第 11 卷第 3 期
1994 年 6 月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol. 11, No. 3
Jun., 1994

Finite-Dimensional Variable Structure Control for a Class of Distributed Parameter Systems*

LI Kamchi

(Department of Electroinc Engineering, Hong Kong Polytechnic·Hong Kong)

LEUNG Tinpui

(Department of Mechanical and Marine Engineering, Hong Kong Polytechnic·Hong Kong)

HU Yueming and LIU Yongqing

(Department of Automation, South China University of Technology·Guangzhou, 510641, PRC)

Abstract: This paper addresses the finite-dimensional variable structure control problem for a class of nonlinear distributed parameter system (DPS). First, a finite-dimensional reduced order model (ROM) is generated via Galerkin procedure; second, finite-dimensional variable structure controller is synthesized to provide the approximate system with desired dynamics; finally, the asymptotic behavior of the actual closed-loop DPS with the designed controller is analyzed and some sufficient conditions are given to guarantee that the actual closed-loop DPS has prescribed asymptotic behavior when the approximate order is predetermined.

Key words: distributed parameter systems; finite-dimensional controller; variable structure control; sliding mode; robustness; stability

1 Introduction

In many industrial processes, such as flexible manipulators, large flexible spacecrafts and chemical processes, the mathematical models for these systems are described by partial differential equations or functional differential equations. Generally the state spaces for these systems are infinite-dimensional. Most of the existing control methods in DPS control theory are difficult or impossible to realize in practical engineering problems because of the following common drawbacks: first, controllers are often designed directly using the complete knowledge of states, and hence these controllers are infinite-dimensional, which make them irrealizable since the implementations are usually done with digital computers, and the available on-line computer capacities are finite; second, structures or parameters of the systems are often supposed to be known exactly, and therefore, in many cases, due to the lack of sufficient knowledge of the system, for example, insufficient modal data in flexible robot arms with varying payloads, the finest control law in theory applied to an actual plant often yields a poor result and even may produce an unstable system. Therefore, to establish a finite-dimensional robust control approach for DPS is an area that merits

* The project was supported in part by the Croucher Foundation of Hong Kong and the National Natural Science Foundation of China.

Manuscript received Feb. 16, 1993, revised Jan. 8, 1994.

investigation.

It is well-known that sliding mode control approach plays an important role in the robust control problems of multivariable uncertain systems, due to its ease implementation and insensitivity to system uncertainties or parameter vairations[3,4,10]. This approach has been applied to the control problems of DPS in recent years such as flexible structure systems, and some finite-dimensional controllers have been designed directly using various approximate techniques such as mode-assumed method and finite-element method, but the stability analysis of the actual closed-loop DPS has been entirely disregarded or only done by computer simulations in these papers. Evidently, the stability conditions for the actual closed loop DPS remain unknown since those controllers are designed only based on finite-dimensional approximate models.

The aim of this paper is to introduce a method for the application of variable structure control to DPS in Hilbert spaces. The synthesizing approach is presented using finite-dimensional approximate technique; sufficient conditions are given to guarantee that the actual closed-loop DPS has desired asymptotic behavior when the approximate order is predetermined. The results in this paper indicate conditions under which finite-dimensional variable structrue controller designed using the Galerkin approximation will produce a closed-loop DPS with desired dynamics.

2 Problem Formulation

Consider the following DPS:

$$\dot{x} = Ax + f(t,x) + Bu, \quad x(0) = x_o. \tag{2.1}$$

where the state variable x belongs to an infinite-dimensional separable Hilbert space V with inner product $(\cdot,\cdot)$ and corresponding norm $\|\cdot\|$; the operator A is a linear closed and unbounded operator with domain dense in V; $f(t,x)$ is Lipschitzian with respect to x and measureable with respect to t.

Control $u(t)$ is applied by m actuators with influence functions b_i in V.

$$Bu = \sum_{i=1}^{m} b_i u_i(t). \tag{2.2}$$

Evidently, the formulation (2.1) and (2.2) represents a wide variety of DPS control problems. In order to design controller using the finite dimensional approximate knwoledge of the state variable, we use the following Galerkin approach.

Let V_N and V_R be the subspaces of V, P_N and P_R be the projections onto V_N and V_R respectively, satisfying the following conditions

1) $V = V_N + V_R$.

2) $\dim(V_N) = N < \infty$; $V_N \subset D(A)$.

3) $V_N \subset V_{N+1} \subset \cdots \subset V$.

4) P_N is monotonously increasing with respect to N[6].

Let $x_N = P_N x, x_R = P_R x$, then (2.1) decomposes into the following form

$$\begin{aligned} \dot{x}_N &= A_N x_N + A_{NR} x_R + f_N(t,x) + B_N u, \\ \dot{x}_R &= A_{RN} x_N + A_R x_R + f_R(t,x) + B_R u \end{aligned} \tag{2.3}$$

with the initial conditions $x_N(0) = \boldsymbol{P}_N x_0;\quad x_R(0) = \boldsymbol{P}_R x_0.$ (2. 4)

where
$$\begin{cases} A_N = \boldsymbol{P}_N A \boldsymbol{P}_N, & A_R = \boldsymbol{P}_R A \boldsymbol{P}_R, \quad A_{NR} = \boldsymbol{P}_N A \boldsymbol{P}_R, \quad A_{RN} = \boldsymbol{P}_R A \boldsymbol{P}_N \\ f_N = \boldsymbol{P}_N f, & f_R = \boldsymbol{P}_R f, \quad B_N = \boldsymbol{P}_N B, \quad B_R = \boldsymbol{P}_R B. \end{cases} \tag{2. 5}$$

The finite-dimensional approximate model of (2. 1) is produced by ignoring the residuals x_R in (2. 3)

$$\dot{\bar{x}}_N = A_N \bar{x}_N + f_N(t, \bar{x}_N) + B_N u, \quad \bar{x}_N = \boldsymbol{P}_N x_0. \tag{2. 6}$$

All parameters except A_R in (2. 5) are bounded operators since $\boldsymbol{P}_N$ has finite rank. The above approximation is often called the Galerkin approximation. Particularly, if the spectrum of A can be separated into two parts $\sigma(A_N)$ and $\sigma(A_R)$, where $\sigma(A_N)$ consists of N isolated eigenvalues of A which can be separated from the rest of the spectrum $\sigma(A_R)$ by a smooth closed curve in the complex plane, then there exist above space decompositions such that A_N and A_R have the spectrum $\sigma(A_N)$ and $\sigma(A_R)$ respectively, and these subspaces are A-invariant, that is

$$A_{RN} = 0 \quad \text{and} \quad A_{NR} = 0. \tag{2. 7}$$

and also called modal subspaces since $V_N = \text{span}\{\varphi_1, \cdots, \varphi_N\}$, where φ_i are the modal shapes or eigenfunctions of the operator A which correspond to the eigenvalues λ_i in $\sigma(A_N)$. For a single-link flexible robot arm, we may use this approach to determine the state space decomposition.

Suppose that (A_N, B_N) is controllable and A_N is an infinitesimal generator of a strongly continuous linear operator semigroup $T_N(t)$ on V_N such that

$$\| T_N(t) \| \leqslant K_N \exp\{-\sigma_N t\}, \quad t \geqslant 0. \tag{2. 8}$$

Otherwise, it only needs to introduce a finite dimensional gain matrix since (A_N, B_N) is assumed to be controllable. Besides, the residual operator A_R is assumed to generate a strongly continuous linear operator semigroup $T_R(t)$ with the following growth property

$$\| T_N(t) \| \leqslant K_R \exp\{-\sigma_R t\}, \quad t \geqslant 0. \tag{2. 9}$$

Using a similar proof to the Theorem 2 in [2], we obtain the following convergence result.

Theorem 1 Under the above assumptions, if

1) $\lim\limits_{N\to\infty} \| A_{RN} x \| = 0, \quad x \in V.$

2) A generates a strongly continuous linear operator semigroup $T(t)$ on V;

3) there exists $L \geqslant 0$ such that $\| f(t,x) \| \leqslant L \| x \|$;

4) control $u(t)$ is absolutely integrable.

Then, over any finite interval of time, we have

$$\lim_{N\to\infty} \| \bar{x}_N(t) - x(t) \| = 0.$$

Therefore, both from the theoretical and practical standpoints, the above Galerkin approximation is proved to be rational. However, for practical control problems, it is difficult to determine a proper approximating order N only using prior knowledge. It is therefore necessary to establish a simple off-line approach of selecting suitable approximate order N for DPS contorl problem. In this paper, we will design the finite-dimensional variable structure controller only using the above approximated model and indicate the conditions under which the actual closed-loop DPS with the proposed controller does have proper asymptotic behavior. It should be noted that, due to the involvement of the variable structure control, the condition (4) in the above theorem 1 is no longer valid. However, we can use the regularization principle (see [1]), and for the regularized system, the above conditions in theorem 1 are then satisfied.

3 Design of Variable Structure Controller

Since the approximated system (2. 6) is finite-dimensional, feedback controller can be designed using the variable structure control theory of finite-dimensional systems[1]. Let the switching manifold be $S=D_N\bar{x}_N=0$, where D_N is an $m\times N$ constant matrix to be determined. From (2. 6) it follows that

$$\dot{S} = D_N A_N \bar{x}_N + D_N f_N(t,x) + D_N B_N u. \tag{3. 1}$$

Using the equivalent control principle[3], we obtain the equivalent control u_e from (3. 1)

$$u_e = -(D_N B_N)^{-1} D_N (A_N \bar{x}_N + f_N(t,\bar{x}_N)) \tag{3. 2}$$

where $D_N B_N$ is assumed to be invertible. Substitute $u=u_e$ (3. 2) into (2. 6), we obtain the sliding mode equation of the approximated system

$$\begin{cases} \dot{\bar{x}}_N = [I_N - B_N(D_N B_N)^{-1} D_N][A_N\bar{x}_N + f_N(t,\bar{x}_N)], \\ D_N \dot{\bar{x}}_N = 0. \end{cases} \tag{3. 3}$$

Since $\mathrm{rank}(B_N)=m$, there exists nonsingular matrix U_N such that

$$U_N B_N = \begin{pmatrix} 0 \\ I_m \end{pmatrix}. \tag{3. 4}$$

Then (3. 3) is reduced to the equivalent form

$$\begin{cases} \dot{z}_{N1} = A_{N1} z_{N1} + A_{N2} z_{N2} + f_{N1}(t, U_N^{-1} z_N), \\ z_{N2} = -\Gamma_N z_{N1} \end{cases} \tag{3. 5}$$

where $z_N=U_N\bar{x}_N=z_{N1}+z_{N2}$, $z_{Ni}\in V_{Ni}\subset V_N$, $i=1,2$. V_{Ni} are the subspaces of V_N such that $V_N=V_{N1}+V_{N2}$, $\dim(V_{N1})=N-m$, $\dim(V_{N2})=m$; $\mathbf{P}_{Ni}$ $(i=1,2)$ are the projections on V_{Ni} $(i=1,2)$ respectively, and

$$A_{Ni} = \mathbf{P}_{N1} U_N A_N U_N^{-1} \mathbf{P}_{Ni}, \quad i = 1,2; \quad f_{N1} = \mathbf{P}_{N1} f_N; \quad \Gamma_N = D_{N2}^{-1} D_{N1}; \quad (D_{N1}, D_{N2}) = D_N U_N^{-1}.$$

Since (A_N, B_N) is controllable, so is (A_{N1}, A_{N2})[1]. Therefore the eigenvalues of $A_{N1}-A_{N2}\Gamma_N$ can be assigned arbitrarily by a proper choice of Γ_N. Let $D_{N2}^{-1}D_{N1}=\Gamma_N$, i. e. $D_{N1}=D_{N2}\Gamma_N$, then we can determine the switching manifold in original coordinates as follows[10]

$$S = D_{N2}(\Gamma_N, I_m) U_N \bar{x}_N = 0$$

so that the linear part of the sliding mode equation (3. 5) has desired exponential decay rate to zero. Without losing of the generality, we suppose that $A_{N1}-A_{N2}\Gamma_N$ generates a strongly continuous linear operator semigroup $T_{N1}(t)$ on V_{N1} such that

$$\| T_{N1}(t) \| \leqslant K_{N1} \exp\{-\sigma_{N1} t\}, \quad t \geqslant 0. \tag{3. 6}$$

where $K_{N1}\geqslant 1$, $\sigma_{N1}\geqslant 0$ are constants which can be assigned properly. Particularly, if the eigenvalues of $A_{N1}-A_{N2}\Gamma_N$ are assigned to be isolated and located in the left complex plane $\{\lambda \mid \mathrm{Re}\lambda \leqslant -\sigma_{N1}\}$, then $K_{N1}=1$.

Writing the solution of (3. 5) in the integral form as follows

$$z_{N1}(t) = T_{N1}(t)z_{N1}(0) + \int_0^t T_{N1}(t-\tau)f_{N1}(\tau, U_N^{-1}z_N(\tau))\mathrm{d}\tau \tag{3.7}$$

and noting that $z_{N2} = -\Gamma_N z_{N1}$, $\overline{\chi}_N = U_N^{-1}z_N = U_N^{-1}\begin{pmatrix} I_{N-m} \\ -\Gamma_N \end{pmatrix} z_{N1} \triangleq \overline{\Gamma}_N z_{N1}$.

we obtain $\| z_{N1}(t) \| \leqslant K_{N1}e^{-\sigma_{N1}t}[\| z_{N1}(0) \|] + L_N\int_0^t e^{\sigma_{N1}\tau} \| \overline{\Gamma}_N \| \; \| z_{N1}(\tau) \| \mathrm{d}\tau]$

where L_N is the Lipschitz constant for f_N. Using Gronwall's inequality it follows that

$$\| z_{N1}(t) \| \leqslant K_{N1}e^{(L_N K_{N1}\|\Gamma_N\| - \sigma_{N1})t} \| z_{N1}(0) \| . \tag{3.8}$$

Therefore, in original coordinates, we know that the sliding mode satisfies

$$\| \overline{\chi}_N(t) \| \leqslant \| \overline{\Gamma}_N \| K_{N1}e^{(L_N K_{N1}\|\Gamma_N\| - \sigma_{N1})(t-t_0)} \| \overline{\chi}_N(t_0) \| , \tag{3.9}$$

Hence, if the following condition is satisfied

$$\gamma_N \triangleq \sigma_{N1} - L_N K_{N1} \| \overline{\Gamma}_N \| > 0. \tag{3.10}$$

then the sliding motion is exponentially stable.

In general, the Lipschitz constant L_N for f_N can be selected as L since $\| \boldsymbol{P}_N \| \leqslant 1$. From (3. 10) we conclude that the sliding motion is stable only if the nonlinear part is relatively small. The robust design of sliding mode will be studied in separated paper.

Second, we design a variable structure controller to guarantee existence of the designed sliding mode, Let

$$u = -(D_N B_N)^{-1}[D_N A_N \overline{\chi}_N + D_N f_N(t, \overline{\chi}_N) + E_N \mathrm{sgn}(S)] \tag{3.11}$$

where $E_N = \mathrm{diag}(\varepsilon_{N1})_{m\times m}$, $\varepsilon_{N1} > 0$, $\mathrm{sgn}(S) = S/\| S \|$. Then from (3. 1) and (3. 11) we have

$$\dot{S} = -E_N \mathrm{sgn}(S) \tag{3.12}$$

which implies existence of the sliding mode[10]. Therefore, the closed-loop system (2. 6) with the designed controller as above has desired properties.

4 Asymptotic Behavior of the Actual Closed-Loop DPS

The finite-dimensional variable structure controller designed as above is only based on the approximated model (2. 6). Of course, it must operate in closed-loop with the actual DPS-not just the approximated system. It is therefore necessary to analyze the stability of the actual closed-loop DPS with the proposed controller.

Let $e_N = \chi_N - \overline{\chi}_N$, then the actual closed-loop DPS is given by

$$\begin{cases} \dot{\overline{\chi}}_N = A_N \overline{\chi}_N + f_N(t, \overline{\chi}_N) + B_N u, \\ \dot{e}_N = A_N e_N + A_{NR}\chi_R + f_N(t, \chi) - f_N(t, \overline{\chi}_N), \\ \dot{\chi}_R = A_{RN}e_N + A_R\chi_R + A_{RN}\overline{\chi}_N + f_R(t, x) + B_R u \end{cases} \tag{4.1}$$

where the control u is given by (3. 11), $\chi = \overline{\chi}_N + e_N + \chi_R$.

First, we analyze the boundedness of the solutions for (4. 1) before the sliding motion occurs. From (3. 11) and (4. 1) it follows that

$$\dot{\overline{\chi}}_N = [I_N - B_N(D_N B_N)^{-1}D_N][A_N\overline{\chi}_N + f_N(t, \overline{\chi}_N)] - B_N(D_N B_N)^{-1}E_N \mathrm{sgn}(S). \tag{4.2}$$

The solution of (4. 2) can be represented in integral form

$$\overline{\chi}_N(t) = \overline{\chi}_N(0) + \int_0^t [I_N - B_N(D_NB_N)^{-1}D_N][A_N\overline{\chi}_N + f_N(t,\overline{\chi}_N)]\mathrm{d}\tau$$
$$- \int_0^t B_N(D_NB_N)^{-1}E_N\mathrm{sgn}(S)\mathrm{d}\tau. \tag{4.3}$$

To note that, before sliding motion occurs, one has $\| S \| \neq 0$, which implies

$$\| \mathrm{sgn}(S) \| \leqslant 1. \tag{4.4}$$

Therefore there are constants C_i $(i=1,2)$ such that

$$\| \overline{\chi}_N(t) \| \leqslant \| \overline{\chi}_N(0) \| + C_1 t + C_2\int_0^t \| \overline{\chi}_N(\tau) \| \mathrm{d}\tau$$

where $C_1 = \| B_N(D_NB_N)^{-1}E_N \|$; $C_2 = \| [I_N - B_N(D_NB_N)^{-1}D_N] \| (\| A_N \| + L_N)$.

Using Gronwall's inequality, we know that $\| \overline{\chi}_N(t) \|$ is bounded, which implies that $u(t)$ is also bounded before the sliding motion occurs.

On the other hand, from (2.8) and (2.9), we know that e_N, χ_R satisfy

$$\begin{cases} e_N(t) = T_N(t-t_0)e_N(t_0) + \int_{t_0}^t T_N(t-\tau)[A_{NR}\chi_R + f_N(\tau,\chi) - f_N(\tau,\overline{\chi}_N)]\mathrm{d}\tau, \\ x_R(t) = T_R(t-t_0)\chi_R(t_0) + \int_{t_0}^t T_R(t-\tau)[A_{NR}e_N + A_{RN}\overline{\chi}_N + f_R(\tau,\chi) + B_Ru]\mathrm{d}\tau \end{cases} \tag{4.5}$$

which implies

$$\| e_N(t) \| \leqslant K_N e^{-\sigma_N(t-t_0)} \| e_N(t_0) \|$$
$$+ \int_{t_0}^t K_N e^{-\sigma_N(t-\tau)}[\| A_{NR} \| \| \chi_R \| + L_N(\| \chi_R \| + \| e_N \|)]\mathrm{d}\tau, \tag{4.6}$$

$$\| \chi_R(t) \| \leqslant K_R e^{-\sigma_R(t-t_0)} \| \chi_R(t_0) \|$$
$$+ \int_{t_0}^t K_R e^{-\sigma_R(t-\tau)}[\| A_{RN} \| (\| e_N \| + \| \overline{\chi}_N \|) + L_R \| \chi \| + \| B_R \| \| u \|]\mathrm{d}\tau$$
$$\leqslant K_R e^{-\sigma_R(t-t_0)} \| \chi_R(t_0) \|$$
$$+ \int_{t_0}^t K_R e^{-\sigma_R(t-t_0)}[(\| A_{RN} \| + L_R)(\| e_N \| + \| \overline{\chi}_N \|)$$
$$+ L_R \| \chi_R \| + \| B_R \| \| u \|]\mathrm{d}\tau, \tag{4.7}$$

where L_R is the Lipschitz constant for f_R such that $L_R \leqslant L$. Let

$$\begin{cases} e(t) = e_N(t) + \chi_R(t), \quad \| e(t) \| = \| e_N(t) \| + \| \chi_R(t) \|, \\ \sigma_{RN} = \min(\sigma_R,\sigma_N), \quad K_{NR} = \max(K_N,K_R). \end{cases} \tag{4.8}$$

Using (4.6) and (4.7), we obtain

$$\| e(t) \| \leqslant K_{NR} e^{-\sigma_{RN}(t-t_0)} \| e(t_0) \| + K_{NR}\int_{t_0}^t e^{-\sigma_{RN}(t-\tau)} \| B_R \| \| u \| \mathrm{d}\tau$$
$$+ K_{NR}\int_{t_0}^t e^{-\sigma_{RN}(t-\tau)}[(\| A_{RN} \| + L_R) \| \overline{\chi}_N \| + (\| A_{NR} \| + L_N + L_R) \| \chi_R \|$$
$$+ (\| A_{NR} \| + L_N + L_R) \| e_N \|]\mathrm{d}\tau.$$

Therefore, using the Gronwall's inequality, one knows that $e_N(t)$ and $\chi_R(t)$ are bounded, since $\overline{\chi}_N(t)$ and $u(t)$ are bounded before the sliding motion occurs.

Second, we analyze the asymptotic behavior after the sliding motion occurs. In this case, the control u takes the value u_e. From (3.2) and (3.9) it follows that

$$\| u_e \| \leqslant \| (D_N B_N)^{-1} D_N \| \, (\| A_N \| + L_N) \, \| \bar{\Gamma}_N \| \, K_{N1} e^{-\gamma_N(t-t_0)} \, \| \bar{\chi}_N(t_0) \| \triangleq G_N e^{-\gamma_N(t-t_0)} \, \| \bar{\chi}_N(t_0) \| . \tag{4.9}$$

From (3.9), (4.7) and (4.9) we have

$$\int_{t_0}^{t} K_R e^{-\sigma_R(t-\tau)} [(\| A_{RN} \| + L_R) \, \| \bar{\chi}_N \| + \| B_R \| \, \| u_e \|] \mathrm{d}\tau$$

$$\leqslant K_R [(\| A_{RN} \| + L_R) \, \| \bar{\Gamma}_N \| \, K_{N1} + G_N] \, \| \bar{\chi}_N(t_0) \| \int_{t_0}^{t} e^{-\sigma_R(t-\tau)} e^{-\gamma_N(t-t_0)} \mathrm{d}\tau$$

$$\triangleq G_{NR} \, \| \bar{\chi}_N(t_0) \| \, e^{-\sigma_R(t-t_0)} g(t) \tag{4.10}$$

where

$$g(t) = \begin{cases} t - t_0, & \text{if } \sigma_R = \gamma_N, \\ e^{(\sigma_R - \gamma_N)(t-t_0)}, & \text{if } \sigma_R \neq \gamma_N, \end{cases} \tag{4.11}$$

$$G_{NR} = \begin{cases} K_R [(\| A_{RN} \| + L_R) \, \| \bar{\Gamma}_N \| \, K_{N1} + G_N], & \text{if } \sigma_R = \gamma_N, \\ \dfrac{K_R [(\| A_{RN} \| + L_R) \, \| \bar{\Gamma}_N \| \, K_{N1} + G_N]}{\sigma_R - \gamma_N}, & \text{if } \sigma_R \neq \gamma_N. \end{cases}$$

Therefore, if $\gamma_N > \sigma_R$, then one has $\| g(t) \| \leqslant 1$. Using (4.7), (4.10) and (4.11) we have

$$\| x_R(t) \| \leqslant e^{-\sigma_R(t-t_0)} [K_R \, \| x_R(t_0) \| + G_{NR} \, \| \bar{x}_N(t_0) \|] + K_R \int_{t_0}^{t} e^{-\sigma_R(t-\tau)} [(\| A_{RN} \| + L_R) \, \| e_N \| + L_R \, \| x_R \|] \mathrm{d}\tau . \tag{4.12}$$

From (4.6) and (4.12) it follows that

$$\begin{aligned} \| e(t) \| \leqslant\ & e^{-\sigma_{RN}(t-t_0)} [K_R \, \| x_R(t_0) \| + G_{NR} \, \| \bar{x}_N(t_0) \| + K_N \, \| e_N(t_0) \|] \\ & + \int_{t_0}^{t} e^{-\sigma_{RN}(t-\tau)} [K_R (\| A_{RN} \| + L_R) + K_N L_N] \, \| e_N \| \, \mathrm{d}\tau \\ & + \int_{t_0}^{t} e^{-\sigma_{RN}(t-\tau)} [K_N (\| A_{NR} \| + L_N) + K_R L_R] \, \| x_R \| \, \mathrm{d}\tau . \end{aligned} \tag{4.13}$$

Let

$$\begin{cases} L_{RN} = \max \{ K_R \, \| A_{RN} \| , K_N \, \| A_{NR} \| \} , \\ \mu = \sigma_{RN} - (L_{RN} + K_N L_N + K_R L_R) . \end{cases} \tag{4.14}$$

By (4.13) and using Gronwall's inequality, we obtain

$$\| e(t) \| \leqslant [K_R \, \| x_R(t_0) \| + K_N \, \| e_N(t_0) \| + G_{NR} \, \| \bar{x}_N(t_0) \|] e^{-\mu(t-t_0)} . \tag{4.15}$$

From the above deductions, we have

Theorem 2 The actual closed-loop DPS with the designed controller (3.11) is exponentially stable if the following conditions are satisfied

1) The operators A_N and A_R satisfy (2.8) and (2.9) respectively;

2) The sliding mode of the approximated system satisfies (3.9);

3) $\gamma_N > \sigma_R$ and $\mu > 0$.

Therefore, when the approximate order N is fixed, the above conditions guarantee the exponential stability of the actual closed-loop DPS (2.1). Evidently, the larger the approximate order N is, the better the provided dynamical property is.

5 Conclusions

In the development of feedback control theory for DPS, it is important to maintain the finite-dimensionality of the controller. If the "modes" of the DPS are known, the natural approach for model reduction is projection onto the modal subspace spanned by a finite number of critical modes. However, in real engineering systems, these modes are never known exactly. In this paper, we develop a novel approach for the design of variable structure controller using only the finite-dimensional approximated model generated by the Galerkin approach. Sufficient conditions are also presented to guarantee that the actual closed-loop DPS with the designed controller has proper dynamic properties when the approximate order is fixed. It therefore provides an implementable robust control approach for the control problems of distributed parameter plants. The application of this approach to flexible robot arms and the finite-dimensional variable structure control problems of DPS using only outputs will be reported in separated paper[9].

References

[1] Orlov, Yu. V. and Utkin, V. I.. Sliding Mode Control in Indefinite-Dimensional Systems. Automatica, 1987, 23(6): 753—757

[2] Balas, M. J.. The Galerkin Method and Feedback Control of Linear Distributed Parameter Systems. J. Math. Analysis and Appl., 1983, 91:527—546

[3] Utkin, V. I.. Sliding Modes in Control and Optimization. Springer-Verlag, New York, 1992

[4] Gao, W. B.. Fundamentals of Variable Structure Control Theory. Science and Technology Press, Beijing, 1990

[5] Hu, Y. M. and Zhou, Q. J.. Variable Structure Control of Parabolic Distributed Parameter Systems. Control Theory and Applications. 1991, 8(1):38—43

[6] Xia, D. X., et al. Real Variable Function Theory and Functional Analysis. Part 2. Higher Education Press, Beijing, 1985

[7] HU, Y. M.. Robust Controller Design for a Class of Distributed Parameter Systems. Proceedings of the CDC'93, Huangshan, P.R.C., 1993, 162—165

[8] Zhao, Y. and Huang, Y.. Nonlinear Distributed Parameter Control Systems Theory. Guangdong Science and Technology Press, Guangzhou, 1990

[9] Hu, Y. M. and Liu, Y. Q.. Variable Structure Control Using Dynamical Output for Distributed Parameter Systems. Proceedings of the first Chinese World Congress on Intelligent Control and Intelligent Automation, Beijing, 1993, 2:2198—2203

[10] Huang, J. Y., Gao, W. B., and Huang, J. C.. Variable Structure Control:A Survey. IEEE Trans. on Indust. Electronics, 1993, 40(1):2—22

一类分布参数系统的有限维变结构控制

李锦赐　　　　梁天培

(香港理工学院电子工程系・香港)　(香港理工学院机械与轮机工程系・香港)

胡跃明　刘永清

(华南理工大学自动化系・广州,510641)

摘要：本文研究了一类非线性分布参数系统的有限维变结构控制问题. 首先采用 Galerkin 近似法给

出了系统的有限维近似模型；进而给出了使近似系统具有良好动态品质的有限维变结构控制器的设计方法；最后分析了闭环系统的渐近性态，给出了在近似阶次一定的情况下，原系统具有良好渐近性态的充分条件.

关键词：分布参数系统；有限维近似；有限维控制器；滑动模；稳定性

本文作者简介

李锦赐 1937年生. 于1970年及1974年先后在美国密执根州立大学取得硕士及博士学位. 曾在美国西雅图PNB电话公司及John Fluke MFG CO. 任职，亦曾在英国STL，ITT，Harlow参与研究工作. 1975年返香港理工学院任职至今. 现任电子工程系主任. 近年研究兴趣为通讯网络，微波电路设计及分析，信号及图像处理，控制系统的计算机辅助设计等.

梁天培 见本刊1994年第2期第152页.

胡跃明 1960年生. 1985年在安徽大学取得硕士学位，1991年华南理工大学取得博士学位，毕业后留校任教. 现在香港理工学院电子工程系作访问研究. 近年来出版专著"分布参数变结构控制系统"(与周其节教授合著)一部及论文三十余篇. 目前研究兴趣为滑动模控制理论及应用，机器人控制，神经网络等.

刘永清 见本刊1994年第1期第11页.

(上接第286页)

国际会议消息

(转载 IFAC NEWSLETTER 1993, No. 6)

Title	1995	Place	Deadline	Further Information
1995 European Control Conference (in cooperation with IFAC)	Sept. 5—8	Rome Italy	1 Oct. 1994	ECC 95 Secretariat DIS, Univ. degli Studi di Roma Via Eudossiana 18 I-00184 Rome, Italy FAX 39/6/44 585367 e-mail: ecc95@itcaspur. caspur. it
IFAC Symposium Low Cost Automation	Sept. 13—15	Buenos Aires Argentina	15 Nov. 1994	IFAC-LCA'95 Secr., AADECA, Eng. Casucci Av. Callao 220 1B 1022 Buenos Aires, Argentina FAX +54/1/463780
IFAC Workshop Motion Control	Oct. 9—11	Munich Germany	31 Jan. 1995	VDI/VDE GMA Graf Recke Strasse 84 D-40239 Düsseldorf, Germany FAX +49/211/6214-575
IFAC Symposium Control of Power Plants and Power Systems	Dec. 4—6	Cancun Mexico	Nov. 1994	Symposium Secretariat Instituto de Investigaciones Electricas AP 475, Cuernavaca, Mor. 62000, Mexico FAX +52/73/189854 e-mail: Sifac@iievmsl. iiecuer. unam. mx