable  $Y(t) = Y(t, t_0, \Phi) = (x_1^T(t), \dots, x_k^T(t))^T$  ( $k \leq m$ ) and  $P(t, t_0) = \text{diag}(p_1(t, t_0), \dots, p_k(t, t_0))$  be the fundamental matrix solution of the isolated subsystems

$$x_i(t) = A_{ii}(t)x_i(t), \quad i = 1, \dots, k. \quad (3.4)$$

**Theorem 1** Assume that

$$i) \|p_i(t, t_0)\| \leq M_i \exp\left(-\int_{t_0}^t \lambda_i(\xi) d\xi\right), \quad i = 1, \dots, k, \quad \text{for } t \geq t_0,$$

where  $M_i$  is constant,  $\lambda_i(t)$  is continuous function on interval  $J$ .

$$ii) \|F_i(t, x_1(t), \dots, x_i(t), x_1(t - \tau_1(t)), \dots, x_i(t - \tau_i(t)))\|$$

$$\leq \sum_{j=1}^i \{a_{ij}(t)\|x_j(t)\| + b_{ij}(t)\|x_j(t - \tau_j(t))\|\}, \quad i = 1, \dots, k,$$

where  $a_{ij}(t)$ ,  $b_{ij}(t)$  are continuous and nonnegative on interval  $J$ .

Let

$$G_1(t) = \exp\left[-\int_{t_0}^t (\lambda_1(s) - \mu_1(s)) ds\right],$$

$$G_i(t) = \left[1 + M \int_{t_0}^t \exp\left(\int_{t_0}^s \lambda_i(\xi) d\xi\right) \sum_{j=1}^{i-1} \{a_{ij}^*(s)G_j(s) + b_{ij}^*(s)G_j(s - \tau_j(s))\} ds\right] \cdot \exp\left[-\int_{t_0}^t (\lambda_i(s) - \mu_i(s)) ds\right], \quad i = 2, \dots, k,$$

where

$$\mu_i(t) = M_i[a_{ii}(t) + (b_{ii}(t) + \|B_{ii}(t)\|)\exp(\int_{t-\tau_i(t)}^t \lambda_i(\xi) d\xi)], \quad (i = 1, \dots, k),$$

$$a_{ij}^*(t) = \|A_{ij}(t)\| + a_{ij}(t), \quad b_{ij}^*(t) = \|B_{ij}(t)\| + b_{ij}(t), \quad (j = 1, \dots, i-1, i = 2, \dots, k),$$

$$M = \max\{M_1, \dots, M_k\}.$$

Then for  $i = 1, \dots, k$ ,  $t \geq t_0$ , the following conditions

$$1) G_i(t) \leq b_i(t_0) = \text{const}, \quad 2) G_i(t) \leq b_i = \text{const}, \quad 3) G_i(t) \rightarrow 0 \quad (t \rightarrow \infty)$$

implies that the trivial solution of the large scale system (3.1) with respect to the partial variable  $Y$  is 1) stable; 2) uniformly stable; 3) asymptotically stable, respectively.

**Proof** When  $i = 1$ , (3.1) means that

$$\dot{x}_1(t) = A_{11}(t)x_1(t) + B_{11}(t)x_1(t - \tau_1(t)) + F_1(t, x_1(s), x_1(t), x_1(t - \tau_1(t))).$$

Using the variation of parameters formula, we have

$$x_1(t) = p_1(t, t_0)x_{10} + \int_{t_0}^t p_1(t, s)[B_{11}(s)x_1(s - \tau_1(s)) + F_1(s, x_1(s), x_1(s - \tau_1(s)))] ds.$$

Therefore

$$\begin{aligned} \|x_1(t)\| &\leq M_1 \|\Phi\| \exp\left(-\int_{t_0}^t \lambda_1(\xi) d\xi\right) + \int_{t_0}^t M_1 [a_{11}(s)\|x_1(s)\| \\ &\quad + (\|B_{11}(s)\| + b_{11}(s))\|x_1(s - \tau_1(s))\|] \exp\left(-\int_s^t \lambda_1(\xi) d\xi\right) ds \end{aligned}$$

which is equivalent to that

$$\|x_1(t)\| \exp\left(\int_{t_0}^t \lambda_1(\xi) d\xi\right)$$

$$\leq M_1 \|\Phi\| + \int_{t_0}^t M_1 [a_{11}(s) \|x_1(s)\| \exp(\int_{t_0}^s \lambda_1(\xi) d\xi) + (b_{11}(s) + \|B_{11}(s)\|) \cdot \exp(\int_{s-\tau_1(s)}^s \lambda_1(\xi) d\xi) \|x_1(s - \tau_1(s))\| \exp(\int_{t_0}^{s-\tau_1(s)} \lambda_1(\xi) d\xi)] ds.$$

Letting

$$W_1(t) = \sup_{t-\tau_1 \leq t_1 \leq t} \{ \|x_1(t_1)\| \exp(\int_{t_0}^{t_1} \lambda_1(\xi) d\xi) \},$$

then

$$\begin{aligned} & \|x_1(t)\| \exp(\int_{t_0}^t \lambda_1(\xi) d\xi) \\ & \leq M_1 \|\Phi\| + \int_{t_0}^t M_1 [a_{11}(s) + (b_{11}(s) + \|B_{11}(s)\|) \exp(\int_{s-\tau_1(s)}^s \lambda_1(\xi) d\xi)] W_1(s) ds. \end{aligned} \quad (3.5)$$

In view of the fact that the right of inequality (3.5) is nondecreasing, we get

$$W_1(t) \leq M_1 \|\Phi\| + \int_{t_0}^t \mu_1(s) W_1(s) ds.$$

By Gronwall-Bellman inequality, we obtain  $W_1(t) \leq M_1 \|\Phi\| \exp(\int_{t_0}^t \mu_1(s) ds)$  which implies that

$$\|x_1(t)\| \leq M_1 \|\Phi\| \exp[-\int_{t_0}^t (\lambda_1(s) - \mu_1(s)) ds] = M_1 \|\Phi\| G_1(t), \quad t \geq t_0. \quad (3.6)$$

When  $i=2$ , (3.1) is that

$$\begin{aligned} \dot{x}_2(t) = & \sum_{j=1}^2 A_{ij}(t) x_j(t) + \sum_{j=1}^2 B_{ij}(t) x_j(t - \tau_j(t)) \\ & + F_2(t, x_1(t), x_2(t), x_1(t - \tau_1(t)), x_2(t - \tau_2(t))). \end{aligned}$$

By the variation of parameters formula, we get

$$\begin{aligned} x_2(t) = & p_2(t, t_0) x_{20} + \int_{t_0}^t p_2(t, s) [A_{21}(s) x_1(s) + \sum_{j=1}^2 B_{ij}(s) x_j(s - \tau_j(s)) \\ & + F_2(s, x_1(s), x_2(s), x_1(s - \tau_1(s)), x_2(s - \tau_2(s)))] ds. \end{aligned}$$

Because of conditions i) and ii), we arrive at

$$\begin{aligned} \|x_2(t)\| \leq & M_2 \|\Phi\| \exp(-\int_{t_0}^t \lambda_2(\xi) d\xi) + \int_{t_0}^t M_2 [(\|A_{21}(s)\| + a_{21}(s)) \|x_1(s)\| \\ & + (\|B_{21}(s)\| + b_{21}(s)) \|x_1(s - \tau_1(s))\|] \exp(-\int_s^t \lambda_2(\xi) d\xi) ds \\ & + \int_{t_0}^t M_2 [a_{22}(s) \|x_2(s)\| + (b_{22}(s) + \|B_{22}(s)\|) \|x_2(s - \tau_2(s))\|] \exp(-\int_s^t \lambda_2(\xi) d\xi) ds, \end{aligned}$$

which is equivalent to that

$$\begin{aligned} & \|x_2(t)\| \exp(\int_{t_0}^t \lambda_2(\xi) d\xi) \\ & \leq M_2 \|\Phi\| + \int_{t_0}^t M_2 [a_{21}^*(s) \|x_1(s)\| + b_{21}^*(s) \|x_1(s - \tau_1(s))\|] \exp(\int_{t_0}^s \lambda_2(\xi) d\xi) ds \\ & + \int_{t_0}^t M_2 [a_{22}(s) \|x_2(s)\| \exp(\int_{t_0}^s \lambda_2(\xi) d\xi) + (b_{22}(s) + \|B_{22}(s)\|)] \end{aligned}$$

$$\cdot \exp\left(\int_{s-\tau_2(s)}^s \lambda_2(\xi) d\xi\right) \|x_2(s - \tau_2(s))\| \exp\left(\int_{t_0}^{s-\tau_2(s)} \lambda_2(\xi) d\xi\right) ds.$$

Letting

$$W_2(t) = \sup_{t-\tau_1 \leq s \leq t} [\|x_2(t_1)\| \exp(\int_{t_0}^s \lambda_2(\xi) d\xi)]$$

and noticing the inequality (3.6), we obtain

$$\begin{aligned} & \|x_2(t)\| \exp\left(\int_{t_0}^t \lambda_2(\xi) d\xi\right) \\ & \leq M_2 \|\Phi\| + \int_{t_0}^t M_1 M_2 \|\Phi\| [a_{21}^*(s) G_1(s) + b_{21}^*(s) G_1(s - \tau_1(s))] \exp\left(\int_{t_0}^s \lambda_2(\xi) d\xi\right) ds \\ & + \int_{t_0}^t M_2 [a_{22}(s) + (b_{22}(s) + \|B_{22}(s)\|)] \exp\left(\int_{s-\tau_2(s)}^s \lambda_2(\xi) d\xi\right) W_2(s) ds. \end{aligned} \quad (3.7)$$

Since the right of inequality (3.7) is nondecreasing, it follows that

$$\begin{aligned} W_2(t) & \leq M_2 \|\Phi\| \{1 + \int_{t_0}^t M [a_{21}^*(s) G_1(s) + b_{21}^*(s) G_1(s - \tau_1(s))] \exp\left(\int_{t_0}^s \lambda_2(\xi) d\xi\right) ds\} \\ & + \int_{t_0}^t \mu_2(s) W_2(s) ds. \end{aligned}$$

By Gronwall-Bellman inequality, we have

$$\begin{aligned} W_2(t) & \leq M_2 \|\Phi\| \{1 + \int_{t_0}^t M [a_{21}^*(s) G_1(s) + b_{21}^*(s) G_1(s - \tau_1(s))] \exp\left(\int_{t_0}^s \lambda_2(\xi) d\xi\right) ds\} \\ & \cdot \exp\left(\int_{t_0}^t \mu_2(s) ds\right), \end{aligned}$$

which implies that

$$\begin{aligned} \|x_2(t)\| & \leq M_2 \|\Phi\| [1 + \int_{t_0}^t M (a_{21}^*(s) G_1(s) + b_{21}^*(s) G_1(s - \tau_1(s))) \exp\left(\int_{t_0}^s \lambda_2(\xi) d\xi\right) ds] \\ & \cdot \exp\left[-\int_{t_0}^t (\lambda_2(s) - \mu_2(s)) ds\right] \\ & = M_2 \|\Phi\| G_2(t), \quad t \geq t_0. \end{aligned} \quad (3.8)$$

It is easy to see that

$$\begin{aligned} \|x_i(t)\| & \leq M_i \|\Phi\| \{1 + \int_{t_0}^t M \sum_{j=1}^{i-1} [a_{ij}^*(s) G_j(s) + b_{ij}^*(s) G_j(s - \tau_j(s))] \exp\left(\int_{t_0}^s \lambda_i(\xi) d\xi\right) ds\} \\ & \cdot \exp\left[-\int_{t_0}^t (\lambda_i(s) - \mu_i(s)) ds\right] \\ & = M_i \|\Phi\| G_i(t), \quad t \geq t_0, \quad i \geq 2. \end{aligned}$$

Consequently we arrive at

$$\|Y(t)\| \leq \sum_{j=1}^k \|x_j(t)\| \leq \sum_{j=1}^k M_j \|\Phi\| G_j(t) = \|\Phi\| \sum_{j=1}^k M_j G_j(t).$$

From this estimate we can see that the conclusion of the theorem holds. The proof is therefore complete.

**Theorem 2** Suppose that the assumptions i), ii) in Theorem 1 hold. Moreover,

$$\int_{t_0}^t a_{ij}^*(s) \exp\left[-\int_{t_0}^s (\lambda_j(\xi) - \lambda_i(\xi) - \mu_j(\xi)) d\xi\right] ds \leq a_{ij} = \text{const},$$

$$\int_{t_0}^t b_{ij}^*(s) \exp\left[\int_{t_0}^s \lambda_i(\xi) d\xi - \int_{t_0}^{s-\tau_j(s)} (\lambda_j(\xi) - \mu_j(\xi)) d\xi\right] \leq b_{ij} = \text{const},$$

for  $j=1, 2, \dots, i-1$ ,  $i=2, \dots, k$ ,  $t \geq t_0$ , where  $a_{ij}^*(s)$ ,  $b_{ij}^*(s)$ ,  $\mu_j(s)$  are defined in the Theorem

1.

Let

$$\mu(t) = \max_{1 \leq j \leq k} \{\mu_j(t)\}, \quad \lambda(t) = \min_{1 \leq j \leq k} \{\lambda_j(t)\}, \quad t \in J.$$

Then, the following conditions

$$1) \int_{t_0}^t [\lambda(\xi) - \mu(\xi)] d\xi \geq b(t_0) = \text{const}, \quad t \geq t_0;$$

$$2) \int_{t_0}^t [\lambda(\xi) - \mu(\xi)] d\xi \geq b = \text{const}, \quad t \geq t_0;$$

$$3) \int_{t_0}^{+\infty} [\lambda(\xi) - \mu(\xi)] d\xi = +\infty$$

implies that the trivial solution of (3.1) in respect to partial variable  $Y(t)$  is 1) stable; 2) uniformly stable; 3) asymptotically stable, respectively.

**Proof** Let  $X(t) = X(t, t_0, \Phi)$  be any solution of system (3.1). By the Theorem 1, we see that

$$\|x_1(t)\| \leq M_1 \|\Phi\| \exp\left[-\int_{t_0}^t (\lambda_1(\xi) - \mu_1(\xi)) d\xi\right], \quad t \geq t_0.$$

From the inequality (3.8), we get

$$\begin{aligned} \|x_2(t)\| &\leq M_2 \|\Phi\| \left\{ 1 + \int_{t_0}^t M_1 [a_{21}^*(s) \exp(-\int_{t_0}^s (\lambda_1(\xi) - \mu_1(\xi)) d\xi) \right. \\ &\quad \left. + b_{21}^*(s) \exp(-\int_{t_0}^{s-\tau_1(s)} (\lambda_1(\xi) - \mu_1(\xi)) d\xi)] \exp(\int_{t_0}^s \lambda_2(\xi) d\xi) ds \right\} \\ &\quad \cdot \exp\left[-\int_{t_0}^t (\lambda_2(s) - \mu_2(s)) ds\right]. \end{aligned}$$

Using the assumptions, we arrive at

$$\begin{aligned} \|x_2(t)\| &\leq M_2 \|\Phi\| [1 + M_1 (a_{21} + b_{21})] \exp(-\int_{t_0}^t (\lambda_2(s) - \mu_2(s)) ds) \\ &= M_2^* \|\Phi\| \exp(-\int_{t_0}^t (\lambda_2(s) - \mu_2(s)) ds). \end{aligned}$$

As a general rule, when  $i \geq 2$ , it follows that

$$\begin{aligned} \|x_i(t)\| &\leq M_i \|\Phi\| \left\{ 1 + \int_{t_0}^t \sum_{j=1}^{i-1} [a_{ij}^*(s) M_j^* \exp(-\int_{t_0}^s (\lambda_j(\xi) - \mu_j(\xi)) d\xi) \right. \\ &\quad \left. + b_{ij}^*(s) M_j^* \exp(-\int_{t_0}^{s-\tau_j(s)} (\lambda_j(\xi) - \mu_j(\xi)) d\xi)] \right. \\ &\quad \left. \cdot \exp(\int_{t_0}^s \lambda_i(\xi) d\xi) \exp(-\int_{t_0}^t (\lambda_i(\xi) - \mu_i(\xi)) d\xi) \right\} \end{aligned}$$

$$\leq M_i \|\Phi\| [1 + \sum_{j=1}^{i-1} M_j^* (a_{ij}^* + b_{ij})] \exp[-\int_{t_0}^t (\lambda_i(\xi) - \mu_i(\xi)) d\xi]$$

$$= M_i^* \|\Phi\| \exp[-\int_{t_0}^t (\lambda_i(\xi) - \mu_i(\xi)) d\xi], \quad t \geq t_0.$$

where  $M_i^* = M_i [1 + \sum_{j=1}^{i-1} M_j^* (a_{ij} + b_{ij})]$ ,  $i=2, \dots, k$ ,  $M_1^* = M_1$ . Therefore

$$\|Y(t, t_0, \Phi)\| \leq \sum_{j=1}^k \|x_j(t)\| \leq (M_1 + \sum_{j=2}^k M_j^*) \|\Phi\| \exp[-\int_{t_0}^t (\lambda(\xi) - \mu(\xi)) d\xi]$$

which implies that the conclusion of the theorem holds. This proves the theorem.

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## 非线性时滞大系统的分解及其部分变元的稳定性

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**摘要:** 本文借助于将一个有向图分解成其强分图等技巧, 首次将非线性时滞大系统分解成一个递阶形式, 进而得到了若干简洁的关于部分变元稳定、渐近稳定的判据。

**关键词:** 分解; 图论; 时滞系统; 部分变元稳定

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