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A New Robust Control Strategy for Robot Tracking

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Abstract: The paper studies the tracking control problem of a robot manipulator subject to parametric uncertainty and external disturbance. By exploiting the important properties of robot dynamics, a new robust control strategy is proposed, based on Lyapunov approach. The novelty lies in that it removes the requirement of a priori bound on parametric uncertainty and weakens the restriction on external disturbance. It is shown that with this control strategy, the asymptotic tracking is guaranteed.

Key words: tracking control; Lyapunov approach; robot; uncertainty

1 Introduction

In this paper, we intend to study a new robust control strategy for a robot manipulator subject to parametric uncertainty and external disturbance. Much work has been done in this area, but almost all proposed control strategies need a priori bound on the parametric uncertainty and assume a strong restriction on external disturbance[1~6]. By exploiting the important properties of robot dynamics, a new robust control strategy is proposed, based on Lyapunov approach. The novelty lies in that it removes the requirement of a priori bound on parametric uncertainty and weakens the restriction on external disturbance, meanwhile, guarantees the asymptotic tracking. It is well known that the manipulator inertia matrix is bounded above and below[7]. Unlike the requirement of "closeness in norm" of the known inertia matrix to the actual one [5,6], the present paper assumes to know only an upper bound (which is not necessary to be the smallest) of the norm of actual inertia matrix.

2 Some Properties of Robot Dynamics

We shall consider the dynamics of a robot to be described by the following nonlinear differential equation

$$\tau = M(q)\ddot{q} + N(q,\dot{q}), \tag{1a}$$

$$N(q,\dot{q}) = V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} + F_s(q) + \tau_d(t,q,\dot{q}), \tag{1b}$$

where q is an $n\times 1$ vector of joint variables, τ is an $n\times 1$ vector of control torque, $M(q)$ is an $n\times n$ inertia matrix, $V_m(q,\dot{q})$ is an $n\times n$ matrix of centripetal and Coriolis terms, $G(q)$ is an $n\times 1$ vector of gravity terms, F_d is an $n\times n$ diagonal matrix of dynamic friction coefficients, $F_s(q)$ is an $n\times 1$ vector of static friction terms, $\tau_d(t,q,\dot{q})$ is an $n\times 1$ vector of external disturbance, which in most papers is assumed to be bounded[1~6], i.e., $\|\tau_d\|\leqslant c$ with c a known or unknown

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constant. However, since τ_d may include various unmodeled effects, it in general depends on t, q and $\dot{q}$, thus, τ_d may be unbounded. [9] assumes $\|\tau_d(t,q,\dot{q})\| \leqslant c_0 + c_1\|q\| + c_2\|\dot{q}\|$ with c_0, c_1, c_2 unknown constants. In this paper, we further weaken restriction on τ_d as follows

$$\|\tau_d(t,q,\dot{q})\| \leqslant c_0 + c_1\|q\| + c_2\|\dot{q}\| + c_3\|q\|^2 + c_4\|\dot{q}\|^2 \tag{2}$$

where c_0, c_1, c_2, c_3, c_4 are unknown constants. Restriction (2) may encompass a broader class of external disturbance.

Let $q^d(t)$ characterize the desired trajectory that the robot should track. Suppose $q^d(t)$, $\dot{q}^d(t)$ and $\ddot{q}^d(t)$ are all bounded. Define error state as $e^{\mathrm{T}} = [e_1^{\mathrm{T}}, e_2^{\mathrm{T}}] = [e_1^{\mathrm{T}}, \dot{e}_1^{\mathrm{T}}]$, where $e_1 = q^d - q$ is the tracking error.

Property 1[7] There exist positive constants $\underline{m}$, $\bar{m}$ and m such that

a) $\underline{m}I \leqslant M(q) \leqslant \bar{m}I$,

b) $\|\dot{M}(q)\| \leqslant m$, $\quad \forall\ q \in \mathrm{R}^n$

where I is an $n \times n$ identity matrix.

Property 2 In terms of (2) and the boundedness of both $q^d(t)$ and $\dot{q}^d(t)$, there exist positive constants β_0, β_1, β_2, β_3, β_4 such that

a) $\|N(q, \dot{q})\| \leqslant \beta_0 + \beta_1\|e\| + \beta_2\|e\|^2$,

b) $\|V_m(q, \dot{q})\| \leqslant \beta_3 + \beta_4\|e\|$, $\quad \forall\ (q, \dot{q}) \in \mathrm{R}^n \times \mathrm{R}^n$.

Property 2 is based on the fact[7] that $V_m(q, \dot{q})\dot{q}$ is quadratic in $\dot{q}$ and both $V_m(q, \dot{q})\dot{q}$ and $G(q)$ depend on q only in terms of sine and cosine functions.

Property 3[7] For any $n \times 1$ vector w, the following relation exists

$$\frac{1}{2} w^{\mathrm{T}} \dot{M}(q) w = w^{\mathrm{T}} V_m(q, \dot{q}) w$$

In this paper, we assume to know some upper bound $\bar{m}^*$ of $\bar{m}$, which is not necessary to be the smallest. All other constants in Property 1 and Property 2 are assumed completely unknown.

3 Derivation of the Control Law

In this paper, we propose the following control law

$$\tau = \tau_1 + \tau_2, \tag{3a}$$

$$\tau_1 = \ddot{q}^d + k_p e_1 + k_v e_2, \tag{3b}$$

$$\tau_2 = \psi_p(e,t)e_1 + \psi_v(e,t)e_2. \tag{3c}$$

where k_p, k_v are positive constants to be determined, $\psi_p(e,t)$, $\psi_v(e,t)$ are functions to be determined.

Substituting the control law (3) into system (1), we obtain error system

$$\dot{e} = Ae + B\Delta A - B\tau_2, \tag{4}$$

where

$$A = \begin{bmatrix} 0 & I \\ -k_p M^{-1} & -k_v M^{-1} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}, \quad \Delta A = [M - I]\ddot{q}^d + N.$$

Our control strategy is based on Lyapunov approach. As a first step, we introduce the following lemma.

Lemma For auxiliary parameters α and β, if $0<\alpha<1$, $\beta>(1-\alpha)\alpha\bar{m}^*$, then matrix $P=\frac{1}{2}\begin{bmatrix}\beta I+\alpha^2 M & \alpha M\\ \alpha M & M\end{bmatrix}$ is positive definite.

Choosing for error system (4) a Lyapunov function candidate $V_1(e)=e^{\mathrm{T}}Pe$ and making use of Property 1~3, we have

Theorem 1 If control law (3) satisfies

$$k_p+\alpha k_v=\beta, \tag{5a}$$

$$k_v=\alpha k_p, \tag{5b}$$

$$\psi_p(e,t)=\alpha\psi_v(e,t). \tag{5c}$$

then there exist positive constants ξ_0, ξ_1, ξ_2 such that

$$\dot{V}_1(e)\leqslant -k\|e\|^2+\xi_0\|e\|+\xi_1\|e\|^2+\xi_2\|e\|^3-\psi(e,t)\|\alpha e_1+e_2\|^2, \tag{6}$$

where $k=k_v$, $\psi(e,t)=\psi_v(e,t)$.

Regarding ξ_0, ξ_1, ξ_2 as unknown constants, we introduce the following algorithm to regulate $\psi(e,t)$

$$\psi(e,t)=\begin{cases}\dfrac{1}{\|\alpha e_1+e_2\|^2}(\hat{\xi}_0\|e\|+\hat{\xi}_1\|e\|^2+\hat{\xi}_2\|e\|^3), & \text{if } \|\alpha e_1+e_2\|\neq 0,\\ 0, & \text{if } \|\alpha e_1+e_2\|=0,\end{cases} \tag{7a}$$

$$\dot{\hat{\xi}}_0=\begin{cases}\gamma_0\|e\|, & \text{if } \|\alpha e_1+e_2\|\neq 0,\\ 0, & \text{if } \|\alpha e_1+e_2\|=0,\end{cases} \tag{7b}$$

$$\dot{\hat{\xi}}_1=\begin{cases}\gamma_1\|e\|^2, & \text{if } \|\alpha e_1+e_2\|\neq 0,\\ 0, & \text{if } \|\alpha e_1+e_2\|=0,\end{cases} \tag{7c}$$

$$\dot{\hat{\xi}}_2=\begin{cases}\gamma_2\|e\|^3, & \text{if } \|\alpha e_1+e_2\|\neq 0,\\ 0, & \text{if } \|\alpha e_1+e_2\|=0,\end{cases} \tag{7d}$$

where γ_0, γ_1, γ_2 are positive constants.

Theorem 2 For error system (4) under the control of (3), (5), (7), the asymptotic tracking is ensured, i. e. , $\lim\limits_{t\to\infty}e(t)=0$.

Proof Define $\tilde{\xi}_0=\hat{\xi}_0-\xi_0$, $\tilde{\xi}_1=\hat{\xi}_1-\xi_1$, $\tilde{\xi}_2=\hat{\xi}_2-\xi_2$. Choose for extended system a Lyapunov function candidate

$$V(e,\tilde{\xi}_0,\tilde{\xi}_1,\tilde{\xi}_2)=V_1(e)+\frac{1}{2}\gamma_0^{-1}\tilde{\xi}_0^2+\frac{1}{2}\gamma_1^{-1}\tilde{\xi}_1^2+\frac{1}{2}\gamma_2^{-1}\tilde{\xi}_2^2.$$

To calculate $\dot{V}(e,\tilde{\xi}_0,\tilde{\xi}_1,\tilde{\xi}_2)$ along the solution of (4), (7), there are two cases to be considered.

Case 1 when e moves outside the submanifold $\alpha e_1+e_2=0$. In terms of (6) and (7), we have

$$\dot{V}=\dot{V}_1+\gamma_0^{-1}\tilde{\xi}_0\dot{\tilde{\xi}}_0+\gamma_1^{-1}\tilde{\xi}_1\dot{\tilde{\xi}}_1+\gamma_2^{-1}\tilde{\xi}_2\dot{\tilde{\xi}}_2\leqslant -k\|e\|^2.$$

Case 2 when e moves on the submanifold $\alpha e_1+e_2=0$. In this case, $\dot{e}_1=e_2=-\alpha e_1$, $\dot{e}_2=-\alpha\dot{e}_1=-\alpha e_2$.

In terms of (7)

$$\begin{aligned}
\dot V &= 2\dot e^{T}Pe + e^{T}\dot P e \\
&= [-\alpha e_1^{T}, -\alpha e_2^{T}]\begin{bmatrix}\beta I+\alpha^2 M & \alpha M\\ \alpha M & M\end{bmatrix}\begin{bmatrix}e_1\\ e_2\end{bmatrix} + \frac{1}{2}e^{T}\begin{bmatrix}\alpha^2\dot M & \alpha\dot M\\ \alpha\dot M & \dot M\end{bmatrix}e \\
&= -\alpha[e_1^{T}, e_2^{T}]\begin{bmatrix}\beta I+\alpha^2(M-\frac{\dot M}{2\alpha}) & \alpha(M-\frac{\dot M}{2\alpha})\\ \alpha(M-\frac{\dot M}{2\alpha}) & M-\frac{\dot M}{2\alpha}\end{bmatrix}\begin{bmatrix}e_1\\ e_2\end{bmatrix} \\
&= -\alpha[e_1^{T}, -\alpha e_1^{T}]\begin{bmatrix}\beta I+\alpha^2(M-\frac{\dot M}{2\alpha}) & \alpha(M-\frac{\dot M}{2\alpha})\\ \alpha(M-\frac{\dot M}{2\alpha}) & M-\frac{\dot M}{2\alpha}\end{bmatrix}\begin{bmatrix}e_1\\ -\alpha e_1\end{bmatrix} \\
&= -\alpha[\beta e_1^{T}, 0][e_1^{T}, -\alpha e_1^{T}]^{T} = -\alpha\beta e_1^{T}e_1.
\end{aligned}$$

Since $\| e \|^2 = [e_1^{T}, -\alpha e_1^{T}][e_1^{T}, -\alpha e_1^{T}]^{T} = (1+\alpha^2)e_1^{T}e_1$, we have

$$\dot V = -\frac{\alpha\beta}{1+\alpha^2}\| e \|^2.$$

Generalizing the two cases, we obtain $\dot V(e, \tilde\xi_0, \tilde\xi_1, \tilde\xi_2) \leqslant -s\, \| e \|^2$ where $s = \min\{k, \frac{\alpha\beta}{1+\alpha^2}\}$.

Thus, from Lassalle's theorem, we obtain $\lim\limits_{t\to\infty} e(t) = 0$.

To make the control law continuous, we introduce a boundary layer around the submanifold $\alpha e_1 + e_2 = 0$[8].

$$\psi(e,t) = \begin{cases}\dfrac{1}{\| \alpha e_1 + e_2 \|^2}(\hat\xi_0 \| e \| + \hat\xi_1 \| e \|^2 + \hat\xi_2 \| e \|^3), & \text{if } \| \alpha e_1 + e_2 \| \geqslant \varepsilon, \\ \dfrac{1}{\varepsilon}(\hat\xi_0 \| e \| + \hat\xi_1 \| e \|^2 + \hat\xi_2 \| e \|^3), & \text{if } \| \alpha e_1 + e_2 \| < \varepsilon,\end{cases} \tag{8a}$$

$$\dot{\hat\xi}_0 = \begin{cases}\gamma_0\|e\|, & \text{if } \|\alpha e_1 + e_2\| \geqslant \varepsilon, \\ 0, & \text{if } \|\alpha e_1 + e_2\| < \varepsilon,\end{cases} \tag{8b}$$

$$\dot{\hat\xi}_1 = \begin{cases}\gamma_1\|e\|^2, & \text{if } \|\alpha e_1 + e_2\| \geqslant \varepsilon, \\ 0, & \text{if } \|\alpha e_1 + e_2\| < \varepsilon,\end{cases} \tag{8c}$$

$$\dot{\hat\xi}_2 = \begin{cases}\gamma_2\|e\|^3, & \text{if } \|\alpha e_1 + e_2\| \geqslant \varepsilon, \\ 0, & \text{if } \|\alpha e_1 + e_2\| < \varepsilon.\end{cases} \tag{8d}$$

Now we summary our control law as follows

$$\tau = \ddot q^{d} + k_p e_1 + k_v e_2 + \alpha\psi(e,t)e_1 + \psi(e,t)e_2 \tag{9}$$

where $0<\alpha<1$, $\beta>(1-\alpha)\alpha\overline{m}^{*}$, k_p and k_v satisfy (5), $\psi(e,t)$ is determined by (8).

4 Simulation Example

A simple two-degree-of-freedom manipulator is simulated to test the proposed control law. The manipulator is modeled as two rigid links of lengths l_1 and l_2 with point masses m_1 and m_2 at the distal ends of links. The dynamic equations of the manipulator can be found in [10]. The parameters are chosen to be $m_1 = m_2 = 1.0$kg, $l_1 = l_2 = 1.0$m, from which $\overline{m} = 2 + 2\sqrt{2}$. The desired trajectory is chosen to be $q_1^{d}(t) = \sin t$, $q_2^{d}(t) = \cos t$.

Suppose we have known an upper bound $\bar{m}^* = 40$ of $\bar{m}$. Choose $\alpha = 0.5$, $\beta = 50 > (1-\alpha)\alpha\bar{m}^*$, $\gamma_0 = \gamma_1 = \gamma_2 = 10$, $\varepsilon = 0.01$, the control law is determined by (9).

In simulation, the initial conditions are chosen to be $q_1(0) = 1$, $q_2(0) = 0$, $\dot{q}_1(0) = \dot{q}_2(0) = 0$, $\hat{\xi}_0(0) = \hat{\xi}_1(0) = \hat{\xi}_2(0) = 0$; the external disturbance is supposed to be $\tau_d = [q_1\dot{q}_1\sin t,\ q_2\dot{q}_2\cos t]^{\mathrm{T}}$. The simulation results are shown in figures.

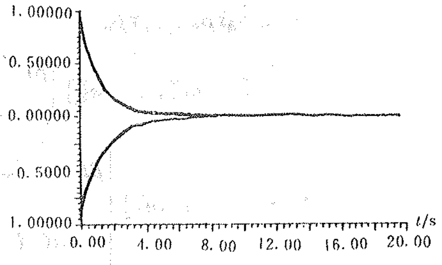

Fig. 1 Position errors of two joints

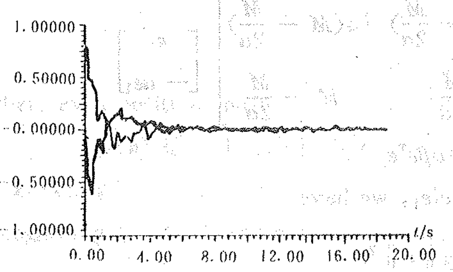

Fig. 2 Velocity errors of two joints

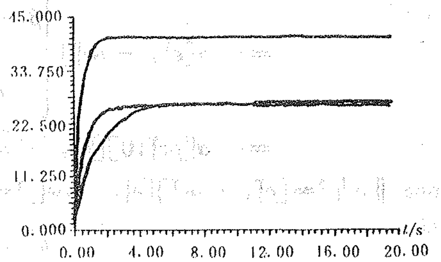

Fig. 3 Transient curves of $\hat{\xi}_0, \hat{\xi}_1$ and $\hat{\xi}_2$

5 Conclusion

By exploiting robot dynamic structure properties, we propose a new robust control strategy, with which the asymptotic tracking is ensured. Since it removes the requirement of a priori bound on parametric uncertainty and weakens the restriction on external disturbance, the proposed strategy is suitable to the case of high number of links.

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一种新的机械手轨迹跟踪鲁棒控制

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摘要：本文考虑存在外扰及参数不确定情况下机械手轨迹跟踪问题，通过利用机械手固有的结构特性，提出一种基于李雅普诺夫方法的新控制算法.该算法的新颖之处在于无需预先知道参数的不确定范围，并且减弱了对外扰的限制.该算法的渐近跟踪性已被证明.

关键词：轨迹跟踪控制；李雅普诺夫方法；机器人；不确定性

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