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On Learning Control of Constrained Robots Modeled by Singular Systems

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Abstract: In this paper, an iterative learning control strategy is presented for a class of constrained robots. The controller design is based on the reduced form of the robot model. The strategy guarantees the perfect motion tracking of the robot with its end-effector moving on a linear and frictionless constrained surface in the presence of unknown bounded disturbances. The bounded and adjustable force tracking error is obtained.

Key words: constrained robot; learning control; tracking

1 Introduction

For many operations of the robot, the robot end-effector is required to make contact with its environment. As a result of contact with a rigid environment, the robot motion is kinematically constrained. In that case, it is necessary in general to control both the position of the end-effector and the contact force between the end-effector and the environment. The mathematical model for the constrained robot, explicitly taking into account the contact force, has been given in [1~3]. This model is in the form of singular systems. McClamroch and Wang[4] transformed the constrained system into the reduced unconstrained subsystems and proposed a modified computed torque method to solve the stability and tracking problem of a robot moving on a frictionless holonomic constrained surface. A number of other approaches have recently been proposed to deal with this problem, e.g. [5~8].

If the dynamics are exactly known for the control of a robot in constrained manipulation, the methods as mentioned above can be used for designing the effective controllers for the constrained motion of a robot. However, if there exist uncertainties, such as unknown disturbances, the controller so designed may give a degraded performance. To enhance the dynamic performance of a robotic system in the presence of uncertainties, we present a learning control strategy in this paper. The learning control is attributed to a class of self-tuning process with the system performance of a specified task being improved based on the previous performance of identical tasks. In many applications, the repeatability of operation is one of main technical features of

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present industrial robots. A number of learning control methods for unconstrained robots have been reported in the literature, such as [9~12]. The learning control methods can serve as effective means for improving a performance index.

The purpose of this paper is to present an iterative learning control strategy for a class of constrained robots. Before developing the strategy, we summarize the ideal principles that underlie the concept of learning control as the following set of postulates[11]:

1) Each operation ends in a fixed finite time $t_f > 0$.

2) A desired output $y_d(t)$ is given a priori over the time duration $t \in [0, t_f]$.

3) Repeatability of the initial setting is satisfied, i.e., the initial state $x^j(0)$ of the system can be set the same at the beginning of each operation in the following way:

$$x^j(0) = x^0, \quad \text{for } j = 1, 2, \cdots$$

where j denotes the trial number of operation.

4) Invariance of the system dynamics is assumed throughout repeated training.

5) Each output trajectory $y(t)$ can be measured without noise and hence the error signal

$$e^j = y_d(t) - y^j(t)$$

can be utilized in construction of the next command input.

6) The next command input $u^{j+1}(t)$ must be composed of a simple and fixed recursive law as follows:

$$u^{j+1}(t) = F(u^j(t), e^j(t)).$$

For convenience of later reference, we state briefly the nonlinear transformation[4] which converts the constrained system into two reduced unconstrained subsystems, and some structural properties of the reduced form of the robot model in the next section.

2 Constrained Robot Dynamics and Main Properties

For a constrained robot having n revolute joints, explicitly incorporating the effects of contact forces, the mathematical model of the robot system, at the jth iteration, is described as

$$M(q^j)\ddot{q}^j + C(q^j, \dot{q}^j)\dot{q}^j + G(q^j) + T_a(t) = u^j + f^j, \tag{1}$$

$$\Phi(q^j) = 0 \tag{2}$$

where $q^j \in R^n$ is the generalised displacement; $M(q^j) \in R^{n\times n}$ is the symmetric positive definite inertia matrix; $C(q^j, \dot{q}^j)\dot{q}^j \in R^n$ is the vector of centripetal and Coriolis torques and $G(q^j) \in R^n$ is the vector of gravitational torques; $T_a(t) \in R^n$ is the unknown bounded disturbance vector that is repetitive for each iteration; $u^j \in R^n$ is the generalised control input; $\Phi(q^j)$ is the m-dimensional constraint vector function; $f^j = J^T(q^j)\lambda^j$; $J(q^j) = [\partial\Phi(q^j)/\partial q^j]$ is an $m \times n$ Jacobian matrix; and $\lambda^j \in R^m$ is the generalised contact force vector associated with the constraints.

The constraints given in (2) are assumed to be holonomic and frictionless. Note that, if $\Phi(q^j) = 0$ is identically satisfied, then also $J(q^j)\dot{q}^j = 0$. Hence, the motion of robot is constrained in the constraint manifold $S \subset R^{2n}$, defined by

$$S = \{(q^j, \dot{q}^j) : \Phi(q^j) = 0, \; J(q^j)\dot{q}^j = 0\}.$$

McClamroch and Wang[4] use the following nonlinear transformation to convert the constrained system into two reduced unconstrained subsystems in which the constraints are satisfied automatically.

Suppose that there exists an open set $V = \mathbb{R}^{n-m}$ and a function $\Omega: V \to \mathbb{R}^m$, such that

$$\Phi(\Omega(q_2^j), q_2^j) = 0 \quad \text{for all } q_2^j \in V.$$

Consider the nonlinear transformation

$$x^j = \begin{bmatrix} x_1^j \\ x_2^j \end{bmatrix} = X(q^j) = \begin{bmatrix} q_1^j - \Omega(q_2^j) \\ q_2^j \end{bmatrix} \tag{3}$$

which is differentiable and has a differentiable inverse transformation $Q: \mathbb{R}^n \to \mathbb{R}^n$, such that

$$q^j = \begin{bmatrix} q_1^j \\ q_2^j \end{bmatrix} = Q(x^j) = \begin{bmatrix} x_1^j + \Omega(x_2^j) \\ x_2^j \end{bmatrix} \tag{4}$$

Let the nonsingular Jacobian matrix of the inverse transformation be

$$T(x^j) = \frac{\partial Q(x^j)}{\partial x^j} = \begin{bmatrix} I_m & \partial\Omega(x_2^j)/\partial x_2^j \\ 0 & I_{n-m} \end{bmatrix}. \tag{5}$$

The constrained system, given in (1) and (2), can be transformed to

$$\overline{M}(x^j)\ddot{x}^j + \overline{C}(x^j, \dot{x}^j)\dot{x}^j + \overline{G}(x^j) + T^T(x^j)T_e(t) = T^T(x^j)u^j + T^T(x^j)\overline{J}^T(x^j)\lambda^j, \tag{6}$$

$$x_1^j = 0. \tag{7}$$

where $\overline{M}(x^j) = T^T(x^j)M(Q(x^j))T(x^j)$,

$\overline{C}(x^j, \dot{x}^j) = T^T(x^j)C(Q(x^j), T(x^j)\dot{x}^j)T(x^j) + T^T(x^j)M(Q(x^j))\dot{T}(x^j)$,

$\overline{G}(x^j) = T^T(x^j)G(Q(x^j))$,

$\overline{J}(x^j) = J(Q(x^j))$.

In this paper, we assume

$$\Phi(q^j) = T_1 q^j + \theta = 0 \tag{8}$$

where $T_1 \in \mathbb{R}^{m\times n}$ is a constant matrix and $\operatorname{rank} T_1 = m$; $\theta \in \mathbb{R}^m$ is a constant vector. Without loss of generality, we can rewrite (8) as

$$\Phi(q^j) = q_1^j + T_2 q_2^j + \theta = 0$$

where $q_1^j \in \mathbb{R}^m$ and $q_2^j \in \mathbb{R}^{n-m}$; $T_2 \in \mathbb{R}^{m\times(n-m)}$ is a constant matrix. Hence, we have

$$\overline{J}(x^j) = [I_m\ T_2]; \quad T(x^j) = \begin{bmatrix} I_m & -T_2 \\ 0 & I_{n-m} \end{bmatrix} \triangleq T$$

and thus

$$\overline{M}(x^j) = T^T M(Q(x^j))T, \tag{9}$$

$$\overline{C}(x^j, \dot{x}^j) = T^T C(Q(x^j), T^T\dot{x}^j)T, \tag{10}$$

$$\overline{G}(x^j) = T^T G(Q(x^j)). \tag{11}$$

By introducing the partitioning of the identity matrix $I_n = [E_1^T\ E_2^T]$, where E_1 is an $m\times n$ matrix and E_2 is an $(n-m)\times n$ matrix, (6) can further be transformed to

$$E_1\overline{M}(x_2^j)E_2^T\ddot{x}_2^j + E_1\overline{C}(x_2^j, \dot{x}_2^j)E_2^T\dot{x}_2^j + E_1\overline{G}(x_2^j) + E_1T^T T_e(t) = E_1T^T u^j + \lambda^j, \tag{12}$$

$$E_2\overline{M}(x_2^j)E_2^T\ddot{x}_2^j + E_2\overline{C}(x_2^j, \dot{x}_2^j)E_2^T\dot{x}_2^j + E_2\overline{G}(x_2^j) + E_2T^T T_e(t) = E_2T^T u^j. \tag{13}$$

where $\overline{M}(x_2^j)$, $\overline{C}(x_2^j, \dot{x}_2^j)$ and $\overline{G}(x_2^j)$ denote respectively $\overline{M}(x^j)$, $\overline{C}(x^j, \dot{x}^j)$ and $\overline{G}(x^j)$ evaluated at $x^j = [0^T\ x_2^{jT}]^T$ and $\dot{x}^j = [0^T\ \dot{x}_2^{jT}]^T$. By considering certain properties inherent to robot dynamics, see, e.g., [13] and [14], we immediately obtain the following properties:

Property 1 The matrix $E_2\overline{M}(x_2^j)E_2^T$ is symmetric and positive definite. There exists a positive constant α so that

$$\alpha I_{n-m} \leqslant E_2\overline{M}(x_2^j)E_2^T \quad \forall\, x_2^j \in \mathrm{R}^n.$$

Property 2 The exist constants k_M, k_c and k_G, such that for any x_2^j, $\dot{x}_2^j$, we have

$$\| \overline{M}(x_2^j) \| \leqslant k_M, \quad \| \overline{C}(x_2^j,\dot{x}_2^j) \| \leqslant k_C \| \dot{x}_2^j \| \quad \text{and} \quad \| \overline{G}(x_2^j) \| \leqslant k_G.$$

Property 3 If $C(q,\dot{q})$ is defined so that

$$x^T(\dot{M}(q) - 2C(q,\dot{q}))x = 0 \quad \forall\, x \in \mathrm{R}^n.$$

is verified, $E_2\overline{M}(x_2^j)E_2^T$ and $E_2\overline{C}(x_2^j,\dot{x}_2^j)E_2^T$ in (13) satisfy

$$y^T(E_2\dot{\overline{M}}(x_2^j)E_2^T - 2E_2\overline{C}(x_2^j,\dot{x}_2^j)E_2^T)y = 0, \quad \forall\, y \in \mathrm{R}^{n-m}.$$

3 Learning Control Design

In this section, we consider the tracking problem of constrained robots. For simplicity, the tracking problem is formulated in terms of robot coordinates. The desired motion and the desired constraint forces, in robot coordinates, are defined by vectors q_d and f_d respectively. For consistency with the imposed constraints, it is necessary that $\Phi(q_d)=0$ and $f_d=J^T(q_d)\lambda_d$ are identically satisfied, for some multiplier λ_d. We assume q_d, $\dot{q}_d$ and $\ddot{q}_d$ to be bounded.

Our objective is that given a desired motion q_d and a desired constraint force f_d, or desired multiplier λ_d, to determine a sequence of control torques which guarantee that the perfect motion tracking, i. e., $\lim_{j\to\infty} q^j=q_d$, is achieved and the force tracking error is bounded and adjustable for the entire span of $t\in[0,\ t_f]$.

Due to $q_1^j=\Omega(q_2^j)$, it is only required $q_2^j\to q_{2d}$, i. e., $x_2^j\to x_{2d}(x_{2d}\triangleq q_{2d})$ as $j\to\infty$ to achieve perfect motion tracking.

We present the control torque for the jth iteration as follows:

$$\begin{aligned} T^T u^j = {} & KE_2^T z^j + \hat{\overline{M}}(x_2^j)E_2^T\ddot{x}_{2d} + \hat{\overline{C}}(x_2^j,\dot{x}_2^j)E_2^T\dot{x}_{2d} + \hat{\overline{G}}(x_2^j) \\ & + a(\overline{M}(x_2^j)E_2^T\dot{e}^j + \overline{C}(x_2^j,\dot{x}_2^j)E_2^T e^j) + H^j + E_1^T G_f(\lambda^j - \lambda_d) - E_1^T\lambda_d \end{aligned} \tag{14}$$

where, K is an $n\times n$ symmetric positive definite matrix;

$$e^j \triangleq x_{2d} - x_2^j;$$

$$z^j \triangleq \dot{e}^j + ae^j, \quad a>0;$$

$$\hat{\overline{M}}(x_2^j) \triangleq \overline{M}(x_2^j) - \overline{M}(x_{2d});$$

$$\hat{\overline{C}}(x_2^j,\dot{x}_2^j) \triangleq \overline{C}(x_2^j,\dot{x}_2^j) - \overline{C}(x_{2d},\dot{x}_{2d});$$

$$\hat{\overline{G}}(x_2^j) \triangleq \overline{G}(x_2^j) - \overline{G}(x_{2d});$$

$H^{j+1}\triangleq H^j+\beta KE_2^T z^j$, $0<\beta<2$, H^1 is set to be bounded for all $t\in[0,\ t_f]$;

G_f is an $m\times m$ constant nonnegative definite feedback gain matrix.

With the learning control law (14), the following theorem ensures that the perfect motion tracking is achieved and the force tracking error is bounded and abjustable for all $t\in[0,\ t_f]$.

Theorem Consider the constrained robot modeled in the reduced form (12), (13), using the learning control law (14), then the following holds:

$$M(q^j)=\begin{bmatrix} m_1l_1^2 & m_1l_1^2+m_1l_1l_2\cos q_2^j \\ m_1l_1^2+m_1l_1l_2\cos q_2^j & (m_1+m_2)l_2^2+m_1l_1^2+2m_1l_1l_2\cos q_2^j \end{bmatrix},$$

$$C(q^j,\dot q^j)=\begin{bmatrix} 0 & m_1l_1l_2\dot q_2^j\sin q_2^j \\ -m_1l_1l_2(\dot q_1^j+\dot q_2^j)\sin q_2^j & -m_1l_1l_2\dot q_1^j\sin q_2^j \end{bmatrix},$$

$$G(q^j)=\begin{bmatrix} -[m_1l_1\cos(q_1^j+q_2^j)]g \\ -[(m_1+m_2)l_2\cos q_1^j+m_1l_1\cos(q_1^j+q_2^j)]g \end{bmatrix},$$

$$T_a(t)=\begin{bmatrix} 0.5+0.5\cos 10t \\ 1+0.5\sin 10t \end{bmatrix}.$$

The constraint surface, in terms of robot coordinates, is expressed as

$$\Phi(q^j)=q_1^j-\gamma=0$$

where γ is a constant. $J^{\mathrm T}(q^j)=[1\ 0]^{\mathrm T}$, and thus $f_2\equiv 0$. In this example, the parameters are assumed to have the following values

$$m_1=m_2=1(\mathrm{kg}),\quad l_1=l_2=1(\mathrm{m}),\quad r=\sqrt{2}\,(\mathrm{m})$$

and thus $\gamma=\pi/2$. The desired trajectory and the desired constraint force are respectively assumed to $q_d=[\frac{\pi}{2}\sin(2t)]^{\mathrm T}$ and $f_d=[7\ 0]^{\mathrm T}$. In the controller design, we choose

$$K=\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix};\quad \alpha=2;\quad \beta=1;$$

$$H^1=1;\quad G_f=4.$$

Fig. 2(a) and (b) depict q_2 at the first iteration and at the 20th iteration respectively. f_1 at the first iteration and at the 20th iteration are displayed in Fig. 2(c).

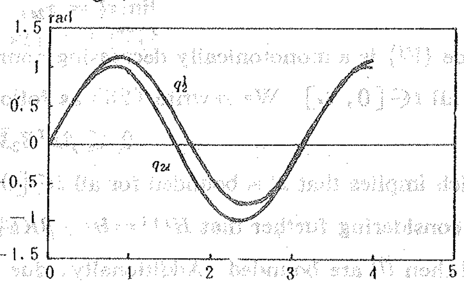

Fig. 2(a) Position tracking error of joint 2, at $j=1$

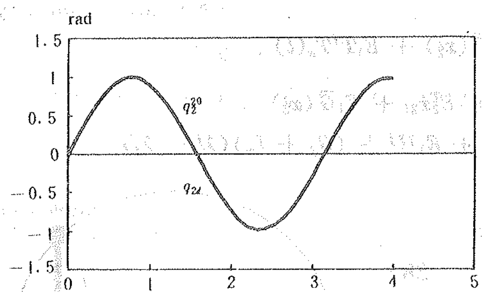

Fig. 2(b) Position tracking error of joint 2, at $j=20$

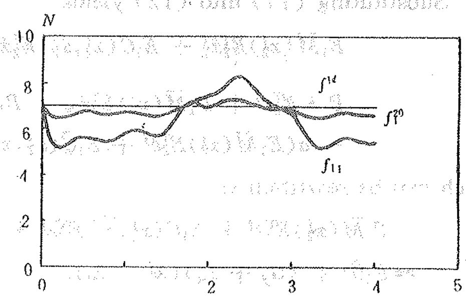

Fig. 2(c) Force tracking errors at $j=1$ and $j=20$

5 Conclusion

We have presented a new iterative learning controller for constrained robots in the presence of unknown bounded disturbances. The controller design is based on the singular system model representation. In the motion tracking of the closed-loop system, the unknown disturbance rejection is achieved for all $t\in[0,t_f]$ and the perfect motion tracking is guaranteed. In addition, the force tracking error remains bounded and the bound can be adjusted by a design matrix G_f. The control input contains no acceleration measurement or estimation.

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用奇异模型描述的受限机器人的学习控制

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摘要：本文给出了一类受限机器人的迭代学习控制方案，控制器的设计是基于机器人模型的降阶形式. 在存在有界未知干扰的情况下，对末端操纵器受线性、无摩擦约束面约束的受限机器人. 本文给出的控制方案保证了机器人系统的完全运动跟踪，同时保证了力跟踪误差是有界的，且界的大小是可调节的.

关键词：受限机器人；学习控制；跟踪

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