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Criterion for Modal Controllability and Observability of 2-D Systems

FANG Yong
(Department of Mathematics, Neijiang Normal College·Sichuan, 641002, PRC)
YANG Chengwu
(Ballistic Research Laboratory of China, Nanjing University of Technology·Nanjing, 210014, PRC)

Abstract: In this paper, the errors of paper [1~3] about criterion for modal controllability and observability of 2-D systems are pointed out and the correct criteria are given.

Key words: 2-D systems; modal controllability; modal observability

1 Introduction and Related Results

It is well known that modal controllability and observability theory[2] of 2-D Roesser model[1] (RM) have become the foundation to search for the minimal state space realization for 2-D RM, since it was raised by Kung et al. in 1977. Many criteria of modal controllability and observability have been given. However, some of them are faulty. In the present paper, the errors of paper [1~3] are discussed and the correct criteria are established.

The criterion for modal controllability of 2-D RM is given in the following lemma.

Lemma 1[1,2] RM is modal controllable if and only if every general point (z_1, z_2) on the irreducible algebraic curve V_i defined by irreducible polynomial $a_i(z_1, z_2)$ satisfies

$$\operatorname{rank}\begin{pmatrix} I_{n_1}z_1 - A_{11} & -A_{12} & B_1 \\ -A_{21} & I_{n_2}z_2 - A_{22} & B_2 \end{pmatrix} = n, \tag{1}$$

where

$$\det\begin{pmatrix} I_{n_1}z_1 - A_{11} & -A_{12} \\ -A_{21} & I_{n_2}z_2 - A_{22} \end{pmatrix} = \prod_{i=1}^{k} a_i(z_1, z_2), \quad z_1, z_2 \in \mathrm{C}. \tag{1a}$$

A_{ij}, $B_i (i, j = 1, 2.)$ are real constant matrices of appropriate dimensions (see [1]), $n = n_1 + n_2$.

Proposition 1[1,3] RM is modal controllable if the matrix pair A_{11}, B_1 and A_{22}, B_2 are controllable in 1-D sense.

Let

$$A(z_2) = A_{11} + A_{12}(I_{n_2}z_2 - A_{22})^{-1}A_{21}, \tag{2}$$

$$B(z_2) = B_1 + A_{12}(I_{n_2}z_2 - A_{22})^{-1}B_2, \tag{3}$$

$$A(z_1) = A_{22} + A_{21}(I_{n_1}z_1 - A_{11})^{-1}A_{12}, \tag{4}$$

$$B(z_1) = B_2 + A_{21}(I_{n_1}z_1 - A_{11})^{-1}B_1. \tag{5}$$

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Proposition 2[1,2] RM is modal controllalbe if and only if the matrix pair $A(z_2)$, $B(z_2)$ is controllable on $R(z_2)$, and $A(z_1)$, $B(z_1)$ is controllable on $R(z_1)$. Where $R(z)$ is the set of proper rational functions with real coefficients.

We will use the following lemma.

Lemma 2 The matrix pair $A(z)$, $B(z)$ is controllable on $R(z)$ if and only if the matrix pair D_A, D_B is controllalbe on R. Where $A(z)\in R^{n\times n}(z)$, $B(z)\in R^{n\times m}(z)$; $D_A=\lim_{z\to\infty}A(z)$, $D_B=\lim_{z\to\infty}B(z)$.

With respect to RM, assume that $A(z_2)$, $B(z_2)$ and $A(z_1)$, $B(z_1)$ are controllable on $R(z_2)$ and $R(z_1)$ respectively. Let

$$\theta = \sigma(A_{22}) \cup \{z_2 : z_2 \in C \text{ and } A(z_2),\ B(z_2) \text{ is uncontrollable on C}\},$$

$$\varphi = \sigma(A_{11}) \cup \{z_1 : z_1 \in C \text{ and } A(z_1),\ B(z_1) \text{ is uncontrollable on C}\},$$

$$T_i = \{(z_1,z_2) : z_1 \in C,\ z_2 \in \theta\} \cap V_i, \tag{6a}$$

$$N_i = \{(z_1,z_2) : z_1 \in \varphi,\ z_2 \in C\} \cap V_i, \tag{6b}$$

where $\sigma(\cdot)$ represents the set of eigenvalues of $(\cdot)$, $i=1,2,\cdots,k$. k is defined as (1a).

2 The Errors of Proposition

Let RM: $A=\left(\begin{array}{c:c}2 & -1\\ \hdashline 1 & 0\end{array}\right)$, $B=\begin{bmatrix}1\\ \frac{1}{2}\end{bmatrix}$, then $\det\begin{pmatrix}z_1-2 & 1\\ -1 & z_2\end{pmatrix}=z_1z_2-2z_2+1=a(z_1,z_2)$. Obviously, $(0,\frac{1}{2})$ is a point on the irreducible algebraic curve $a(z_1,z_2)=0$. Because

$$\operatorname{rank}\begin{bmatrix}0-2 & 1 & 1\\ -1 & \frac{1}{2}-0 & \frac{1}{2}\end{bmatrix}=1<2.$$

Applying Lemma 1, RM is modal uncontrollable.

On the other hand, 2, 1 and 0, $\frac{1}{2}$ are controllable in 1-D sense. According to proposition 1, RM is modal controllable. Therefore, proposition 1 is not true. The error comes from the using of the following faulty result in the proof of proposition 1.

If $\operatorname{rank}(I_{n_1}z_1-A_{11}\quad B_1)=n_1$, $z_1\in C$ and $\operatorname{rank}(I_{n_2}z_2-A_{22}\quad B_2)=n_2$, $z_2\in C$, then

$$\operatorname{rank}\begin{bmatrix}I_{n_1}z_1-A_{11} & -A_{12} & B_1\\ -A_{21} & I_{n_2}z_2-A_{22} & B_2\end{bmatrix}=n_1+n_2,\quad z_1,z_2\in C.$$

By (2)~(5), we obtain $\lim_{z_1\to\infty}A(z_1)=A_{22}$, $\lim_{z_1\to\infty}B(z_1)=B_2$, $\lim_{z_2\to\infty}A(z_2)=A_{11}$ and $\lim_{z_2\to\infty}B(z_2)=B_1$. If A_{11}, B_1 and A_{22}, B_2 are controllable, based on Lemma 2, we have that $A(z_1)$, $B(z_1)$ and $A(z_2)$, $B(z_2)$ are controllable on $R(z_1)$ and $R(z_2)$ respectively. By Proposition 2 RM is modal controllable. Since Proposition 1 is faulty, then Proposition 2 is also incorrect. It is caused by the following faulty result.

Assume that $A(z_2)$, $B(z_2)$ is controllable on $R(z_2)$, then there exists matrix P and Q with elements from $R(z_2)[z_1]$ such that

$$(I_{n_1}z_1-A(z_2))P+B(z_2)Q=I_{n_1}. \tag{7}$$

For RM as above, $A(z_2)=2-\frac{1}{z_2}$, $B(z_2)=1-\frac{1}{2z_2}$ is controllable. If there exists matrix P and Q such that (7) holds, then

$$(z_1 - 2 + \frac{1}{z_1})P + (1 - \frac{1}{2z_2})Q = 1.$$

Let $z_2=\frac{1}{2}$, we get $z_1 P(z_1, \frac{1}{2})=1$. Hence, $P \notin R^{n_1 \times n_1}(z_2)[z_1]$.

3 Main Result

Theorem 1 RM is modal controllable if and only if 1) $A(z_2)$, $B(z_2)$ and $A(z_1)$, $B(z_1)$ are controllable on $R(z_2)$ and $R(z_1)$ respectively, 2) For every general point on the T_i and N_i, $\operatorname{rank}\begin{bmatrix} I_{n_1}z_1-A_{11} & -A_{12} & B_1 \\ -A_{21} & I_{n_2}z_2-A_{22} & B_2 \end{bmatrix}=n$, where T_i and N_i are defined as (6).

Proof According to [1,2] and Lemma 1, necessity is obtained immediately.

Sufficiency: Let $A(z_2)$, $B(z_2)$ be controllable on $R(z_2)$. then for an arbitrary $z_2^0 \notin \theta$, $A(z_2^0)$, $B(z_2^0)$ is controllable on C. By [4], there exists matrix P and Q with elements from $C[z_1]$ such that

$$(I_{n_1}z_1 - A(z_2^0))P + B(z_2^0)Q = I_{n_1}, \tag{8}$$

hence

$$\begin{bmatrix} I_{n_1}z_1 - A_{11} & -A_{12} \\ -A_{21} & I_{n_2}z_2^0 - A_{22} \end{bmatrix} \cdot$$

$$\left[\begin{array}{c:c} P & PA_{12}(I_{n_2}z_2^0 - A_{22})^{-1} \\ (I_{n_2}z_2^0 - A_{22})^{-1}(A_{21}P - B_2Q) & (I_{n_2}z_2^0 - A_{22})^{-1}(A_{21}P - B_2Q)A_{12}(I_{n_2}z_2^0 - A_{22})^{-1} + (I_{n_2}z_2^0 - A_{22})^{-1} \end{array}\right]$$

$$+ \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}(Q \ \vdots \ QA_{12}(I_{n_2}z_2^0 - A_{22})^{-1}) = \begin{bmatrix} I_{n_1} & 0 \\ 0 & I_{n_2} \end{bmatrix}$$

and this implies that $\begin{bmatrix} I_{n_1}z_1-A_{11} & -A_{12} \\ -A_{21} & I_{n_2}z_2^0-A_{22} \end{bmatrix}$, $\begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$ is left prime to each other on $C[z_1]$. That is, for any $z_1 \in C$, $z_2^0 \notin \theta$, then

$$\operatorname{rank}\begin{bmatrix} I_{n_1}z_1 - A_{11} & -A_{12} & B_1 \\ -A_{21} & I_{n_2}z_2^0 - A_{22} & B_2 \end{bmatrix} = n.$$

By the similar way we can get dual result, According to condition 2) and Lemma 1, RM is modal controllable. Therefore, the proof of Theorem 1 is complete.

By using Lemma 2, the condition 1) of Theorem 1 implies the condition of Proposition 2. Corresponding to Proposition 1, the following theorem is obtained immediately.

Theorem 2 If RM is modal controllable, then A_{11}, B_1 and A_{22}, B_2 are controllable in 1-D sense.

By applying duality principle, we can get the similar results about modal observability of RM.

4 Conclusion

This paper pointed out the errors of paper [1～3] about criteria for modal controllability and observability of 2-D systems, and established the correct criterions. These criteria are good for judging the modal uncontrollability and unobservability of RM.

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2-D 系统模能控性和模能观性判据

方　勇　　　　杨成梧

(内江师范专科学校数学系·四川,641002)　(南京理工大学八系·南京,210014)

摘要：本文指出并纠正了文[1～3]关于 2-D 系统模能控性和模能观性判据的错误.

关键词：2-D 系统；模能控；模能观

本文作者简介

方　勇　1964 年生. 1984 年毕业于四川师范大学数学系,获理学学士学位,1990 年获华东工学院工程热物理与飞行力学系自动控制专业硕士学位. 现为内江师范专科学校数学系讲师,并兼任电教中心副主任. 主要研究方向为 2-D 系统,非线性系统.

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