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Asymptotically Unbiased Estimation of Transfer Functions with Colored Noises and Unmodeled Dynamics*

ZHANG Ying and FENG Chunbo

(Research Institute of Automation, Southeast University·Nanjing, 210018, PRC)

Abstract: In the presence of the unmodeled dynamics, one common and important concern is how to remove the noise-induced bias in the estimate of the transfer function. An extension to the newly established BELS method is made in this paper. When the unmodeled dynamics is decribed by a finite impulse response (FIR) model, it can achieve a unbiased estimate of the transfer function without the priori knowledge about the probability density functions (PDFs) of the noise and the dynamics.

Key words: identification; parameter estimation; least-squares estimation

1 Introduction

In general, the transfer function of a practical system is difficult to determined very accurately. Usually only a low-order nominal part of the transfer function can be identified by using the input and output data. Therefore, the presence of unmodeled dynamics is inevitable, and it will affect the process of identification. How to accurately identify the nominal part of a transfer function in the presence of noise and unmodeled dynamics is an interesting task. This problem was studied in [1] under the assumption that the unmodeled dynamics can be sufficiently closely approximated by a FIR model. A maximum-likelihood technique was developed in [1] based on the priori knowledge that the probability density functions (PDFs) of the noise and the unmodeled dynamics are known. However, it is usually difficult or even impossible to obtain these PDFs in practice. In [1] only the bound of the unmodeled dynamics can be known. In this paper, it will be shown that when the unmodeled dynamics is described by a FIR model, not only the nominal model but also the FIR model can be identified unbiasedly without priori knowledge about the PDFs of the noise and the unmodeled dynamics.

In order to obtain the consistent estimates of the system parameters, a bias-eliminated least square method (BELS) was developed in [2～4]. In this method, a known prefilter was inserted into the identified system so that the augmented system has some known zeros which can, based on asymptotic analysis, be used for eliminating the colored-noise-induced biases in the ordinary LS estimates. Since the unbiased estimates of system parameters can be obtained by the BELS method without any priori information of the noise model, this method possesses good robustness with respect to the measurement noise. In this paper, the BELS method is used to deal with the

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problem in [1].

2 Problem Formulation

The aim of identification is to establish a model for a dynamic system by using the observed data of the input-output data sequence $Z^N=[u_k, y_k]$, where the observed data Z^N are assumed to be generated by the system described as

$$y_k = G_T(q^{-1})u_k + H(q^{-1})e_k. \tag{1}$$

Here $G_T(q^{-1})$ and $H(q^{-1})$ are rational transfer functions in the backward shift operator q^{-1}. Both of them are assumed to be asymptotically stable and have no poles in the region $|q|\geqslant 1$. The disturbance sequence e_k and the input sequence u_k are assumed to be quasistationary according to [5]. They are also assumed to be uncorrelated with each other. Without loss of generality, e_k is assumed to have a zero expectation, i. e. $E[e_k]=0$, where $E[\cdot]$ denotes expectation operator.

The stable transfer function $G_T(q^{-1})$ to be identified can be divided into two parts: a simple nominal low-order parametric part $G(q^{-1}, \theta_0)$ and a more complex part of unmodeled dynamics $G_\Delta(q^{-1})$, namely,

$$G_T(q^{-1}) = G(q^{-1}, \theta_0) + G_\Delta(q^{-1}). \tag{2}$$

In general the transfer function $G(q^{-1}, \theta_0)$ can be characterized by using a form as

$$G(q^{-1}, \theta_0) = \frac{B(q^{-1})}{A(q^{-1})} = \frac{b_1q^{-1} + b_2q^{-2} + \cdots + b_mq^{-m}}{1 + a_1q^{-1} + a_2q^{-2} + \cdots + a_nq^{-n}}. \tag{3}$$

Thus the parameters to be identified are the coefficients, denoted by the vector $\theta_0=[a_1, a_2, \cdots, a_n, b_1, \cdots, b_m]^{\mathrm{T}} \in \mathrm{R}^{n+m}$, in $G(q^{-1}, \theta_0)$.

If the unmodeled dynamics can be described by a FIR model [1], i. e.

$$G_\Delta(q^{-1}) = q^{-d}(\eta_1^0 q^{-1} + \eta_2^0 q^{-2} + \cdots + \eta_L^0 q^{-L}), \tag{4}$$

then the system equation (1) can now be rewritten in a signal-regressive form as follows:

$$y_k = \varphi_k^{\mathrm{T}}\theta_0 + \psi_k^{\mathrm{T}}\eta + v_k, \tag{5}$$

where

$$\varphi_k = [y_{k-1}, y_{k-2}, \cdots, y_{k-n}, u_{k-1}, \cdots, u_{k-m}]^{\mathrm{T}} \in \mathrm{R}^{n+m}, \tag{6}$$

$$\psi_k = [u_{k-d-1}, u_{k-d-2}, \cdots, u_{k-d-n-L}]^{\mathrm{T}} \in \mathrm{R}^{n+L}, \tag{7}$$

$$\eta = [\eta_1, \eta_2, \cdots, \eta_{n+L}]^{\mathrm{T}} \in \mathrm{R}^{n+L}, \tag{8}$$

$$v_k = A(q^{-1})H(q^{-1})e_k. \tag{9}$$

and $\eta_i (i=1,2,\cdots,n+L)$ is the coefficient of the term q^{-i} in the following equation:

$$\pi(q^{-1}) = G_\Delta(q^{-1})A(q^{-1}) = \eta_1q^{-1} + \eta_2q^{-2} + \cdots + \eta_{n+L}q^{-(n+L)}. \tag{10}$$

Using the notations

$$\Phi^{\mathrm{T}} = [\varphi_1, \varphi_2, \cdots, \varphi_N], \tag{11}$$

$$\Psi^{\mathrm{T}} = [\psi_1, \psi_2, \cdots, \psi_N], \tag{12}$$

$$Y^{\mathrm{T}} = [y_1, y_2, \cdots, y_N]. \tag{13}$$

Vector $\hat{\theta}_N$ is the LS estimate of θ_0 and is given by

$$\hat{\theta}_N = (\Phi\Phi^{\mathrm{T}})^{-1}\Phi^{\mathrm{T}}Y. \tag{14}$$

When N tends to infinity, from the asymptotic property of the LS estimate we can obtained [4]

$$\theta^* = \lim_{N\to\infty} \hat{\theta}_N = \theta_0 + R_{\varphi\varphi}^{-1} R_{\varphi v} + R_{\varphi\varphi}^{-1} R_{\varphi\psi} \eta, \tag{15}$$

with

$$R_{\varphi\varphi} = E[\varphi_k \varphi_k^{\mathrm{T}}], \quad R_{\varphi v} = E[\varphi_k v_k], \quad R_{\varphi\psi} = E[\varphi_k \psi_k^{\mathrm{T}}], \tag{16}$$

where θ_0 is the true parameter vector.

(15) shows that the LS estimate for θ_0 in the presence of a colored noise and unmodeled dynamics is certainly convergent but asymptotically biased. The asymptotic bias vector is given by

$$\Delta\theta = R_{\varphi\varphi}^{-1} R_{\varphi v} + R_{\varphi\varphi}^{-1} R_{\varphi\psi} \eta. \tag{17}$$

It follows from (17) that the bias consists of two parts; one is caused by the noise, the other is caused by the unmodeled dynamics. If there is only noise and no unmodeled dynamics, the biased-eliminated-least-squares (BELS) method proposed in [2~4] can be directly used to obtain the unbiased estimate of transfer functions. In this paper, the BELS method will be used to estimate the transfer functions described above. It will be shown that in this case the bias caused by noise can also be eliminated. Furthermore, it will be shown that the unmodeled dynamics can also be correctly estimated if it is defined by a FIR of known order.

3 Bels Method

Note that (15) can be rewritten as

$$\theta_0 = \lim_{N\to\infty} \hat{\theta}_N - \Delta\theta. \tag{18}$$

It shows clearly that an asymptotically unbiased estimate for θ_0 can be obtained if the bias term $\Delta\theta$ is extracted from the LS estimate of the unknown parameter vector. The main point of this section is to study the method for estimating $\Delta\theta$. In the expression of $\Delta\theta$ given by (17). η and $R_{\varphi v}$ are unknown vectors where η is defined by (8) and $R_{\varphi v}$ is given in [4] as

$$R_{\varphi v} = [r_{yv}(1), r_{yv}(2), \cdots, r_{yv}(n), 0, \cdots, 0]^{\mathrm{T}} \in \mathrm{R}^{n+\bar{m}} \tag{19}$$

Therefore, η and $R_{\varphi v}$ should be estimated in order to estimate the $\Delta\theta$.

In the same way as in [2], a digital filter $F^{-1}(q^{-1})$ is connected to the system at the input terminal, wher $F(q^{-1})$ is defined as

$$F(q^{-1}) = 1 + f_1 q^{-1} + f_2 q^{-2} + \cdots + f_{2n+L} q^{-(2n+L)} \tag{20}$$

$F(q^{-1})$ is designed to be stable, namely, the polynomial

$$F^*(q^{-1}) = q^{(2n+L)} + f_1 q^{(2n+L-1)} + \cdots + f_{(2n+L)} = q^{(2n+L)} F(q^{-1}) \tag{21}$$

has all its zeros located strictly inside the unit disc.

The filter $F^{-1}(q^{-1})$ is inserted to form an augmented system artificially. The augmented system thus obtained is expressed by the model

$$A(q^{-1}) y_k = \bar{B}(q^{-1}) \bar{u}_k + v_k + q^{-d} \bar{\Pi}(q^{-1}) \bar{u}_k, \tag{22}$$

where

$$\begin{aligned}\bar{B}(q^{-1}) &= F(q^{-1}) B(q^{-1}) \\ &= \bar{b}_1 q^{-1} + \bar{b}_2 q^{-2} + \cdots + \bar{b}_{\bar{m}} q^{-\bar{m}}, \quad \bar{m} = m + 2n + L,\end{aligned} \tag{23}$$

$$\begin{aligned}\bar{\Pi}(q^{-1}) &= F(q^{-1})\pi(q^{-1}) \\ &= \bar{\eta}_1 q^{-1} + \bar{\eta}_2 q^{-2} + \cdots + \bar{\eta}_{\bar{L}} q^{-\bar{L}}, \quad \bar{L} = 3n + L_2,\end{aligned} \tag{24}$$

$$\bar{u}_k = \frac{1}{F(q^{-1})} u_k. \tag{25}$$

Rewriting (22) in a signal regressive form gives

$$y_k = \bar{\varphi}_k^{\mathrm{T}} \bar{\theta}_0 + \bar{\psi}_k^{\mathrm{T}} \bar{\eta} + v_k, \tag{26}$$

with

$$\begin{aligned}\bar{\varphi}_k &= [y_{k-1}, y_{k-2}, \cdots, y_{k-n}, \bar{u}_{k-1}, \cdots, \bar{u}_{k-\bar{m}}]^{\mathrm{T}} \in \mathrm{R}^{n+\bar{m}}, \\ \bar{\psi}_k &= [\bar{u}_{k-d-1}, \bar{u}_{k-d-2}, \cdots, \bar{u}_{k-d-\bar{L}}]^{\mathrm{T}} \in \mathrm{R}^{\bar{L}}, \\ \bar{\theta}_0 &= [a_1, a_2, \cdots, a_n, \bar{b}_1, \cdots, \bar{b}_{\bar{m}}]^{\mathrm{T}} \in \mathrm{R}^{n+\bar{m}}, \\ \bar{\eta} &= [\bar{\eta}_1, \bar{\eta}_2, \cdots, \bar{\eta}_{n+L}]^{\mathrm{T}} \in \mathrm{R}^{\bar{L}}.\end{aligned}$$

In a similar way, the LS estimate of the parameters of the new model (22), based on N observed data $Z^N = [\{\bar{u}_k, \{y_k\}]$, can be obtained as follows

$$\hat{\bar{\theta}}_{\mathrm{LS}}(N) = (\bar{\Phi}\bar{\Phi}^{\mathrm{T}})^{-1}\bar{\Phi}^{\mathrm{T}} Y \in \mathrm{R}^{n+\bar{m}}, \tag{27}$$

and an asymptotic analysis gives

$$\begin{aligned}\lim_{N\to\infty} \hat{\bar{\theta}}_{\mathrm{LS}}(N) &= \bar{\theta}_0 + \Delta\theta \\ &= \bar{\theta}_0 + R_{\bar{\varphi}\bar{\varphi}}^{-1} R_{\bar{\varphi}v} + R_{\bar{\varphi}\bar{\varphi}}^{-1} R_{\bar{\varphi}\bar{\psi}} \bar{\eta},\end{aligned} \tag{28}$$

where

$$R_{\bar{\varphi}v} = [R_{yv}^{\mathrm{T}}; 0]^{\mathrm{T}} = [r_{yv}(1), \cdots, r_{yv}(n); 0, \cdots, 0]^{\mathrm{T}} \in \mathrm{R}^{n+\bar{m}}, \tag{29}$$

$$\bar{\Phi}^{\mathrm{T}} = [\bar{\psi}_1, \bar{\psi}_2, \cdots, \bar{\psi}_N]^{\mathrm{T}} \in \mathrm{R}^{(n+\bar{m})\times N}, \tag{30}$$

$$R_{\bar{\varphi}\bar{\varphi}} = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} (\bar{\varphi}_k \bar{\varphi}_k^{\mathrm{T}}) \in \mathrm{R}^{(n+\bar{m})\times(n+\bar{m})}, \tag{31}$$

$$R_{\bar{\varphi}\bar{\psi}} = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} (\bar{\varphi}_k \bar{\psi}_k^{\mathrm{T}}) \in \mathrm{R}^{(n+\bar{m})\times \bar{L}}. \tag{32}$$

It is to be noticed that the $(2n+L)$ known zeros of the augmented system are just the zeros of $F^*(q)$. By using these known zeros, $R_{\bar{\varphi}v}$ and $\bar{\eta}$ can now be estimated. Let $\lambda_i (i=1,2,\cdots,2n+L)$ be the zeros of $F^*(q)$. This implies that the following equations hold

$$\begin{aligned}\bar{B}^*(\lambda_i) &= F^*(\lambda_i) B^*(\lambda_i) \\ &= \bar{b}_1 \lambda_i^{\bar{m}-1} + \bar{b}_2 \lambda_i^{\bar{m}-2} + \cdots + \bar{b}_{\bar{m}} = 0, \quad i = 1,2,\cdots,2n+L,\end{aligned} \tag{33}$$

$$\begin{aligned}\bar{\Pi}^*(\lambda_i) &= F^*(\lambda_i)(\eta_1 \lambda_i^{n+L-1} + \eta_2 \lambda_i^{n+L-2} + \cdots + \eta_{n+L} \\ &= \bar{\eta}_1 \lambda_i^{\bar{L}-1} + \bar{\eta}_2 \lambda_i^{\bar{L}-2} + \cdots + \bar{\eta}_{\bar{L}} = 0, \quad i = 1,2,\cdots,2n+L.\end{aligned} \tag{34}$$

where $B^*(q) = q^m B(q^{-1})$, $\bar{B}^*(q) = q^{\bar{m}} \bar{B}(q^{-1})$ and $\bar{\Pi}^*(q) = q^{\bar{L}} \bar{\Pi}(q^{-1})$.

Let us introduce matrices H_1 and H_2 defined by these known zeros as follows:

$$H_1^{\mathrm{T}} = \left[\begin{array}{c|cccc} & \lambda_1^{\bar{m}-1}, & \cdots, & \lambda_1, & 1 \\ 0 & \cdots, & \cdots, & \ddots, & \cdots \\ & \lambda_{2n+L}^{\bar{m}-1}, & \cdots, & \lambda_{2n+L}, & 1 \end{array}\right] \in \mathrm{R}^{(2n+L)\times(n+\bar{m})}, \tag{35}$$

$$H_2^{\mathrm{T}} = \begin{bmatrix} \lambda_1^{\bar{L}-1}, & \cdots, & \lambda_1, & 1 \\ \cdots, & \cdots, & \cdots, & \cdots \\ \lambda_{2n+L}^{\bar{L}-1}, & \cdots, & \lambda_{2n+L}, & 1 \end{bmatrix} \in \mathrm{R}^{(2n+L)\times \bar{L}}. \tag{36}$$

Then (33) and (34) can be expressed in a compact form

$$H_1^T\bar{\theta}_0 = 0, \tag{37}$$

$$H_2^T\bar{\eta} = 0. \tag{38}$$

Multiplying the matrix H_1^T in both sides of (28) gives

$$H_1^T \lim_{N\to\infty} \hat{\bar{\theta}}_{LS}(N) = H_1^T R_{\bar{\varphi}\bar{\varphi}}^{-1} R_{\bar{\varphi}v} + H_1^T R_{\bar{\varphi}\bar{\varphi}}^{-1} R_{\bar{\varphi}\bar{\varphi}}\bar{\eta}. \tag{39}$$

Equations (38) and (39) can also be rewritten in a matrix form

$$\begin{bmatrix} H_2^T & 0 \\ H_1^T R_{\bar{\varphi}\bar{\varphi}}^{-1} R_{\bar{\varphi}\bar{\varphi}} & H_1^T R_{\bar{\varphi}\bar{\varphi}}^{-1} \end{bmatrix} \begin{bmatrix} \bar{\eta} \\ R_{\bar{\varphi}v} \end{bmatrix} = \begin{bmatrix} 0 \\ H_1^T \lim\limits_{N\to\infty} \hat{\bar{\theta}}_{LS}(N) \end{bmatrix}. \tag{40}$$

It is straightforward to conclude from (40) that

$$\begin{bmatrix} \bar{\eta} \\ R_{yv} \end{bmatrix} = \begin{bmatrix} H_2^T & 0 \\ H_1^T R_{\bar{\varphi}\bar{\varphi}}^{-1} R_{\bar{\varphi}\bar{\varphi}} & H_{11} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ H_1^T \lim\limits_{N\to\infty} \hat{\bar{\theta}}_{LS}(N) \end{bmatrix}. \tag{41}$$

with

$$H_1^T R_{\bar{\varphi}\bar{\varphi}}^{-1} = [H_{11}; H_{12}], \quad H_{11} \in \mathrm{R}^{(2n+L)\times n}. \tag{42}$$

Asymptotic bias $\Delta\bar{\theta}$ can be determined from (41), (42), (28) and (29). The calculations presented above can be performed by the following algorithm.

Algorithm 1

1) Design a suitable, stable filter $F^{-1}(q^{-1})$ of order $2n+L$, and connect it to the input terminal. Thus the original system is augmented.

2) Estimate the parameters of the augmented systm by using the ordinary LS method, which gives

$$\hat{\bar{\theta}}_{LS}(N) = R_{\bar{\varphi}\bar{\varphi}}^{-1}(N) R_{\bar{\varphi}y}(N). \tag{43}$$

3) Calculate the correlation vector $R_{\bar{\varphi}v}(N)$ and the parameter vector $\bar{\eta}(N)$ by

$$\begin{bmatrix} \bar{\eta}(N) \\ R_{yv}(N) \end{bmatrix} = \begin{bmatrix} H_2^T & 0 \\ H_1^T R_{\bar{\varphi}\bar{\varphi}}^{-1}(N) R_{\bar{\varphi}\bar{\varphi}}(N) & H_{11}(N) \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ H_1^T \hat{\bar{\theta}}_{LS}(N) \end{bmatrix}. \tag{44}$$

$$H_1^T R_{\bar{\varphi}\bar{\varphi}}^{-1}(N) = [H_{11}(N); H_{12}(N)], \quad H_{11}(N) \in \mathrm{R}^{(2n+L)\times n}, \tag{45}$$

$$R_{\bar{\varphi}v}(N) = [R_{yv}^T(N); 0]^T. \tag{46}$$

4) Calculate the bias vector and estimated parameter vector as

$$\Delta\hat{\bar{\theta}}_{BELS}(N) = R_{\bar{\varphi}\bar{\varphi}}^{-1}(N)[R_{yv}^T(N); 0]^T + R_{\bar{\varphi}\bar{\varphi}}^{-1}(N) R_{\bar{\varphi}\bar{\varphi}}(N)\bar{\eta}(N), \tag{47}$$

$$\hat{\bar{\theta}}_{BELS}(N) = \hat{\bar{\theta}}_{LS}(N) - \Delta\hat{\bar{\theta}}_{BELS}(N). \tag{48}$$

5) Compute the estimate $\hat{\theta}_{BELS}(N)$ of the original system parameter vecotr θ_0 from $\hat{\bar{\theta}}_{BELS}(N)$ (see (33) and (34)).

In the above algorithm, $R_{\bar{\varphi}\bar{\varphi}}^{-1}(N)$, $R_{\bar{\varphi}\bar{\varphi}}(N)$ and $R_{\bar{\varphi}v}(N)$ are the estimate of $R_{\bar{\varphi}\bar{\varphi}}^{-1}$, $R_{\bar{\varphi}\bar{\varphi}}$ and $R_{\bar{\varphi}v}$ respectively. They are defined similarly as in [4]. The algorithm can be carried out recursively.

Algorithm 2

1) Design a suitable, stable filter $F^{-1}(q^{-1})$ of order $2n+L$, and use it to conduct real-time

filtering of the input data, which is equivalent to inserting the filter into the identified system.

2) Set the recursive initial values for $\hat{\bar{\theta}}_{LS}(0)$ and $\bar{P}_0$ properly with $t=0$.

3) Estimate the parameters of the augmented system by using the standard recursive LS method, which gives

$$\hat{\bar{\theta}}_{LS}(t) = \hat{\bar{\theta}}_{LS}(t-1) + \bar{P}_t\bar{\varphi}_t[y(t) - \bar{\varphi}_t^T\hat{\bar{\theta}}_{LS}(t-1)], \tag{49}$$

$$\bar{P}_t = \bar{P}_{t-1} - \bar{P}_{t-1}\bar{\varphi}_t[1 + \bar{\varphi}_t^T\bar{P}_{t-1}\bar{\varphi}_t]^{-1}\bar{\varphi}_t^T\bar{P}_{t-1}, \tag{50}$$

where

$$\bar{P}_t = [\sum_{k=1}^{t}\bar{\varphi}_k\bar{\varphi}_k^T]^{-1} \tag{51}$$

4) Calculate the bias vector and estimated parameter vector as

$$\begin{bmatrix} \bar{\eta}(t) \\ R_{\bar{y}v}(t) \end{bmatrix} = \begin{bmatrix} H_2^T & 0 \\ H_1^T(t\bar{P}_t)R_{\bar{\varphi}\bar{\varphi}}(t) & H_{11}(t) \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ H_1^T\hat{\bar{\theta}}_{LS}(t) \end{bmatrix}, \tag{52}$$

$$\Delta\hat{\bar{\theta}}_{BELS}(t) = t\bar{P}_t[R_{\bar{y}v}^T(t);0]^T + t\bar{P}_tR_{\bar{\varphi}\bar{\varphi}}(t)\bar{\eta}(t), \tag{53}$$

$$\hat{\bar{\theta}}_{BELS}(t) = \hat{\bar{\theta}}_{LS}(t) - \Delta\hat{\bar{\theta}}_{BELS}(t). \tag{54}$$

5) Compute the estimate $\hat{\theta}_{BELS}(N)$ of the original system parameter vecotr θ_0 from $\hat{\theta}_{BELS}(N)$ (see (33) and (34)).

6) Repeat step 3)~5) until some convergence criterion has been reached.

4 Convergence Analysis

Before proceeding to analyse the convergence property of the BELS algorithm presented above, let us first recall a conclusion given in [2].

Proposition 1 As N approaches infinity, $\hat{\theta}_{BELS}(N)$ is a consistent estimate of the parameter vector θ_0 of the original system shown by Eq. (1) or (5) if $\hat{\bar{\theta}}_{BELS}(N)$ is a consistent estimate of the parameter vector $\bar{\theta}_0$ of the augmented system (22) or (26).

In the view of Proposition 1 it can be seen that it is necessory to prove that $\hat{\bar{\theta}}_{BELS}(N)$ converges to $\bar{\theta}_0$. For this the following theorem can be proved.

Theorem 1 When the size of sampled data tends to infinity, the estimated parameter vector $\hat{\bar{\theta}}_{BELS}(N)$ obtained from the algorithms is an asymptotically consistent estimate of $\bar{\theta}_0$, namely

$$\lim_{N\to\infty}\hat{\bar{\theta}}_{BELS}(N) = \bar{\theta}_0, \quad \text{w. p. 1} \tag{55}$$

where "w. p. 1" dentoes "with probability one"

Proof It follows from the properties of quasi-stationary sequences given in [5] that

$$\lim_{N\to\infty}\begin{bmatrix} H_2^T & 0 \\ H_1^TR_{\bar{\varphi}\bar{\varphi}}^{-1}(N)R_{\bar{\varphi}\bar{\varphi}}(N) & H_{11}(N) \end{bmatrix}^{-1} = \begin{bmatrix} H_2^T & 0 \\ H_1^TR_{\bar{\varphi}\bar{\varphi}}^{-1}R_{\bar{\varphi}\bar{\varphi}} & H_{11} \end{bmatrix}^{-1} = Q, \quad \text{w. p. 1} \tag{56}$$

Thus it can be concluded by using (41) and (44) that

$$\lim_{N\to\infty}\begin{bmatrix} \bar{\eta}(N) \\ R_{\bar{\varphi}v}(N) \end{bmatrix} = Q\begin{bmatrix} 0 \\ H_1^T\lim_{N\to\infty}\hat{\bar{\theta}}(N) \end{bmatrix} = \begin{bmatrix} \bar{\eta} \\ R_{\bar{y}v} \end{bmatrix}, \quad \text{w. p. 1} \tag{57}$$

Taking the limits on both sides of (44) and then substituting (56) and (57) into the limits gives

$$\lim_{N\to\infty} \Delta \hat{\bar{\theta}}_{\mathrm{BELS}}(N) = R_{\bar{\varphi}\bar{\varphi}}^{-1} R_{\bar{\varphi}v} + R_{\bar{\varphi}\bar{\varphi}}^{-1} R_{\bar{\varphi}\bar{\phi}} \bar{\eta}. \quad \text{w. p. 1} \tag{58}$$

Therefore

$$\lim_{N\to\infty} \hat{\bar{\theta}}_{\mathrm{BELS}}(N) = \bar{\theta}_0, \quad \text{w. p. 1} \tag{59}$$

Thus the theorem is proved.

Table 1 Simulation results of the example 1

$N=200$	a_1	a_2	b_1	b_2	η_1^0	η_2^0
true	−1.5	0.7	1.0	0.5	2.0	−1.0
estimate	−1.485	0.686	0.993	0.488	1.987	−0.983

5 Simulation Example

To verify the preceding conclusions, a simulation example is presented.

Example Consider a system represented by Eq. (1)～(4), where the nominal part is

$$G(q^{-1},\theta^0) = \frac{1.0q^{-1} + 0.5q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}}, \tag{60}$$

and the unmodeled dynamics is described by

$$G_\Delta(q^{-1}) = 2.0q^{-1} - 1.0q^{-2}. \tag{61}$$

The correlated disturbance v_k is simulated by

$$v_k = e_k - 1.0e_{k-1} + 0.2e_{k-2}. \tag{62}$$

u_k is taken as a pseudorandom binary signal (PRBS) of an unit magnitude and e_k is white noise with zero mean.

In this example the filter is designed as

$$F(q^{-1}) = 1.0 - 1.4q^{-1} + 0.48q^{-2}. \tag{63}$$

The simulation results are listed in Table 1.

6 Conclusion

This paper deals with the problem of unbiased identification of transfer function with the unmodeled dynamics described by FIR, and thus extends our results [2～4]. It is shown that the idea of BELS method can also be exploited to get satisfactory identification results even when there is not much more priori information on the noise and the unmodeled dynamics. This kind of identification method is expected to be a feasible and effective mean to improve the identification accuracy.

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存在噪声和未建模动态系统传递函数的渐近无偏估计

张 颖 冯纯伯

(东南大学自动化所·南京，210018)

摘要：存在未建模动态时，如何去除系统传递函数估计中噪声引起的偏差是系统辨识关心的重要问题. 本文将我们已提出的BELS方法进行推广得出：当系统未建模动态可用一FIR模型描述时，应用本文的方法可得到系统传递函数的无偏辨识，而不需知道有关噪声和未建模动态概率分布函数的先验知识.

关键词：系统辨识；无偏估计；最小二乘法；未建模动态

本文作者简介

张 颖 1967年生. 分别于1989年和1992年在东南大学获学士和硕士学位. 目前在东南大学自动化研究所攻读博士学位. 主要研究方向是系统辨识，信号处理等.

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