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Robustness of Reduced-Order Multivariable State-Space Self-Tuning Controller

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Abstract: In this paper, we present a quantitative analysis of the robustness of a reduced-order pole-assignment state-space self-tuning controller for a multivariable adaptive control system whose order of the real process is higher than that of the model used in the controller design. The result of stability analysis shows that, under a specific bounded modelling error, the adaptively controlled closed-loop real system via the reduced-order state-space self-tuner is BIBO stable in the presence of unmodelled dynamics.

Key words: adaptive control; reduced-order controller; robust control; state-space self-tuner

1 Introduction

The problem of robustness of an adaptive control system has recently been studied by many authors[1~5]. This is because the development of adaptive controllers for adaptive control systems is based on the assumption that the model used in the controller design is an accurate representation of the real process; however, the degree of most real processes is often higher than that of the model used in practice. As a result, a stability problem may occur due to a mismatch of the orders of the modeled processes and the real processes[1]. Hence, a study of robust stability of the utilized algorithms for the controller design is necessary.

During the last decade, vast amount of research was devoted to quantitative analysis of the robustness of self-adaptive algorithms such as the development of conic sector theory and normalized system scheme[2,3]. In reference [4], the normalized parameter estimation approach combined with a dead-zone method in which the modelling errors are treated as a bounded disturbance and utilized as a parameter adaptation stopping criterion to guarantee global stability was developed. In contrast, in reference [5], the robust stability of a multivariable adaptive controller based on a factorization approach was established, which is useful for the robust stability analysis of adaptive algorithms.

In this paper, we are concerned with the robust stability of the multivariable adaptive control system via the reduced-order state-space self-tuning controller developed in reference [6]. Our approach to quantitative analysis of the robust stability of the adaptive control system[6] can

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be described as follows. First, we utilize the normalized parameter estimation scheme[4] to carry out the parameter estimation with the presence of unmodelled dynamics. Then, we use the analytical method developed in reference [5] to resolve the robust stability of the adaptive control system via the reduced-order state-space self-tuner[6]. Finally, we determine the bound of the modelling error with which the self-tuner can be tolerated.

2 System Description

In this paper, both of the plant and the reduced-order model are assumed controllable and observable.

Consider the following m-input-output block observer-type discrete-time stochastic linear plant:

$$x_o^*(k) = A_o^* x_o^*(k-1) + B_o^* u(k-1) + K_o^* e^*(k-1), \tag{1a}$$

$$y(k) = C_o^{*\mathrm{T}} x_o^*(k) + e^*(k) \tag{1b}$$

where

$$A_o^* = \begin{bmatrix} -A_{o1} & I_m & 0_m & \cdots & 0_m \\ -A_{o2} & 0_m & I_m & \cdots & 0_m \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -A_{or} & 0_m & 0_m & \cdots & 0_m \end{bmatrix}, \quad B_o^* = \begin{bmatrix} B_{o1} \\ B_{o2} \\ \vdots \\ B_{or} \end{bmatrix}, \quad x_o^*(k) = \begin{bmatrix} x_{o1}^*(k) \\ x_{o2}^*(k) \\ \vdots \\ x_{or}^*(k) \end{bmatrix},$$

$$C_o^{*\mathrm{T}} = [I_m\ 0_m\ 0_m\ \cdots\ 0_m]_{m\times mr},$$

$u(k)\in \mathrm{R}^m$ and $y(k)\in \mathrm{R}^m$ are input and output vectors, respectively; block elements A_{oi}, $B_{oi}\in \mathrm{R}^{m\times m}(i=1,2,\cdots,r)$ are constant matrices, $x_{oi}^*(k)\in \mathrm{R}^m(i=1,2,\cdots,r)$ $e^*(k)\in \mathrm{R}^m$ is the innovation process which is a white noise process with zero mean and covariance $R_e^*\in \mathrm{R}^{m\times m}$ and

$$\sup_{0<k<\infty} \| e^*(k) \| \leqslant \delta_o \tag{2}$$

with $\delta_o>0$, $K_o^*\in \mathrm{R}^{mr\times m}$ is the Kalman gain matrix.

$$A^*(z^{-1}) = I_m + A_{o1}z^{-1} + A_{o2}z^{-2} + \cdots + A_{or}z^{-r}, \tag{3a}$$

$$B^*(z^{-1}) = B_{o1}z^{-1} + B_{o2}z^{-2} + \cdots + B_{or}z^{-r}, \tag{3b}$$

$$D^*(z^{-1}) = I_m + D_{o1}z^{-1} + D_{o2}z^{-2} + \cdots + D_{or}z^{-r}, \tag{3c}$$

and

$$D_{oi} = A_{oi} + K_{oi}, \quad i = 1,2,\cdots,r. \tag{3d}$$

It is observed from (3d) that the Kalman gain K_{oi} can be directly computed from the estimated parameters D_{oi} and A_{oi}. An alternate representation of the original system in (1) can be described as follows:

$$y(k) = \theta^{*\mathrm{T}}\Phi^*(k) + e^*(k), \tag{4a}$$

where

$$\theta^{*\mathrm{T}} = [A_{o1},\cdots,A_{or},B_{o1},\cdots,B_{or},D_{o1},\cdots,D_{or}], \tag{4b}$$

$$\Phi^*(k) = [-y_1(k-1),\cdots,-y_m(k-1),\cdots,-y_m(k-r),u_1(k-1),\cdots,$$
$$u_m(k-1),\cdots,u_m(k-r),e_1^*(k-1),\cdots,e_m^*(k-1),\cdots,e_m^*(k-r)]^{\mathrm{T}}. \tag{4c}$$

The θ^* in (4) is the parameter matrix of the original system. For a reduce-order controller design, a reduced-order observable model is required as

$$x_o(k) = A_o x_o(k-1) + B_o u(k-1) + K_o e(k-1), \tag{5a}$$

$$y(k) = C_o^{\mathrm{T}} x_o(k) + e(k), \tag{5b}$$

where

$$A_o = \begin{bmatrix} -A_{o1} & I_m & 0_m & \cdots & 0_m \\ -A_{o2} & 0_m & I_m & \cdots & 0_m \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -A_{on} & 0_m & 0_m & \cdots & 0_m \end{bmatrix}, \quad B_o = \begin{bmatrix} B_{o1} \\ B_{o2} \\ \vdots \\ B_{on} \end{bmatrix}, \quad K_o = \begin{bmatrix} K_{o1} \\ K_{o2} \\ \vdots \\ K_{on} \end{bmatrix}, \quad x_o(k) = \begin{bmatrix} x_{o1}(k) \\ x_{o2}(k) \\ \vdots \\ x_{on}(k) \end{bmatrix},$$

$$C_o^{\mathrm{T}} = [I_m \ 0_m \ 0_m \ \cdots \ 0_m]_{m \times mn},$$

where $n \leqslant r$, and $e(k)$ is the innovation process of the model.

The equivalent observable ARMAX model of (5) is

$$A(z^{-1})y(k) = B(z^{-1})u(k) + D(z^{-1})e(k), \tag{6}$$

The alternate form of the model in (6) can also be rewritten as follows:

$$y(k) = \theta^{\mathrm{T}}\Phi(k) + e(k), \tag{7a}$$

where

$$\theta^{\mathrm{T}} = [A_{o1}, \cdots, A_{on}, B_{o1}, \cdots, B_{on}, D_{o1}, \cdots, D_{on}], \tag{7b}$$

$$\Phi(k) = [-y_1(k-1), \cdots, -y_m(k-1), \cdots, -y_m(k-n), u_1(k-1), \cdots, u_m(k-1), \cdots, u_m(k-n), e_1^*(k-1), \cdots, e_m^*(k-1), \cdots, e_m^*(k-n)]^{\mathrm{T}}. \tag{7c}$$

where θ is the parameter matrix of the reduced-order model. The $e(k)$ in (6) can be decomposed into two terms,

$$e(k) = \bar{e}(k) + e^*(k), \tag{8a}$$

where $e^*(k)$ is the innovation process of the original system and

$$\bar{e}(k) = \theta^{*\mathrm{T}}\Phi^*(k) - \theta^{\mathrm{T}}\Phi(k), \tag{8b}$$

In reference [6], it was assumed that $\bar{e}(k)$ in (8) is a zero-mean stochastic sequence and statistically independent of $e^*(k)$. In this paper, the assumption in reference [6] is relaxed so that $\bar{e}(k)$ is not a zero-mean stochastic sequence and can be represented as

$$D(z^{-1})\bar{e}(k) = \Delta\theta^{\mathrm{T}}\Delta\Phi(k), \tag{9a}$$

where

$$\Delta\theta^{\mathrm{T}} = \begin{cases} [A_{on+1}, \cdots, A_{or}, B_{on+1}, \cdots B_{or}, D_{on+1}, \cdots, D_{or}]_{m \times 3m(r-n)}, & r > n, \\ 0, & r = n, \end{cases} \tag{9b}$$

$$\Delta\Phi(k) = [-y_1(k-n-1), \cdots, -y_m(k-n-1), \cdots, -y_m(k-r), u_1(k-n-1), \cdots, u_m(k-n-1), \cdots, u_m(k-r), e_1^*(k-n-1), \cdots, e_m^*(k-n-1), \cdots, e_m^*(k-r)]^{\mathrm{T}}. \tag{9c}$$

Then, from (9a), it is reasonable to make the following assumption:

Assumption 1 Assume that there exists a $\mu > 0$, such that for $k \geqslant 0$, the unmodelled error satisfies the constraint:

$$\| \bar{e}(k) \| \leqslant \mu \| \Delta\Phi(k) \|, \tag{10a}$$

and further, we have

$$\| \bar{e}(k) \| \leqslant \mu \| \Phi^*(k) \|, \tag{10b}$$

where $\Phi^*(k)$ is related to the plant input and output sequences. Eq. (10b) shows that the mod-

elling error $\bar{e}(k)$ is relatively bounded; therefore, μ can be considered a measure of the relative magnitude of the modelling error. Note our primary interest is to find a relative bound of the modelling error with which the adaptive controller can be tolerated.

3 Normalized Parameter Estimation

In order to develop an adaptive control law, we shall first introduce the parameter estimation algorithm for the model in (6).

Defining the parameter estimation error, $\bar{\theta}(k)=\hat{\theta}(k)-\theta$, the estimated output $\hat{y}(k)=\hat{\theta}(k-1)\hat{\Phi}(k)$, and the innovation process, $\varepsilon(k)=y(k)-\hat{y}(k)$.

The reduced-order model equation to be estimated is written as follows:

$$y(k) = \hat{\theta}^{\mathrm{T}}(k-1)\hat{\Phi}(k) + \varepsilon(k), \tag{11a}$$

$$\hat{\theta}(k-1) = [\hat{A}_{o1},\cdots,\hat{A}_{on},\hat{B}_{o1},\cdots,\hat{B}_{on},\hat{D}_{o1},\cdots,\hat{D}_{on}]^{\mathrm{T}}, \tag{11b}$$

$$\hat{\Phi}(k) = [-y_1(k-1),\cdots,-y_m(k-1),\cdots,-y_m(k-n),u_1(k-1),\cdots, u_m(k-1),\cdots,u_m(k-n),\varepsilon_1(k-1),\cdots,\varepsilon_m(k-1),\cdots,\varepsilon_m(k-n)]^{\mathrm{T}}. \tag{11c}$$

As far as the parameter identification is concerned, in the case of bounded disturbances, the dead zone technique is utilized to prevent parameter drift. On the other hand, in the case of unmodelled dynamics, the identification error may grow without bound; hence, the dead zone technique of the bounded distrurbances can not be applied directly. This leads to the use of the parameter normalization technique which allows unmodelled dynamics to be treated as bounded disturbances. In this paper, a normalized parameter estimation scheme[4] is used to estimate the parameter A_{oi}, B_{oi}, and D_{oi} in (6).

The normalized variables of the process are defined as

$$y^n(k) = \frac{y(k)}{n(k)},\ \Phi^n(k) = \frac{\Phi(k)}{n(k)},\ \hat{\Phi}^n(k) = \frac{\hat{\Phi}(k)}{n(k)},\ \Delta\Phi^n(k) = \frac{\Delta\Phi(k)}{n(k)},\ \varepsilon^n(k) = \frac{\varepsilon(k)}{n(k)}, \tag{12a}$$

$$n(k) = \max\Big(\max_{1\leqslant i\leqslant 3mr}|\Phi_i^*(k)|, r_o\Big), \tag{12b}$$

where r_o is a pre-selected positive constant, and Φ_i^* is the i-th element of Φ^*. Using these normalized variables, the model in (7a) can be rewritten as follows:

$$y^n(k) = \theta^{\mathrm{T}}\Phi^n(k) + \Delta^n(k), \tag{13a}$$

where

$$\Delta^n(k) = \frac{[\bar{e}(k) + e^*(k)]}{n(k)}. \tag{13b}$$

It can be shown in the following lemma that the sequence $\{\Delta^n(k)\}$ in (13b) is bounded.

Lemma 1 The normalized sequence of the perturbed signals $\{\Delta^n(k)\}$ in (13b) is bounded.

Proof

$$\|\Delta^n(k)\| \leqslant \frac{\mu\|\Delta\Phi(k)\|}{n(k)} + \frac{\|e^*(k)\|}{n(k)} \leqslant \mu[3(r-n)m]^{1/2} + \frac{\delta_o}{r_o}, \tag{13c}$$

we are able to estimate an upper bound of $\Delta^n(k)$.

Define

$$\Delta_b = [\Delta_{b1}, \cdots, \Delta_{bm}]^T,$$

where $\Delta_{bi} > 0$ is an estimate of an upper bound of $|\Delta_i^*(k)|$, $i = 1, \cdots, m$. Also, define $\bar{\Delta}_b = \| \Delta_b \|$, then $\bar{\Delta}_b$ is an estimate of an upper bound of $\| \Delta^*(k) \|$.

Based on the identification algorithm in [4], we can obtain the convergence properties of the posterior estimation error and of the parameters. A posterior estimation error is defined as

$$\bar{\varepsilon}(k) \triangleq y(k) - \hat{\theta}^T(k)\hat{\Phi}(k). \tag{14}$$

Lemma 2 According to the esimation algorithm in [4], we have:

i) $\| \hat{\theta}(k) \|$ is uniformly bounded, which implies that there exists a constant $M > 0$, and $\hat{\theta}(k) \in \Omega = \{\theta : \| \theta \| \leqslant M\}$, where Ω is a closed subset of R^{3mn};

ii) $\lim\limits_{k\to\infty}[\| \hat{\theta}(k) \|^2 - \| \hat{\theta}(k-1) \|^2] = 0$;

iii) $\lim\limits_{k\to\infty}[\| \hat{\theta}(k) \| - \| \hat{\theta}(k-h) \|] = 0$, h is a limited positive integer;

iv) There exists a positive integer K_t such that

$$\| \bar{\varepsilon}(k) \| \leqslant 2\bar{\Delta}_b \max \| \Phi^*(k) \|, r_o), \quad \text{as } k > K_t.$$

This proof can be found in reference [4].

4 Multivariable State-Space Self-Tuning Controller

Once the system parameters $\hat{\theta}(k)$ are obtained, the adaptive control law can be determined as follows[6]:

The estimated $\hat{x}_o(k)$ with $\hat{\theta}(k)$ can be written as

$$\hat{x}_o(k) = \hat{A}_o(k)\hat{x}_o(k-1) + \hat{B}_o(k)u(k-1) + \hat{K}_o(k)\hat{\varepsilon}(k-1), \tag{15a}$$

$$y(k) = C_o^T\hat{x}_o(k) + \hat{\varepsilon}(k), \tag{15b}$$

where $\hat{A}_o(k)$, $\hat{B}_o(k)$ and $\hat{K}_o(k)$ are the kth step estimation of A_o, B_o and K_o, respectively.

Assumption 2 Let $G(\hat{\theta}) = [\hat{A}_o^{n-1}\hat{B}_o, \hat{A}_o^{n-2}\hat{B}_o, \cdots, \hat{A}_o\hat{B}_o, \hat{B}_o]$. Assume that there exists a positive real constant $\gamma_o > 0$ such that $|\det G(\hat{\theta})| \geqslant \gamma_o$. Then: the state-feedback control law is given by

$$u(k) = H_c r(k) - F_c^T T_c \hat{x}_o(k), \tag{16a}$$

where $r(k) \in R^m$ is a reference input vector with an input gain matrix $H_c \in R^{m\times m}$, $F_c^T \in R^{m\times nm}$

$$F_{ci} = \alpha_{ci} - A_{ci}, \quad i = 1, \cdots n, \tag{16b}$$

where A_{ci} comes from the controllable model of (15), and

$$\hat{\alpha}_c(\lambda) = \sum_{i=0}^{n} \alpha_{ci}\lambda^{n-i} = \prod_{i=1}^{n}(\lambda I_m - P_i), \tag{16c}$$

$$\det(\lambda I_{nm} - \hat{A}_o - \hat{B}_o F_c^T T_c)] = \prod_{i=1}^{n} \det(\lambda I_m - P_i). \tag{16d}$$

5 Extended Dynamic System Description

In this section, we reformulate the adaptive control system developed in previous sections into a composite dynamic system which is suitable for robust stability analysis using the theory developed in the next section.

First, let

$$Z^T(k) = [\Phi^{*T}(k), \hat{x}_o^T(k)], \tag{17}$$

where $\Phi^{*\mathrm{T}}(k)$ and $\hat{x}_o^{\mathrm{T}}(k)$ are defined in (4c) and (15), respectively. Next, we rewrite $\Phi^*(k)$:

$$\Phi^*(k+1) = S\Phi^*(k) + \Phi_y(k) + \Phi_u(k) + \Phi_{e^*}(k), \tag{18a}$$

where

$$S = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ I_{m(r-1)} & 0 & 0 & \cdots & 0 \\ 0 & I_{m(r-1)} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \tag{18b}$$

$$\Phi_y^{\mathrm{T}}(k) = [\overbrace{-y^{\mathrm{T}}(k), 0_m^{\mathrm{T}}, \cdots, 0_m^{\mathrm{T}}}, \overbrace{0_m^{\mathrm{T}}, \cdots, 0_m^{\mathrm{T}}}, \overbrace{0_m^{\mathrm{T}}, \cdots, 0_m^{\mathrm{T}}}], \tag{18c}$$

$$\Phi_u^{\mathrm{T}}(k) = [0_m^{\mathrm{T}}, \cdots, 0_m^{\mathrm{T}}, u^{\mathrm{T}}(k), 0_m^{\mathrm{T}}, \cdots, 0_m^{\mathrm{T}}, 0_m^{\mathrm{T}}, \cdots, 0_m^{\mathrm{T}}], \tag{18d}$$

$$\Phi_{e^*}^{\mathrm{T}}(k) = [0_m^{\mathrm{T}}, \cdots, 0_m^{\mathrm{T}}, 0_m^{\mathrm{T}}, \cdots, 0_m^{\mathrm{T}}, e^{*\mathrm{T}}(k), 0_m^{\mathrm{T}}, \cdots, 0_m^{\mathrm{T}}], \tag{18e}$$

where $0_m^{\mathrm{T}} = [0, \cdots, 0]_{1\times m}$. Then, from (15b) and (16a), Eq. (18a) becomes

$$\Phi^*(k+1) = S\Phi^*(k) + D_1(k)\hat{x}_o(k) + D_2(k)\bar{\varepsilon}(k) + D_3(k)e^*(k) + D_4(k)r(k), \tag{19a}$$

where

$$D_1 = \begin{bmatrix} -C_o^{\mathrm{T}} \\ 0 \\ -F_c^{\mathrm{T}}T_c \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} -I_m \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad D_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ I_m \\ 0 \end{bmatrix}, \quad D_4 = \begin{bmatrix} 0 \\ 0 \\ H_o \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{19b}$$

Substituting (16a) into (15a) gives

$$\begin{aligned} \hat{x}_o(k+1) &= (\hat{A}_o(k) - \hat{B}_o(k)F_c^{\mathrm{T}}T_c)\hat{x}_o(k) + \hat{K}_o(k)\bar{\varepsilon}(k) + \hat{B}_o(k)H_c r(k) \\ &= F_1(k)\hat{x}_o(k) + \hat{K}_o(k)\hat{\varepsilon}(k) + \hat{B}_o(k)H_c r(k), \end{aligned} \tag{20a}$$

$$F_1(k) = \hat{A}_o(k) - \hat{B}_o(k)F_c^{\mathrm{T}}T_c, \tag{20b}$$

and combining the resulting equations (19a) and (20a) yields the composite dynamic equation of the closed-loop system:

$$Z(k+1) = E_1(k)Z(k) + E_2(k)\bar{\varepsilon}(k) + E_3(k)e^*(k) + E_4(k)r(k) \tag{21a}$$

where

$$E_1(k) = \begin{bmatrix} S & D_1(k) \\ 0 & F_1(k) \end{bmatrix}, \quad E_2(k) = \begin{bmatrix} D_2(k) \\ \hat{K}_o(k) \end{bmatrix}, \quad E_3(k) = \begin{bmatrix} D_3(k) \\ 0 \end{bmatrix}, \quad E_4(k) = \begin{bmatrix} D_4(k) \\ \hat{B}_o(k)H_c \end{bmatrix}. \tag{21b}$$

In the following section, we carry out the robust stability analysis using (21a).

6 The Robustness of Self-Tuning Adaptive Controller

In this section, we state two lemmas as follows.

Lemma 3 Consider the time-varying difference equation

$$x(t+1) = A(t)x(t) + f(t,x), \quad x(t_1) = x_1 \in \mathrm{R}^n, \tag{22a}$$

where $f(t,x)$ is bounded as

$$\| f(t,x) \| \leqslant (d_0 + d_1(t)) \| x(t) \| + r_2(t), \tag{22b}$$

$$d_0 \geqslant 0, \quad 0 \leqslant d_1(t) \in l^1, \text{ and } 0 \leqslant r_2(t) \in l^\infty.$$

Suppose that the zero-input system in (22) of the form

$$x(t+1) = A(t)x(t) \tag{23}$$

is exponentially stable, i. e. , there exist some constants $\alpha_1 \geqslant 1$ and $1 \geqslant \alpha_2 \geqslant 0$ such that

$$\| \Phi(t,t_1) \| \leqslant \alpha_1 \alpha_2^{(t-t_1)} \tag{24}$$

for any $t_1 \geqslant 0$ and $t \geqslant t_1$, where $\Phi(t,t_1)$ is the state transition matrix of the system in (23). Then, if the following inequality is satisfied:

$$0 \leqslant d_0 < \frac{(1-\alpha_2)}{\alpha_1}, \tag{25}$$

we can conclude that

i) $x(t) \in l^\infty$, and in addition, $\| x(t) \| \leqslant v(t)$ as $t \to \infty$, where $v(t)$ is the output of the system with the transfer function $G'(z^{-1}) = \alpha_1/(z-(d_0\alpha_1+\alpha_2))$, driven by $r_2(t)$.

ii) Whenever $r_2(t) \in l^p$, and $p \in [1,\infty)$, then $x(t) \in l^p$ which implies $\| x(t) \| \to 0$ as $t \to \infty$.

Proof see reference [5].

Lemma 4[7] Consider the system

$$x(t+1) = A(t)x(t) \tag{26}$$

which has the following properties:

a) $\| A(t) \|$ is uniformly bounded;

b) There exist $0 < \varepsilon_0 < 1$, such that

$$\max |\lambda_j(A(t))| \leqslant 1 - \varepsilon_0 < 1 \quad \text{for all} \quad t \geqslant t_0;$$

c) $\sup \| A(t+1) - A(t) \|$ is sufficiently small.

where t_0 is a positive constant, and $\lambda_j(A(t))$ denotes the jth eigenvalue of the matrix $A(t)$. Then, the system is exponentially stable.

In order to show the system in (21a) to be exponentially stable, we proceed through the following steps.

Step 1 Show that $E_1(k)$ satisfies part a) of Lemma 4.

From Lemma 2-i), we know that $\| \hat{A}_o(k) \|$ and $\| \hat{B}_o(k) \|$ are bounded. From Assumption 2, we know that $\| F_c^{\mathrm{T}} \|$ and $\| T_c \|$ are also bounded. Thus, $\| E_1(k) \|$ is bounded.

Step 2 Show that $E_1(k)$ satisfies part b) of Lemma 4.

Since

$$\begin{aligned} \det(\lambda I - E_1(k)) &= \det(\lambda I - S)\det(\lambda I - F_1(k)) \\ &= \det(\lambda I - S) \prod_{i=1}^{m} \det(\lambda I_m - P_i) \end{aligned} \tag{27}$$

the roots of $\det(\lambda I - S)$ are zero. If we choose P_i, $i = 1, \cdots, m$ such that $\det(\lambda I - P_i)$ has no roots lying outside the circular disk, i. e. , $0 \leqslant |\lambda| < 1 - \varepsilon_o$ and $0 < \varepsilon_o < 1$, then $E_1(k)$ satisfies

part b) of Lemma 4.

Step 3 Show that $E_1(k)$ satisfies prat c) of Lemma 4.

The above fact can obviously be verified from Lemma 2-iii) and Assumption 2.

Based on the results shown in the above three steps, we coclude that $E_1(k)$ is exponentially stable. This implies that an association of the α_1 and α_2 with the exponentially stable $E_1(k)$ is evident.

Next, we explore the norm bounded property of the forcing terms, $E_2\bar{\varepsilon}(k)$, $E_3e^*(k)$, and $E_4r(k)$, in (21a) as follows.

By virtue of Lemma 2-i), Assumption 2, Lemma 2-iv), and (2), we have

$$\begin{aligned}&\| E_2\bar{\varepsilon}(k) + E_3e^*(k) + E_4r(k) \|\\ \leqslant& K_1 \| \bar{\varepsilon}(k) \| + K_2 \| e^*(k) \| + K_3 \| r(k) \|\\ \leqslant& 2K_1\bar{\Delta}_b[\| \Phi^*(k) \| + r_o] + K_2\delta_o + K_3 \| r(k) \|, \end{aligned} \tag{28a}$$

where $K_1 = \sup\limits_{0<k<\infty} \| E_2(k) \|$, $K_2 = \sup\limits_{0<k<\infty} \| E_3(k) \|$, $K_3 = \sup\limits_{0<k<\infty} \| E_4(k) \|$. From (13c), we know that $\mu[3(r-n)m]^{1/2}+\delta_o/r_o$ is an upper bound of $\| \Delta^*(k) \|$, and $\bar{\Delta}_b$ is an estimate of an upper bound of $\| \Delta^*(k) \|$. Thus, if we can choose $\bar{\Delta}_b \leqslant \mu[3(r-n)m]^{1/2}+\delta_o/r_o$ for the estimation in reference [4], then

$$\begin{aligned}\| E_2\bar{\varepsilon}(k) + E_3e^*(k) + E_4r(k) \| \leqslant& 2K_1[\mu[3(r-n)m]^{1/2} + \frac{\delta_o}{r_o}][\| \Phi^*(k) \| + r_o]\\ &+ K_2\delta_o + K_3 \| r(k) \| \\ \leqslant& 2K_1[\mu[3(r-n)m]^{1/2} + \frac{\delta_o}{r_o}][\| Z(k) \| + r_o]\\ &+ K_2\delta_o + K_3 \| r(k) \|. \end{aligned} \tag{28b}$$

Now, comparing (22b) with (28b) yields

$$d_0 = 2K_1[\mu[3(r-n)m]^{1/2} + \frac{\delta_o}{r_o}],$$

$$d_1(k) = 0, \quad r_2(k) = 2K_1[\mu[3(r-n)m]^{1/2} + \frac{\delta_o}{r_o}]r_o + K_2\delta_o + K_3 \| r(k) \|.$$

Then, applying Lemma 3, we have the following main results for the adaptive controller.

Theorem 1 With Assumptions 1 and 2, if the μ in (10a) (a measure of the relative magnitude of the modelling error) is bounded as

$$0 \leqslant \mu < \frac{1}{2K_1[3(r-n)m]^{1/2}}\left[\frac{1-\alpha_2}{\alpha_1} - \frac{2K_1\delta_o}{r_o}\right], \tag{29}$$

and the $\bar{\Delta}_b$ in the estimation algorithm[4] satisfies

$$0 < \bar{\Delta}_b \leqslant \mu[3(r-n)m]^{1/2} + \frac{\delta_o}{r_o}, \tag{30}$$

then the adaptively controlled close-loop system via the reduced-order controller is BIBO stable for any initial condition in both the plant and the adaptive controller irrespective of the presence of unmodelled dynamics.

Remark 1 The BIBO stable system has the following properties:

i) $Z(k)\in l^\infty$, then $y(k)\in l^\infty$, and $u(k)\in l^\infty$;

ii) $\| Z(k) \| \leqslant v(k)$, as $k\to\infty$, where $v(k)$ is the output of the system with the transfer function

$$G'(z^{-1}) = \frac{\alpha_1}{z-(\alpha_1 d_0+\alpha_2)},$$

driven by $r_2(k)$.

iii)

$$\| Z(k) \| \leqslant v'(k) + \left[\frac{\alpha_1}{1-(\alpha_1 d_0+\alpha_2)}\right](2K_1 d_o r_o + K_2\delta_o), \tag{31}$$

where $v'(k)$ is the output of the system with the transfer function $G'(z^{-1})$, driven by $K_3\| r(k) \|$.

Remark 2 The value $\frac{1}{2K_1[3(r-n)m]^{1/2}}\left[\frac{1-\alpha_2}{\alpha_1}-\frac{2K_1\delta_o}{r_o}\right]$ in (29) is obviously a measure of the robustness of the adaptive controller. It implies that the adaptive controller is allowed to be perturbed by the modelling error $\bar{e}(t)$ satisfying the μ in (29). For the pole-assignment algorithm in this paper once we have selected the desired closed-loop polynomial matrix $\hat{a}_c(\lambda)$ in (16c), it is possible to determine α_1 and α_2 a priori. As a result, we have the knowledge of the degree of robust stability for the controller to be designed.

7 Conclusions

This paper has demonstrated that the state-feedback pole-assignment self-tuning controller[6] has a certain stability robustness. As a result, the reduced-order model can be used to design a reduced-order self-tuner with suitable conditions and the adaptively controlled closed-loop original system via the designed reduced-order self-tuner is BIBO stable in the presence of unmodelled dynamics.

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降阶多变量状态空间自校正控制器的鲁棒性

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摘要: 本文建立了降阶极点配置状态空间自校正控制器的鲁棒性定量分析. 在这个多变量自适应控制系统中,控制器设计中所用模型的阶低于实际过程的阶. 稳定性分析结果表明,当建模误差满足一定条件时,在降阶控制器作用下的自适应闭环系统是输入输出有界的.

关键词: 自适应控制; 降阶控制器; 鲁棒控制; 状态空间自校正

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袁著祉 1937年生. 教授,博士生导师. 1962年于南开大学数学系获理学学士学位. 后在该校数学系、计算机与系统科学系担任教学与科研工作. 主要研究领域为自适应控制. 曾获国家教委科技进步一等奖.

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