第 11 卷第 1 期
1994 年 2 月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol. 11, No. 1
Feb., 1994

Force Control for Tracking a Set of Tasks in Presence of Constraints

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Abstract: A new approach to tracking tasks by robots in the presence of environmental constraints is proposed. Since the dynamic equations of a rigid object are written in a component form with respect to axes fixed with the object, a new model of constraints is given in a very simple form as linear functions of velocity components. As a result, the system of differential equations of motion of the object is automatically decomposed into two subsystems: equations of motion and equations of reactive force. Two force feedback schemes are developed. A hierarchical architecture is established for the overall control strategy such that the robot can track a set of different objects for different desired motions.

Key words: robot control with constraints; control of the reactive force; tracking a set of tasks; hybrid control; hierarchical control

1 Introduction

The study of tracking task with environmental constraints is an important issue of robot control from both theoretical and practical points of view. Here reactive forces at the contact points of the held object with the constraints have to be controlled desirably. The system is usually written as

$$M(q)\ddot{q} + N(q,\dot{q}) = \tau + J^{T}F, \tag{1}$$

$$c(P_0) = 0 \quad \text{or} \quad C(q) = 0, \tag{2}$$

where q is the joint variable vector. $M(q)$ the inertial matrix. $N(q,\dot{q})$ a vector including centrifugal and Coriolis forces as well as gravity, τ the control vector on the joints, F the reactive force from the constraints and J the Jacobian matrix formed from the relation

$$P_0 = h(q), \quad J = \frac{\partial h}{\partial q}, \tag{3}$$

where P_0 denotes the position vector of the end-effector in base coordinate system. Existence of contact constraints (2) implies that components of both $q(t)$ and F are not all known. If $n=6$, so is the number of all known components of both $q(t)$ and F in general case. Therefore, great efforts have been devoted to decompose (1) into a motion subsystem and a force subsystem to establish motion/force controls or hybrid controls[1~7].

However, the procedure for such decomposition is huge complicated and needs much computation. Furthermore, as the object is considered joined with the end-effector fixedly, invariance

Manuscript received Jul. 15, 1992, revised Jan. 8, 1993.

against change of either the object or desired trajectory requires the established control law to be adaptive or robust.

All these issues are treated in a simple way in this paper. First, we consider the object and the manipulator as two separate units, even though their motions are determined each other. Second, the equation of motion of the object, by means of Newton's law and Euler's momentum law, is described by differential equations relative to central principal inertial axes which is not an inertial system. Third, as a consequence the mathematical model of the constraints may be in many cases represented in terms of velocity components for simplicity. Fourth, the control law is in the hierarchical architecture where the low layer provides a control for the manipulator itself; the middle layer generates a control to track the object and the high layer is an adaptive one available to different objects.

This paper is organized as follows. Preliminary are introduced about force systems in section 2 and about kinematics in section 3, respectively. In section 4 the mathematical equation of the object-manipulator system is established and a new model of constraints is presented in section 5. Sequently, in section 6 control problems are formulated. After this, in sections 7 and 8 the overall hierarchical control strategy is proposed. Then simulation results are shown. Sections 6 ~ 9 provide the main results of this paper.

2 Representation and Transfer of Force Systems[8]

Suppose a given force system

$$F = \{(f_1, p_1), \cdots, (f_n, p_n)\}$$

is exerted on a body where f_i is the force vector and p_i the radius vector of the point of application P_i of f_i and this point may be replaced by any other point on the force line when the body is rigid. This being the case, we can transfer this force system to some point O to get a simplified force system consisting of a force f acting at O and a corresponding moment (torque) m_O, i. e.

$$F = \{f, m_O\}, \quad f = \sum_{i=1}^{n} f_i, \quad m_O = \sum_{i=1}^{n} r_i \times f_i,$$

where r_i is the radius vector of point P_i relative to O.

Similarly, transfer the force system to point O' we have

$$F = \{f, m_{O'}\}, \quad f = \sum_{i=1}^{n} f_i, \quad m_{O'} = \sum_{i=1}^{n} r'_i \times f_i,$$

where r'_i is the radius vector of P_i relative to O'.

Let $Oxyz$ and $O'x'y'z'$ be two coordinate systems fixed to the body. We have the following equivalent representations of the same given force system:

$$F_O = [f_x, f_y, f_z, m_{Ox}, m_{Oy}, m_{Oz}]^{\mathrm{T}}, \quad F_{O'} = [f_{x'}, f_{y'}, f_{z'}, m_{O'x'}, m_{O'y'}, m_{O'z'}]^{\mathrm{T}}$$

The components of the last two 6×1 vectors are projections of the f, m_O and f, $m_{O'}$, on the corresponding coordinate axes. Let $R_O^{O'}$ be the 3×3 orientation matrix of $Oxyz$ relative to $O'x'y'z'$ and its inverse be $R_{O'}^{O}$. We have the following transfer relation between the two representations

$$F_{O'} = SF_O, \tag{4}$$

where
$$S = \begin{bmatrix} R & 0 \\ PR & R \end{bmatrix}, \quad R = R_O^{O'}, \quad P = \begin{bmatrix} 0 & -\zeta & \eta \\ \zeta & 0 & -\xi \\ -\eta & \xi & 0 \end{bmatrix}, \tag{5}$$
and $[\xi,\eta,\zeta]$ is the coordinates of point O in coordinate system $O'x'y'z'$. It is obvious S is nonsingular.

Suppose on the robot manipulator a joint force/torque system $F_1=\tau=[\tau_1,\tau_2,\cdots,\tau_n]^{\mathrm{T}}$is applied and on the end-effector another force system $F_2=[f_x, f_y, f_z, m_x, m_y, m_z]^{\mathrm{T}}$is applied to balance F_1. Clearly, force systems F_1 and $-F_2$ are not statically equivalent because there are reactive forces acting on the manipulator from the basement of the robot which have not been taken into account. But F_1 and F_2 cause the same effects on the manipulator, hence we have
$$F_1 = -G^{\mathrm{T}}F_2 \tag{6}$$
where G is a matrix defined in (14). When $n=6$, J is a nonsingular matrix of variable q except at a few singular points.

3 Kinematic Representations and Transformation[8]

The position of a body is determined by a 6×1 matrix.
$$P_o = [x_o, y_o, z_o, \varphi_o, \theta_o, \psi_o]^{\mathrm{T}}$$
where x_o, y_o, z_o are the coordinates of the point o and angles φ_o, θ_o, ψ_o are the orientation angles which may be chosen, for example, as the Euler's angles. The drawback of this representation is that the time derivative of P_o has no direct meaning in sense of "velocity". As is well-known, the Euler's kinematic equations
$$\begin{cases} \omega_x = \dot{\varphi}\sin\theta\sin\psi + \dot{\theta}\cos\psi, \\ \omega_y = \dot{\varphi}\sin\theta\cos\psi - \dot{\theta}\sin\psi, \\ \omega_z = \dot{\varphi}\cos\theta + \psi \end{cases} \tag{7}$$
give the relations between Euler's angles and the angular velocities about x, y, z axes and relation (7) may also represented as
$$\omega = W\dot{\varepsilon}, \quad \omega = [\omega_x, \omega_y, \omega_z]^{\mathrm{T}}, \quad \varepsilon = [\varphi,\theta,\psi]^{\mathrm{T}}, \tag{8}$$
where
$$W = \begin{bmatrix} \sin\theta\sin\psi & \cos\psi & 0 \\ \sin\theta\cos\psi & -\sin\psi & 0 \\ \cos\theta & 0 & 1 \end{bmatrix}.$$

Now we have two representations
$$\dot{P}_o = [V_{OX}, V_{OY}, V_{OZ}, \dot{\varphi}_o, \dot{\theta}_o, \dot{\psi}_o]^{\mathrm{T}} \tag{9}$$
and
$$V_o = [V_{ox}, V_{oy}, V_{oz}, \omega_x, \omega_y, \omega_z]^{\mathrm{T}} \tag{10}$$
to denote the "velocity" of the same body. Clearly, neither of them is adequate in all circumstances. It is easy to show that they are interrelated by
$$V_o = X\dot{P}_o, \quad X = \begin{bmatrix} U & 0 \\ 0 & W \end{bmatrix}, \quad U = R_B^o \tag{11}$$
where B stands for the base frame and o a frame fixed with the body.

Also, consider the transformation between any two kinematic representations of the same body at o and o'. It is true that

$$V_{o'} = YV_o, \quad Y = \begin{bmatrix} R & PR \\ 0 & R \end{bmatrix}, \quad R = R_o^{o'}. \tag{12}$$

Related to the object-robot system, let us consider a six-link manipulator grasping fixedly a moving body. Thus the position of the mainpulator is uniquely determined by that of the body and vice versa. Differentiating both sides of the first equation in eq. (3) yields[9]

$$\dot{P}_o = \frac{\partial f}{\partial q}\dot{q} = J\dot{q} \tag{13}$$

where the subscript 0 to P is referred to as the coordinate frame fixed to the end-effector of the manipulator. Another kinematic relation connecting $\dot{q}$ and the velocity of the end-effector, V_E, is[9]

$$V_E = G\dot{q} \tag{14}$$

in this equation the physical meaning of $V_o = [v_{ox}, v_{oy}, v_{oz}, \omega_{ox}, \omega_{oy}, \omega_{oz}]^{\mathrm{T}}$ and $\dot{q} = [\dot{q}_1, \cdots, \dot{q}_6]^{\mathrm{T}}$ are both clear and simple, but in order to get the trajectory $q(t)$ from either of them, we need to perform integration of the Euler's equation (7).

Now with the knowledge of motion of the manipulator we can uniquely determine motion of the object from (12) and (14) by means of a series of transformations:

$$V_o = YV_E, \quad V_E = G\dot{q}, \tag{15}$$

Eq. (15) is the kinematic transition chain which presents itself in a recursive form, and relates $\dot{q}$ and velocities of the object with respect to those frames which are attached to the end-effector, center of mass, constraint point with constraints, etc.

4 Dynamics of Robotic Systems

The dynamics of a robotic system consists of equations of motions of the body and the manipulator. The equations of motion of the body are given by the Newton's law of motion and the Euler's equation of momentum:

$$\begin{aligned} \dot{V}_x &= L_x + m^{-1}f_x, & \dot{\omega}_x &= H_x(\omega) + J_x^{-1}m_x, \\ \dot{V}_y &= L_y + m^{-1}f_y, & \dot{\omega}_y &= H_y(\omega) + J_y^{-1}m_y, \\ \dot{V}_z &= L_z + m^{-1}f_z, & \dot{\omega}_z &= H_z(\omega) + J_z^{-1}m_z \end{aligned} \tag{16}$$

where

$$\begin{cases} L_x = \omega_z V_y - \omega_y V_z, \\ L_y = \omega_x V_z - \omega_z V_x, \\ L_z = \omega_y V_x - \omega_x V_y, \\ H_x = J_x^{-1}(J_y - J_z)\omega_y\omega_z, \\ H_y = J_y^{-1}(J_z - J_x)\omega_z\omega_x, \\ H_z = J_z^{-1}(J_x - J_y)\omega_x\omega_y. \end{cases} \tag{17}$$

m is the mass of the moving body, $oxyz$ is the coordinate system fixed to the body with origin o at the center of gravity and coordinate axes along the principal axes of inertia. f_x, f_y, f_z, and m_x, m_y, m_z are the components of the principal vector and the principal momentum of the force system resulting from all external forces acting on the body, such as reactive froces from the end-effector of the manipulator, gravity and reactions from the environments.

Note that since oxyz is a moving coordinate system, hence in the equations of motion such terms as L_x, L_y, L_z appear. The reason why we choose a moving coordinate system is that otherwise J_x, J_y, J_z will not remain constant during the motion such that the equations of motion consequently become nonstationary and much more complicated.

Eqs. (16) and (17) can be rewritten as a compact from

$$\frac{\mathrm{d}}{\mathrm{d}t}V = K + F, \tag{18}$$

where

$$K = [L_x,\ L_y,\ L_z,\ H_x,\ H_y,\ H_z]^{\mathrm{T}}, \tag{19}$$

$$F = EF',\quad F' = [f_x,\ f_y,\ f_z,\ m_x,\ m_y,\ m_z]^{\mathrm{T}}, \tag{20}$$

$$E = \mathrm{diag}[m^{-1}, m^{-1},\ m^{-1},\ J_x^{-1},\ J_y^{-1},\ J_z^{-1}]^{\mathrm{T}}. \tag{21}$$

Now turn to the equation of motion of the robot manipulator. We have

$$M(q)\ddot{q} + N(q,\dot{q}) = \tau + G^{\mathrm{T}}F_B, \tag{22}$$

where $q=[q_1,q_2,\cdots,q_6]^{\mathrm{T}}$ is the joint vector and $M(q)$ the inertia matrix; $N(q,\dot{q})$ denotes a vector including gravitational, centrifugal and Coriolis forces; $\tau=[\tau_1,\cdots,\tau_6]^{\mathrm{T}}$is the joint force/todrque vector and $F_B=[f_x,\ f_y,\ f_z,\ m_x,\ m_y,\ m_z]^{\mathrm{T}}$ the reactive force system exerted on the end-effector by the moving body; G is known as the Jacobian matrix defined in (14).

For empty payload case: $F_B=0$, eq. (22) is now reduced to

$$M(q)\ddot{q} + N(q,\dot{q}) = \tau^0,$$

where τ^0 is referred to as the empty payload control.

5 Constraints on the Moving Body

In most previous works dealing with compliance control[1~7], the constraints are applied on the end-effector and described abstractly by

$$c(P_o) = 0,\quad c:\mathrm{R}^n \to \mathrm{R}^r$$

where P_o is either the radius vector $[x,y,z]^{\mathrm{T}}$ or the position vecotr of the end-effector $[x,y,z,\varphi,\theta,\psi]^{\mathrm{T}}$, described in the base space. Compared with this popularly accepted mathematical model, for constraints describing contact of a rigid body held by the end-effector with other bodies while sliding, its seems better to employ another representation in terms of velocity components on *oxyz* fixed with the moving rigid body. This new constraint model may be given by

$$L(V) = L_o(t),\quad L:\mathrm{R}^6 \to \mathrm{R}^r,\quad L_o:\mathrm{R} \to \mathrm{R}^r, \tag{23}$$

$$V = [V_x,\ V_y,\ V_z,\ \omega_x,\ \omega_y,\ \omega_z]^{\mathrm{T}}$$

where $r<6$. For symmetrical bodies the principal axes x, y, z coincide with the axes of symmetry. In many cases the constraints can be represented in a linear form:

$$\alpha^{\mathrm{T}}V = \alpha_1(t)V_x + \alpha_2(t)V_y + \alpha_3(t)V_z = \bar{\alpha}(t), \tag{24}$$

$$\beta^{\mathrm{T}}\omega = \beta_1(t)\omega_x + \beta_2(t)\omega_y + \beta_3(t)\omega_z = \bar{\beta}(t). \tag{25}$$

The simplest case is described by some of the following equations:

$$v_x = 0,\quad v_y = 0,\quad v_z = 0,\quad \omega_x = 0,\quad \omega_y = 0,\quad \omega_z = 0. \tag{26}$$

For example, when the moving body is a rectangular cube with constraint surface being a plane perpendicular to the x-axis, then the constraints are:

$$V_x = 0, \quad \omega_y = 0, \quad \omega_z = 0. \tag{27}$$

This new model of constraints possesses several advantages. Especially, it is simple and remains the same in most practical cases, no matter whether constraints result from a plane or a curve surface and are stationary or nonstationary.

6 Control Problem

Let us investigate a typical case of constraint given by (27)

$$V_x = 0, \quad \omega_y = 0, \quad \omega_z = 0.$$

Obviously, in this case the reactive force from the constraint on the object should be

$$r_x^f, \ r_y^f = r_z^f = 0, \quad r_x^m = 0, \quad r_y^m, \ r_z^m. \tag{28}$$

Now from (17)~(21) the differential equations of our object-robot system become

$$\frac{\mathrm{d}V}{\mathrm{d}t} = K + F, \tag{29}$$

$$V = [0, \ v_y, \ v_z, \ \omega_x, \ 0, \ 0]^{\mathrm{T}},$$

$$K = [0, \ \omega_x v_z, \ -\omega_x v_z, \ 0, \ 0, \ 0]^{\mathrm{T}},$$

$$F = EF' = [m^{-1}f_x, \ m^{-1}f_y, \ m^{-1}f_z, J_x^{-1}m_x, \ J_y^{-1}m_y, \ J_z^{-1}m_z]^{\mathrm{T}},$$

$$F' = [f_x, \ f_y, \ f_z, \ m_x, \ m_y, \ m_z]^{\mathrm{T}} = F_{oE} + S^2F_o + S^3r$$

$F_{oE}=S^1F_E$: force system description of F_E from the end-effector at the center of mass of the object;

F_o: external known force system described at the certer of mass of the object;

$r=[r_x^f, \ 0, \ 0, \ 0, \ r_y^m, \ r_z^m]^{\mathrm{T}}$, the reaction force system from the contraints represented at the center of mass of the object.

Among the transition matrices, only S^1 depends upon the joint vector q. In our case $S^3=I_6$.

Expension of (29) into components turns out that

$$\begin{cases} 0 = m^{-1}(S_1^2F_o + F_{oE1} + r_x^f), \\ \dfrac{\mathrm{d}v_y}{\mathrm{d}t} = \omega_x v_z + m^{-1}(S_2^2F_o + F_{oE2}), \\ \dfrac{\mathrm{d}v_z}{\mathrm{d}t} = -\omega_x v_y + m^{-1}(S_3^2F_o + F_{oE3}), \\ \dfrac{\mathrm{d}\omega_x}{\mathrm{d}t} = J_x^{-1}(S_4^2F_o + F_{oE4}), \\ 0 = J_y^{-1}(S_5^2F_o + F_{oE5} + r_y^m), \\ 0 = J_z^{-1}(S_6^2F_o + F_{oE6} + r_z^m). \end{cases} \tag{30}$$

The control task is to determine F_{oE}, the dynamic load, and in turn the control on the robot such that the constraint reaction force approaches the desired value $r^d(t)$ and the motion of the object (robot too) approaches the desired motion represented by

$$V_o(t) \to V_o^d(t), \quad \text{or} \quad P_o(t) \to P_o^d(t), \quad \text{or} \quad q(t) \to q^d(t).$$

under the constraints in our case: $V^d(t) = [0, \ v_{oy}^d(t), \ v_{oz}^d(t), \ \omega_{oy}^d, \ 0, \ 0]^{\mathrm{T}}$.

Consider two schemes of force control as follows.

Programmed Force Scheme

$$r(t)=r^d(t)=[r_z^{fd}(t),\ 0,\ 0,\ 0,\ r_y^{md}(t),\ r_z^{md}(t)]^T \tag{31}$$

where r_z^{fd} is a given positive-valued function of time or a positive constant, while $r_y^{md}(t)$ and $r_z^{md}(t)$ are given functions of time or constants.

Force Feedback Scheme

$$r(t)=r^d(t)-D\int_0^t e_r(t)\mathrm{d}t,\quad e_r=r(t)-r^d(t), \tag{32}$$

where
$$r(t)=[r_z^f(t),\ 0,\ 0,\ 0,\ r_y^m(t),\ r_z^m(t)]^T,$$
$$r^d(t)=[r_z^{fd}(t),\ 0,\ 0,\ 0,\ r_y^{md}(t),\ r_z^{md}(t)]^T,$$
$$D=\mathrm{diag}[d_1,\ 0,\ 0,\ 0,\ d_5,\ d_6],\quad d_i>0,\quad (i=1,5,6).$$

For simplicity, $r_z^{fd}(t)$ is assumed to be a given positive constant and $r_y^{md}(t)$, $r_z^{md}(t)$ some given constants.

On the other hand, for establishing the control strategy we often need the current value of the dynamic reaction force at the end-effector. i. e. the dynamic load on the robot, F_B. From (29) we have

$$F_B=(S^1)^{-1}(F'-S^2F_o-S^3r),\quad F'=E^{-1}(\dot{V}-K).$$

Hence
$$F_B=(S^1)^{-1}[E^{-1}(\dot{V}-K)-S^2F_o-S^3r] \tag{33}$$

where
$$\dot{V}=[0,\ \dot{v}_y,\ \dot{v}_z,\ \dot{\omega}_x,\ 0,\ 0]^T.$$

7 Programmed Force Scheme

Consider the first scheme when the reaction force takes programmed desired values (31). A hierarchical control can be established in three layers as follows (Fig. 1).

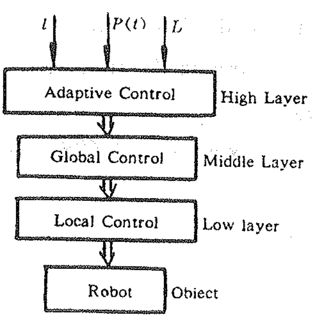

Fig. 1 Hierarchy of control

1) Lower layer——local control (a special case of multiple robot system[10] where each robot is a subsystem): empty payload robot control .

For the robot described by

$$M(q)\ddot{q}+N(q,\dot{q})=\tau,$$

employing the computed torque mehtod, or other methods such as variable structure control and so on, we have

$$\tau^L=N(q,\dot{q})+M(q)[\ddot{q}^d-K_v\dot{e}-K_pe], \tag{34}$$

where
$$e=q(t)-q^d(t).$$

This control part in the local layer of our hierarchical control drives the robot itself to the desired motion, i. e. $e(t)\to 0$, or $q(t)\to q^d(t)$.

2) Middle layer——global control: task control.

This control part τ^M is dedicated to the tracking activity for the object. Based on the force transition chain as in (6) and (33), we get

$$\tau^M=G^TF_B=G^T(S^1)^{-1}[E^{-1}(\dot{V}-K)-S^2F_o-S^3r]. \tag{35}$$

3) Higher layer——adaptive control: Adaptivity to object change.

In almost all cases, it is desirable to develop a control strategy for the robot such that its ability to implement tracking tasks is not affected by the mass, size and desired trajectory of the held object. The mechanical parameters of the object may be described by inertial parameter vector l involving mass m, moments of inertia J_x, J_y, J_z, and some geometric parameter vector L, and the desired motion $P_o^d(t)$. By considering these values as input to a control unit to get the control law for the current object in the higher adaptive layer, a three-layer hierarchical control strategy is built up as shown in Fig. 1. Now we obtain the overall control

$$\tau = \tau^L + \tau^M. \tag{36}$$

Generally, the objects to be tracked are limited in variety with measurable inertial and geometric parameters l and L and they are all known. For the sake of finishing this set tasks, a control strategy implemented by a parameter-adjustable computer unit is probably superior to other techniques such as robust control strategy, on-line identification and the like.

8 Force Feedback Scheme

Solving from (35) for r, we obtain

$$r = (S^3)^{-1}[E^{-1}(\dot{V} - K) - S^2F_o] - (S^3)^{-1}s^1(G^T)^{-1}\tau^M \tag{37}$$

Let

$$v = -(S^3)^{-1}s^1(G^T)^{-1}\tau^M$$

and recall Eq. (32), we get

$$r^d(t) - D\int_0^t e_r dt = (S^3)^{-1}[E^{-1}(\dot{V} - K) - S^2F_0] + v \tag{38}$$

where v is the new control vector obtained form τ^M by a transformation. Now letting

$$v = r^d(t) - (S^3)^{-1}[E(\dot{V} - K) - S^2F_o],$$

which implies
$$\tau^M = -(S^3)^{-1}\{r^d(t) - (S^3)^{-1}[E^{-1}(\dot{V} - K) - S^2F_o]\}, \tag{39}$$

we arrive at
$$e_r(t) + D\int_0^t e_r(t)dt = 0. \tag{40}$$

Equation (40) means that the reaction force is asymptotically stable as a solution of a first order dynamic system. The control τ^M not only guarantees the asymptotically approaching property $r(t) \to r^d(t)$, but also bear the dynamic load necessary to track the object. Eventually we obtain the control $\tau = \tau^L + \tau^M$ where τ^M is given by (39) and τ^L is defined as in (34).

9 Example and Simulation

A three-link planar robot shown in Fig. 2 is investigated, where the constraint is a vertical line which moves sinucoidally with

$$D(t) = 1.5 + 0.5\sin t.$$

The initial condition are given below

$$x(0) = 1.5, \quad y(0) = 0.3, \quad \theta(0) = 0,$$
$$\dot{x}(0) = 0.5, \quad \dot{y}(0) = 0.2, \quad \dot{\theta}(0) = 0,$$
$$r_x^f(0) = 10, \quad r_y^f = 0, \quad r_z^m = 0.5$$

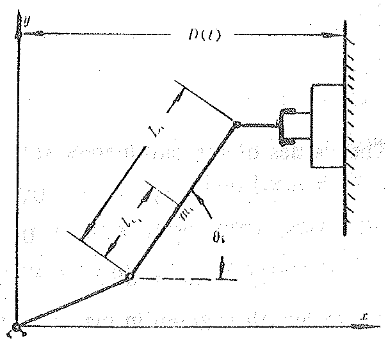

Fig. 2 Robot with moving constraints

under the desired task of the object described as

$$r_z^{fd} = 15, \quad r_z^{md} = 1, \quad x^d(t) = 1.5 + 0.5\sin t,$$

$$y^d = 0.5\sin[\frac{\pi}{2}(t-1)], \quad \dot{\theta}(t) = 0,$$

Now the dynamic equation of the robot is written as

$$M(q)\ddot{q} + N(q,\dot{q}) = \tau - G^T F_E,$$

where

$$M(q) = \begin{bmatrix} m_1 l_{c1}^2 + m_2 L_1^2 + m_3 L_1^3 + l_1 & (m_2 L_1 l_{c2} + m_3 L_1 L_2)\cos(q_2 - q_1) & m_3 L_1 l_{c3}\cos(q_3 - q_1) \\ (m_2 L_1 l_{c2} + m_3 L_1 L_2)\cos(q_2 - q_1) & m_2 l_{c2}^2 + m_3 L_2^2 + l_2 & m_2 L_2 l_{c3}\cos(q_3 - q_2) \\ m_3 L_1 l_{c3}\cos(q_3 - q_1) & m_3 L_2 l_{c3}\cos(q_3 - q_2) & m_3 l_{c3}^2 + l_3 \end{bmatrix},$$

$$N(q,\dot{q}) = \begin{bmatrix} (m_2 L_1 l_{c2} + m_3 L_1 L_2)\sin(q_1 - q_2)\dot{q}_2^2 + m_3 L_1 l_{c3}\sin(q_1 - q_3)\dot{q}_3^2 \\ (m_2 L_1 l_{c2} + m_3 L_1 L_2)\sin(q_1 - q_2)\dot{q}_1^2 + m_3 L_2 l_{c3}\sin(q_3 - q_2)\dot{q}_3^2 \\ m_3 L_1 l_{c3}\sin(q_1 - q_3)\dot{q}_1^2 + m_3 L_2 l_{c3}\sin(q_3 - q_2)\dot{q}_2^2 \end{bmatrix}$$

$$+ \begin{bmatrix} (m_1 l_{c1} + m_2 L_1 + m_3 L_1) g\cos q_1 \\ (m_2 l_{c2} + m_3 L_2) g\cos q_2 \\ m_3 g l_{c3}\cos q_3 \end{bmatrix},$$

$$G = \begin{bmatrix} L_1\sin(q_3 - q_1) & L_2\sin(q_3 - q_2) & 0 \\ L_1\cos(q_3 - q_1) & L_2\cos(q_3 - q_2) & L_3 \\ 0 & 0 & 1 \end{bmatrix}, \quad F_E = [f_{Ex}, f_{Ey}, m_{Ez}]^T, \quad \tau = [\tau_1, \tau_2, \tau_3]^T.$$

Note that G is the matrix defined by relation $V_E = G\dot{q}$, where V_E is the velocity and (x_E, y_E) the coordinates of the end-effector.

Denote the mass and moment of inertia of the body by m and l respectively, then the equation of motion of the body will be

$$\frac{d}{dt}\begin{bmatrix} v_x \\ v_y \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} m^{-1} & 0 & 0 \\ 0 & m^{-1} & 0 \\ 0 & 0 & l^{-1} \end{bmatrix} ((Y^T)^{-1}\begin{bmatrix} f_{Ex} \\ f_{Ey} \\ m_{Ez} \end{bmatrix} + \begin{bmatrix} r_x^f \\ r_y^f \\ r_z^m \end{bmatrix}) + \begin{bmatrix} -g\sin\theta \\ -g\cos\theta \\ 0 \end{bmatrix}$$

where (x, y, θ) is the coordinates of the body and $r = [r_x^f, r_y^f, r_z^m]^T$ the constraint force represented in the frame fixed with center of the mass of the object. In this case, $r_y^f = 0$. The velocity of the body is represented by

$$V = YV_E, \quad Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & d \\ 0 & 0 & 1 \end{bmatrix},$$

The values of the parameters are

$$m_1 = m_2 = 10, \quad m_3 = 2, \quad l_1 = l_2 = 0.8, \quad l_3 = 0.5,$$
$$L_1 = L_2 = 1.0, \quad L_3 = 0.5, \quad l_{c1} = l_{c2} = 0.5, \quad l_{c3} = 0.25,$$
$$m = 30, \quad l = 2.5, \quad d = 0.2,$$

where length is given in meters, time in second, mass in kg, moment of inertia in kgm^2 and force in Newtons. To use the two proposed control laws, we choose $K_v = 40$, $K_P = 400$.

Simulation results are shown in Fig. 3 and Fig. 4. Fig. 3 gives the results in case of programmed force constraints, while Fig. 4 the case of force feedback control. Figures with alphabet a shows the tracking position and velocity errors, *b* the errors of constraint force and momentrum, and *c* the components of the torque on the robot.

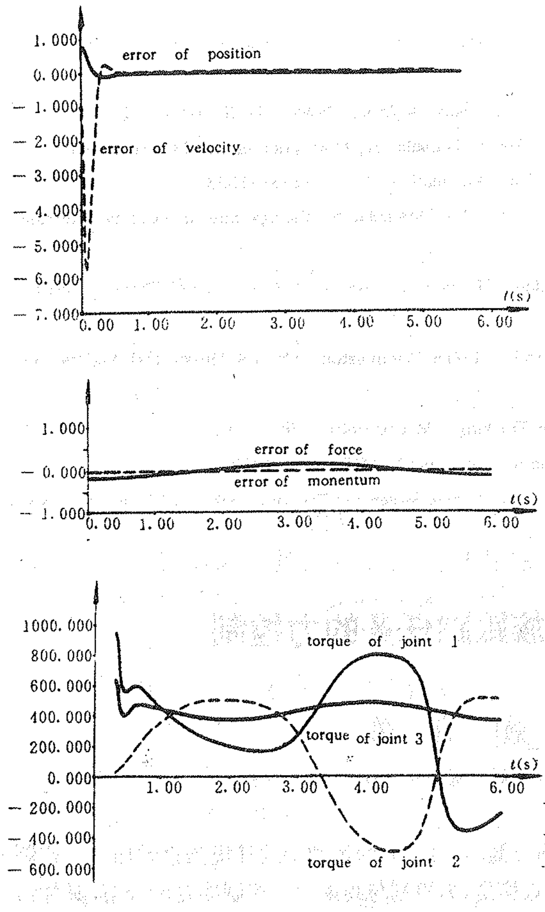

Fig. 3 Simulations of programmed force scheme

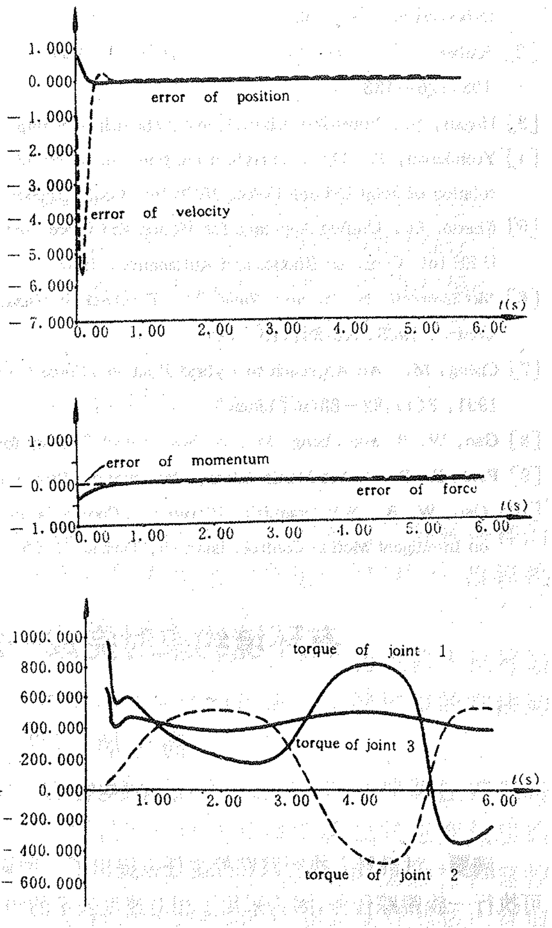

Fig. 4 Simulations of force feedback scheme

Simulations state that the design procedures of both control schemes are simple. In case of programmed force control the static force error does not vanish but oscilates slightly. In the case of force feedback the static force error approaches zero quickly.

10 Conclusion

In the previous sections, by using local coordinate systems attached to the object, a new type of 6×6 matrix representations for static, kinematic and dynamic quantities of a object-manipulator system[8] is proposed. This is a powerful tool, especially when the object is constrained. It is also shown that a hierarchical control stratetgy is tracking a set of objects. For force control two simple control schemes are investigated. Besides, simulations indicate that the proposed control laws are acceptable.

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有环境约束时完成一族跟踪任务的力控制

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摘要: 对机器人执行跟踪控制任务提出了一种新方案.当机器人的运动受到环境约束时用这一控制可执行一族跟踪任务,因为采用了相对速度表示的约束模型,力控制与约束运动控制能自动分解.具体给出了两种力控制律.此外,对于复杂的控制任务建立了递阶的控制结构.

关键词: 有约束机器人控制; 反作用力控制; 一族跟踪控制; 混合控制; 递阶控制

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