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A Design Method for Control Systems Possessing Integrity*

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Abstract: This paper is concerned with the design method of the state feedback control system which possesses integrity. A control law which maintains the closed-loop system asymptotically stable against arbitrary sensor failure is derived by utilizing the solution of a type of matrix equation and constraint condition. The simplicity of this method is demonstrated with two numerical examples.

Key words: integrity; feedback control; sensor failure

1 Introduction

A multivariable feedback control system may become unstable when the feedback signals are switched off by a failure in the actuator or the sensor. We say that the system possesses integrity if it still remains stable in the presence of such failures. This is an inherent property of multi-input multi-output control systems: the over closed-loop system under the failure is not necessarity stable even if the open-loop system is stable.

Some work has previously been done on integrity. In the state-space approach, Shimemura and Fujita[1] studied a state feedback system which possesses integrity against actuator failures by solving a matrix equation. Furthermore, Ni and Wu[2] gave a designing approach for multivariable control systems possessing integrity by using Lyapunov equation and Riccati equation. These methods yield a sufficient stability margin, but there is no guide for control system design possessing integrity against sensor failures.

In this paper, a new method for state feedback system which possesses integrity against sensor failures is proposed. Firstly, we introduce about the questions of the sensor failures. Secondly, the design method for linear continuous and discrete with state feedback systems possessing integrity against the sensor failures is derived. Finally, the effectiveness of the proposed mehtod is illustrated with two numerical examples.

2 Statement of the Problem

We consider the linear time-invariant dynamical system

$$\dot{x} = Ax + Bu \tag{2.1}$$

where x is an n-dimensional vector, u is an r-dimensional input vector, and A and B are $n \times n$ and $n \times r$ constant matrices, respectively. We assume throughout this paper.

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a) (A,B) is a controllable pair, and b) A is asymptotically stable.

Now we discuss a method of deciding the state feedback gain matrix K by which the closed-loop system still remains asymptotically stable, even if one or more feedback loop failures. System having such a property is called "integrity".

Assume that all state variables are measureable. To represent the sensor failure in the feedback loops, we define a "switching matrix"

$$L = \mathrm{diag}(\delta_1, \delta_2, \cdots, \delta_n) \quad (2.2)$$

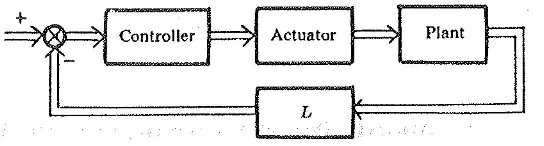

Fig 1 Switch matrix

where δ_i $(i=1,2,\cdots,n)$ is either 1 or 0. When this matrix L is placed between the plant and the controller, as in Fig. 1, we have a model of the failure in the sensor channel: $\delta_i=0$ corresponds to the situation of the ith channel being "open" as the result of the failure, while $\delta_i=1$ corresponds to the normal situation of the ith channel. We require the plant to be asymptotically stable, even when all δ_i are zero, which means the complete disconnection of the plant from feedback loops. The feedback control may be regarded as

$$u = KLx.$$

Now we consider the feedback system

$$\dot{x} = (A + BKL)x$$

instead of the system (2.1). Then the problem of integrity is formulated as finding the feedback gain matrix K to ensure the asymptotic stability of the system matrix $A+BKL$ for all $L \in \mathscr{L}$, where $\mathscr{L}$ is a set of L matrices for all the possible combinations of 1 and 0 in the diagonal elements.

3 Main Result for Linear Systems Possessing Integrity

Now we consider a method of synthesis of a state feedback system possessing integrity. Define the following concept of matrix throughout this section.

Definition 3.1 Let

$$A = \{a_{ij}\} \in \mathrm{R}^{n\times n}.$$

The notation $|A|$ denotes the matrix which elements are $|a_{ij}|(i,j=1,\cdots,n)$.

Definition 3.2 Let

$$A = \{a_{ij}\}, \quad B = \{b_{ij}\} \in \mathrm{R}^{n\times n}.$$

The notation $A \geqslant B$ denotes that $a_{ij} \geqslant b_{ij}$.

The analysis of 3.1 and 3.2 also make use the following result.

Lemma 3.1[3] If $A \leqslant |A| \leqslant B$ ($\leqslant$and $|\cdot|$ are defined by Definition 3.1, 3.2), $A,B \in \mathrm{R}^{n\times n}$, then, we have following inequality

$$\rho(A) \leqslant \rho(|A|) \leqslant \rho(B)$$

where $\rho(\cdot)$ is spectral radius.

3.1 Linear Continuous System

Consider the linear continuous system (2.1). Applying the feedback control

$$u = KLx.$$

We have the closed-loop system described by

$$\dot{x}(t) = (A + BKL)x(t). \tag{3.1}$$

Now consider close-loop system (3. 1). Then we obtain the following results on the stability of (3. 1) Note that the following symbols $\leqslant$ and $|\cdot|$ are defined as above.

Theorem 3. 1. 1 Assume the conditions (a)～(b) be held. If, for given positive definite matrix Q,

$$\rho(2|PBB^{\mathrm{T}}P|) < \lambda_{\min}(Q) \tag{3.2}$$

where P is the solution of matrix equation

$$A^{\mathrm{T}}P + PA = -Q \tag{3.3}$$

then the perturbed system (3. 1) with $K = -B^{\mathrm{T}}P$ is asymptotically stable, i. e. the perturbed system possesses integrity.

Proof Consider a Lyapunov function defined by

$$V(x) = x^{\mathrm{T}}Px.$$

The time derivative of $V(x)$ along the solution of (3. 1) is given

$$\begin{aligned}\dot{V}(x(t)) &= \dot{x}^{\mathrm{T}}Px + x^{\mathrm{T}}P\dot{x} = x^{\mathrm{T}}(A^{\mathrm{T}} + LK^{\mathrm{T}}B^{\mathrm{T}})Px + x^{\mathrm{T}}P(A + BKL)x \\ &= x^{\mathrm{T}}(A^{\mathrm{T}}P + PA - LPBB^{\mathrm{T}}P - PBB^{\mathrm{T}}PL)x \\ &= x^{\mathrm{T}}(-Q - LPBB^{\mathrm{T}}P - PBB^{\mathrm{T}}PL)x.\end{aligned}$$

If the following inequality

$$\lambda(-Q - LPBB^{\mathrm{T}}P - PBB^{\mathrm{T}}PL) < 0 \tag{3.4}$$

holds, we have

$$\dot{V}(x(t)) < 0.$$

It is known that

$$\lambda(-Q - LPBB^{\mathrm{T}}P - PBB^{\mathrm{T}}PL) \leqslant \lambda_{\max}(-Q) + \lambda_{\max}(-LPBB^{\mathrm{T}}P - PBB^{\mathrm{T}}PL).$$

Note that Q is positive definite and $\lambda_{\max}(-Q) = \lambda_{\min}(Q)$.

If

$$\lambda_{\max}(-LPBB^{\mathrm{T}}P - PBB^{\mathrm{T}}PL) < \lambda_{\min}(Q) \tag{3.5}$$

then (3. 4) will be satisfied. Note that $|-L| \leqslant I$ (I is unit matrix), then

$$(-L)PBB^{\mathrm{T}}P + PBB^{\mathrm{T}}P(-L) \leqslant |(-L)PBB^{\mathrm{T}}P + PBB^{\mathrm{T}}P(-L)| \leqslant 2|PBB^{\mathrm{T}}P|.$$

From Lemma 3. 1, we have

$$\begin{aligned}\rho((-L)PBB^{\mathrm{T}}P + PBB^{\mathrm{T}}P(-L)) &\leqslant \rho(|(-L)PBB^{\mathrm{T}}P + PBB^{\mathrm{T}}P(-L)|) \\ &\leqslant \rho(2|PBB^{\mathrm{T}}P|).\end{aligned}$$

Since the condition (3. 2) is satisfied, we have conclusion

$$\rho((-L)PBB^{\mathrm{T}}P + PBB^{\mathrm{T}}P(-L)) < \lambda_{\min}(Q).$$

The result (3. 5) follows and hence system (3. 1) is asymptotically stable. Q. E. D.

We propose the design procedure of the continuous system possessing integrity as following.

Step 1 Choose an appropriate Q.

Step 2 Calculate the solution P of the equation (3. 3).

Step 3 If the condition of Theorem 3. 1. 1 is not satisfactory, then repeat Step 1.

Step 4 Calculate feedback gain matrix $K=-B^{T}P$.

Remark 1 If condition (3. 2) is selected as $Q=I$, then it is easy to see that (3. 2) is $\rho(2|PBB^{T}P|)<1$.

Remark 2 Q should be positive definite and satisfy the constraint condition (3. 2).

3.2 Linear Discrete System

Consider the linear discrete system

$$x(t+1) = Ax(t) + Bu(t) \tag{3.6}$$

where $A\in R^{n\times n}$, $B\in R^{n\times p}$. Assume the condition (a), (b) be held.

Now we apply a feedback control with the matrix L which represents the sensor states.

$$u = KLx$$

for system (3. 6). The colsed-loop system is

$$x(t+1) = (A+BKL)x(t). \tag{3.7}$$

Theorem 3. 2. 1 Assume the conditions (a), (b) be held. If, for given positive definite matrix Q,

$$\rho(|APPA^{T}| + |BB^{T}P(I+P)PBB^{T}|) < \lambda_{\min}(Q) \tag{3.8}$$

where I is unit matrix and P is the solution of matrix equation

$$APA^{T} - P + Q = 0 \tag{3.9}$$

then the perturbed system (3. 7) with the feedback gain $K=-B^{T}P$ is asymptotically stable, i. e., the perturbed system possesses integrity.

Proof It is clear that the stability of the system (3. 7) is equivalent to the stability of system

$$Y(t+1) = (A^{T} - LPBB^{T})Y(t).$$

Let Lyapunov function

$$V(Y) = Y(t)^{T}PY(t)$$

then

$$\begin{aligned}\Delta V(Y) &= Y(t)^{T}(APA^{T} + AP(-L)PBB^{T} + BB^{T}P(-L)PA^{T} + BB^{T}PLPLPBB^{T} - P)Y(t)\\ &= Y(t)^{T}(-Q + AP(-L)PBB^{T} + BB^{T}P(-L)PA^{T} + BB^{T}PLPLPBB^{T})Y(t).\end{aligned}$$

For two real vectors $\alpha=[\alpha_1, \alpha_2, \cdots, \alpha_n]^{T}$ and $\beta=[\beta_1, \beta_2, \cdots, \beta_n]^{T}$, it can be shown that $\alpha^{T}\alpha+\beta^{T}\beta-2\alpha^{T}\beta\geqslant 0$ and hence $2\alpha^{T}\beta\leqslant\alpha^{T}\alpha+\beta^{T}\beta$. Adopting this, it follows that

$$\begin{aligned}&Y(t)^{T}(AP(-L)PBB^{T} + BB^{T}P(-L)PA^{T})Y(t)\\ &\leqslant Y(t)^{T}(AP(-L)(-L)PA^{T} + BB^{T}PPBB^{T})Y(t)\\ &= Y(t)^{T}(APL^{2}PA^{T} + BB^{T}PPBB^{T})Y(t)\\ &\leqslant Y(t)^{T}(APPA^{T} + BB^{T}PPBB^{T})Y(t).\end{aligned}$$

Thus $$\Delta V(Y) \leqslant Y(t)^{T}(-Q + APPA^{T} + BB^{T}P(I+LPL)PBB^{T})Y(t).$$

If $$\lambda(-Q + APPA^{T} + BB^{T}P(I+LPL)PBB^{T}) < 0 \tag{3.10}$$

we have $\Delta V(x)<0$. It is clear that

$$\lambda(-Q + APPA^{T} + BB^{T}P(I+LPL)PBB^{T}) \leqslant \lambda_{\max}(-Q) + \lambda_{\max}(APPA^{T} + BB^{T}P(I+LPL)PBB^{T}).$$

If

$$\lambda_{\max}(APPA^T + BB^TP(I + LPL)PBB^T) < \lambda_{\min}(Q) \tag{3.11}$$

then (3. 10) will be satisfied.

By using a similar procedure to that of Theorem 3. 1. 1, we have

$$APPA^T + BB^TP(I + LPL)PBB^T \leqslant |APPA^T| + |BB^TP(I + P)PBB^T|.$$

From Lemma 3. 1

$$\rho(APPA^T + BB^TP(I + LPL)PBB^T) \leqslant \rho(|APPA^T| + |BB^TP(I + P)PBB^T|).$$

Since condition (3. 8) is satisfied, we have conclusion

$$\rho(APPA^T + BB^TP(I + LPL)PBB^T) < \lambda_{\min}(Q).$$

Thus, (3. 11) is satisfied, and the system (3. 7) is asymptotically stable. Q. E. D.

4 Applications

Example 1 We consider a design problem possessing integrity for linear state continuous feedback systems

$$\dot{x} = \begin{bmatrix} -1 & -2 & 0 \\ 1 & -3 & 0 \\ -0.4 & 1 & -1 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0.5 & 0.5 \\ -0.5 & 1 \end{bmatrix} u. \tag{4.1}$$

We assume that all state variables are measureabe. Suppose Q is chosen as

$$Q = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

which is positive definite. We obtain

$$P = \begin{bmatrix} 0.037 & -0.013 & -0.003 \\ -0.013 & 0.03 & 0.014 \\ -0.003 & 0.014 & 0.05 \end{bmatrix}.$$

Check the condition (3. 2)

$$\rho(2|PBB^TP|) = 0.009932 < \lambda_{\min}(Q) = 0.1$$

which is satisfied. Thus, the state feedback gain matrix can be obtained as follows:

$$K = -B^TP = -\begin{bmatrix} 0.032 & -0.005 & -0.021 \\ -0.0035 & 0.001 & -0.043 \end{bmatrix}.$$

Since Theorem 3. 1. 1 conditions are satisfied, we say that the system (4. 1), with $u = KLx$, possesses integrity if the sensor appear failures. The integrity of the designed system can also be checked from the following analysis.

The eigenvalue of the close-loop system is

$$\lambda(A + BK) = \{-2.022 \pm 0.996j, -0.952\}.$$

It is clear that $A+BK$ is stable. Let ith sensor failure, the eigenvalues of the close-loop system

$$i = 1, \quad \lambda(A + BKL) = \{-2.006 \pm 0.994j, -0.953\}, \tag{4.2}$$

$$i = 2, \quad \lambda(A + BKL) = \{-2.023 \pm 0.997j, -0.953\}, \tag{4.3}$$

$$i = 3, \quad \lambda(A + BKL) = \{-2.015 \pm 1.000j, -1\}, \tag{4.4}$$

$$i=1,2,\quad \lambda(A+BKL)=\{-2.007\pm 0.995j,\ -0.952\}, \tag{4.5}$$

$$i=2,3,\quad \lambda(A+BKL)=\{-2.016\pm 1.001j,\ -1\}, \tag{4.6}$$

$$i=1,3,\quad \lambda(A+BKL)=\{-1.999\pm 0.998j,\ -1\}. \tag{4.7}$$

From (4.2)~(4.7), we can ascertain that the state feedback system possesses integrity.

Example 2 Consider the control problem against sensor failures for a linear discrete system given by the following equation:

$$x(t+1)=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}x(t)+\begin{bmatrix}0.2\\ 0.5\end{bmatrix}u(t). \tag{4.8}$$

We give the positive definite matrix

$$Q=\begin{bmatrix}0.48 & 0\\ 0 & 0.48\end{bmatrix}.$$

Applying $APA^{\mathrm{T}}-P+Q=0$, we have

$$P=\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix}.$$

We may check

$$\rho(|APPA^{\mathrm{T}}|+|BB^{\mathrm{T}}P(I+P)PBB^{\mathrm{T}}|)=0.034<\lambda_{\min}(Q)=0.48.$$

The matrix of the close-loop system is

$$A+BK=\begin{bmatrix}0.18 & -0.05\\ -0.05 & 0.1875\end{bmatrix},\quad K=[-0.1\ \ -0.25].$$

It is obvious that $|\lambda(A+BK)|<1$.

Let ith sensor failure, the eigenvalues of the close-loop system

$$i=1,\quad \lambda=\{0.2,\ 0.075\},$$

$$i=2,\quad \lambda=\{0.2,\ 0.18\}.$$

Thus system (4.8) still keep the stable.

5 Conclusion

In this paper, we give a method of a state feedback system, which satisfies the integrity requirement. The presented method is rather simple.

Acknowledgment Sincere thanks are due to Professor Zhang zhongjun and Professor Han zhengzhi for their many helpful suggestions and discussions.

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控制系统具有完整性的一种设计方法

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摘要：本文研究了对任意传感器失效反馈系统仍保持稳定性的问题，给出的状态反馈律对传感器故障具有完整性．反馈律的设计是通过解 Lyapunov 方程以及考虑一个约束条件而获得的，并给出了连续和离散系统两个结果．最后用两个例子说明了本方法的有效性．

关键词：完整性；反馈控制；传感器故障

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3）重要内容以表格方式进行总结

在写作风格上，作者根据他们多年的经验，采用每章结束后将重要内容进行表格式总结的写法．全书共有100多个总结表格使学生学习使用．一方面，通过每章末之表格使学习者一目了然所学过之重要内容，便于记忆；另一方面，也便于从事实际工作的技术人员快速查阅所学过的重要知识；

4）提供习题解答与试题手册

本书提供精选习题之解答与仿真实验指导．此外，还为教师提供八套试题及其详解．毫无疑问，这将为教师在使用此书时提供大的方便；

5）PCET 教学软件

作者们为了使学习者能加深理解书中的内容，特开发了这套教学软件 PCET(Process Control Engineering Teachware)．该软件包括了时域分析，频域分析，根轨迹法直到工业过程应用等有关内容．(关于 PCET 之详细介绍已刊登在 The International Journal of Engineering Education Vol. 8, No. 3, 1992)该软件的使用极为方便，只要按照屏幕上的提示进行就行，可以免去学生花太多时间去阅读使用手册，因此深受学生的欢迎．在加拿大 Alberta 大学化工系的教学实践已经证明．这套软件对加深理解过程控制的基本理论与应用是大有裨益的．

(Process Control Engineering)一书的编写与出版，无疑对国内外的过程控制工程教学与研究会起到积极的推动作用．

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