

## Unbiased Identification of Systems with Nonparametric Uncertainty\*

ZHANG Ying and FENG Chunbo

(Research Institute of Automation, Southeast University • Nanjing, 210018, PRC)

**Abstract:** The set-membership identification of systems with parametric and nonparametric uncertainty is studied in this paper. The bias induced by the additive noise is eliminated by using the Bias-Eliminated Least Squares Method proposed in [5]. A prefilter is connected to the input terminal of the system, so that some zeros are inserted to the system. By using the information obtained from these known zeros the bias induced by the additive noise is eliminated.

**Key words:** system identification; least-squares method; set-membership identification; uncertainty

### 1 Introduction

In general, it is difficult to model the disturbance in a system exactly. A realistic approach is to assume that the disturbance is unknown but bounded (UBB). Based on this assumption, the method of set-membership identification is proposed in [1] and further developed to deal with identification for robust control in [2] and [3]. Taking the unmodeled dynamics to be UBB, these methods can get a model set whose elements are consistent with the observed data. But in these methods, the measurement noise and the unmodeled dynamics are both considered to be UBB disturbances. As a result, only a conservative estimate of the model set can be reached. The recent results of set-membership identification can hardly be said to be satisfactory.

It is noticeable that the measurement noise, in contrast with the unmodeled dynamics, is a kind of disturbance with strong statistic property. In principle its effect can be eliminated or at least reduced. In view of this, this paper is to study the problem of how to extract and eliminate the bias induced by measurement noise in the estimate. In [4] and [5] a bias-eliminated least squares method (BELS) was proposed and has been shown to be effective in dealing with the noise with the UBB property. This paper will extend the idea of [4] and [5] to the elimination of the noise-induced bias in the setmembership estimate. Here the measurement noise and unmodeled dynamics are treated separately. Then the effect of noise is eliminated so that a less conservative model set can be obtained. To this end, a designed digital filter is inserted artificially into the identified system at the input terminal so that the resulting augmented system has some known ze-

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ros which can, based on asymptotic analysis, be used for eliminating the bias caused by a colored noise in the model set estimate. It may be seen that the idea of this paper is of potential value because it provides us with a new approach to draw much more useful information from the sample data so as to get better results in identification.

## 2 Problem Statement

Assume that the observations of the input-output data sequence  $Z^N = [\mu_k, y_k]$  is generated by a system which can be modeled as follows:

$$y_k = \frac{B(q^{-1})}{A(q^{-1})}(1 + \Delta^{-1}(q^{-1})W(q^{-1}))u_k + v_k, \quad (1)$$

where

$$A(q^{-1}) = 1 + a_1 q^{-1} + \dots + a_n q^{-n}, \quad (2)$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \dots + b_m q^{-m} \quad (3)$$

and  $q^{-1}$  is the unit-delay operator.  $u_k, y_k$  and  $v_k$  denote the system input, output and the UBB disturbance at the output terminal respectively. It is also assumed that both  $u_k$  and  $y_k$  have finite power, that is,  $r_{yy} < \infty$  and  $r_{uu} < \infty$ .

The system  $W(q^{-1})\Delta(q^{-1})$  denotes the multiplicative nonparametric uncertainty, which is characterized by an uncertain but unity-bounded stable transfer function  $\Delta(q^{-1})$ , i. e.  $\|\Delta\|_\infty \leq 1$ , and by a known stable transfer function  $W(q^{-1})$ .

As will be shown, the result of identification does not depend on the model of noise  $v_k$ . Therefore the system (1) can be formulated as in [6]:

$$A(q^{-1})y_k = B(q^{-1})(1 + \Delta(q^{-1})W(q^{-1}))u_k + v_k, \quad (4)$$

where  $v_k = A(q^{-1})v'_k$  is also unknown but bounded, and uncorrelated with the input.

Introduce the parameter vector  $\theta^T = [a_1, a_2, \dots, a_n, b_0, b_1, \dots, b_m]$ , the model set of the system (4) can be described by

$$\Theta^* = \{\theta: \left\| \frac{Ay - Bu - v}{BW} \right\|_\infty \leq 1\}. \quad (5)$$

This equation gives the constraints between the nominal model and the unmodeled dynamics. Geometrically, if the center of the  $\Theta^*$  stands for the nominal model, then the bound of unmodeled dynamics can be specified by the radius of  $\Theta^*$ . Moreover, if there is no noise, the definition of the limit parameter set given by (5) is consistent with that in [2] and [3]. But, in the presence of noise, it gives a much more accurate estimate than that in [2] and [3]. That is the reason why Eq. (5) is adopted here.

The estimation of the set  $\Theta^*$  in the case with  $v_k = 0$  has been thoroughly studied in [2] and [3]. In this paper the estimation of the set  $\Theta^*$  in the presence of a colored noise will be discussed.

The sample average of a quasi-stationary sequence  $\{x(t)\}$  is defined as

$$\varepsilon_k(x(t)) = \frac{1}{k} \sum_{t=1}^k x(t). \quad (6)$$

The truncated  $l_2$  norm is taken as

$$\|x\|_k = \left( \sum_{t=1}^k x^2(t) \right)^{\frac{1}{2}}. \quad (7)$$

### 3 Estimating the Model Set $\Theta^*$ in the Presence of Noise

The following theorem gives the result of estimating the  $\Theta^*$  based on the observed data sequence  $Z^N$ .

**Theorem 1** For system (4), we have

$$\text{i) } \Theta_N \rightarrow \Theta_\infty \text{ w. p. 1 as } N \rightarrow \infty. \quad (8)$$

$$\text{ii) } \Theta^* \subset \Theta_\infty. \quad (9)$$

where

$$\Theta_N = \{\theta: \|A(q^{-1})y - B(q^{-1})u - v\|_N \leq \|W(q^{-1})B(q^{-1})u\|_N\}, \quad (10)$$

$$\Theta_\infty = \lim_{N \rightarrow \infty} \Theta_N. \quad (11)$$

The proof of this theorem is similar to the proof given in [2] and is omitted here.

This theorem proposes an approach to estimate  $\Theta^*$  in the presence of noise. It can be pointed out that the estimate of  $\Theta^*$  by this theorem is less conservative than that in [2] and [3]. In fact, in [2] and [3] the estimate of  $\Theta^*$  is  $\Theta_N = \{\theta: \|Ay - Bu\|_N \leq \|WBu\|_N + \|v\|_N\}$ , it is obvious that  $\Theta_N \subset \Theta_N$ .

Define a vector  $\Phi$  whose elements are the following sequences:

$$\Phi = [\Phi_y^T, \Phi_u^T]^T, \quad (12)$$

$$\Phi_y = [y(k-1), y(k-2), \dots, y(k-n)]^T, \quad (13)$$

$$\Phi_u = [u(k), u(k-1), u(k-2), \dots, u(k-m)]^T. \quad (14)$$

The results given in the following theorem present a convenient form for computing  $\Theta_N$ .

**Theorem 2** For system (4), we have,

i)  $\Theta_N$  can be expressed in the quadratic form

$$\Theta_N = \{\theta: \theta^T \Gamma_N \theta - 2\beta_N^T \theta + \alpha_N \leq 0\}, \quad (15)$$

where

$$\alpha_N = \varepsilon_N(y^2(t)) - \varepsilon_N(v^2) \in \mathbb{R}, \quad (16)$$

$$\beta_N = \varepsilon_N(\Phi y) - \varepsilon_N(\Phi v) \in \mathbb{R}^{n+m}, \quad (17)$$

$$\Gamma_N = \varepsilon_N(\Phi \Phi^T) = \begin{bmatrix} 0 & 0 \\ 0 & \varepsilon_N((W\Phi_u)(W\Phi_u)^T) \end{bmatrix}. \quad (18)$$

ii) Provided  $\Gamma_N^{-1}$  exists, then

$$\Theta_N = \{\theta: (\theta - \hat{\theta}_N)^T \Gamma_N (\theta - \hat{\theta}_N) \leq V_N\}, \quad (19)$$

where

$$\hat{\theta}_N = \Gamma_N^{-1} \beta_N, \quad (20)$$

$$V_N = \beta_N^T I_N^{-1} \beta_N - \alpha_N. \quad (21)$$

**Proof** The system (4) can be rearranged as follows:

$$A(q^{-1})y_k - B(q^{-1})u_k - v_k = \Delta(q^{-1})B(q^{-1})W(q^{-1})u_k. \quad (22)$$

Taking autocorrelation of the both sides at  $\tau=0$ , we get

$$\varepsilon_N((Ay - Bu - v)^2) = \varepsilon_N((\Delta BWu)^2). \quad (23)$$

Rewriting (22) in autoregressive form gives

$$\varepsilon_N((y - \Phi^T \theta - v)^2) = \varepsilon_N((\Delta \theta^T \begin{bmatrix} 0 \\ W\Phi_n \end{bmatrix})^2). \quad (24)$$

Now take the supremum of the right-hand side to obtain the set  $\Theta_N$ ,

$$\begin{aligned} \Theta_N &= \{\theta : \varepsilon_N((y - \Phi^T \theta - v)^2) \leq \sup_{\|\Delta\|_\infty \leq 1} (\varepsilon_N(\Delta \theta^T \begin{bmatrix} 0 \\ W\Phi_n \end{bmatrix})^2)\} \\ &= \{\theta : \varepsilon_N((y - \Phi^T \theta - v)^2) \leq \varepsilon_N((\theta^T \begin{bmatrix} 0 \\ W\Phi_n \end{bmatrix})^2)\}. \end{aligned} \quad (25)$$

By the definition (6), the quadratic form of  $\Theta_N$  follows immediately. This proves part i) of the theorem. When  $I_N^{-1}$  exists, part ii) is the alternative form of part i).

**Remark 1** The formula (20) in Theorem 2 indicates that the nominal model can be estimated by the traditional least-squares method. For this nominal model Eq. (21) specifies the bound of the unmodeled dynamics and the noise.

**Remark 2** Eqs. (16), (17), (20) and (21) reveal that the quantities  $\varepsilon_N(\Phi v)$  and  $\varepsilon_N(v^2)$  are the noise-induced biases in the estimates of the nominal model parameters and the bound induced by the unmodeled dynamics. They are unknown and need to be determined.

**Remark 3** If there is no unmodeled dynamics, we can take  $W(q^{-1})=0$ . In this case Theorem 2 gives the result of traditional least-squares identification. According to the condition of persistent excitation from Eq. (18) we can see that  $I_N > 0$ . In the case with  $W(q^{-1}) \neq 1$ , Eq. (18) also shows that  $I_N > 0$  if the system is persistently excited. It is evident that when  $I_N > 0$ ,  $\Theta_N$  forms an ellipsoid in  $R^{n+m+1}$ .

#### 4 Bias-Eliminated Set-Membership Identification

In above sections we have proposed a set-membership identification method in the form of Theorem 2. But  $\varepsilon_N(v^2)$  and  $\varepsilon_N(\Phi v)$  remain to be determined. In this section we will discuss the problem of calculating these unknowns. It is known from [4] that the bias induced by noise can be calculated if some zeros of the identified system are known. A prefilter is designed and inserted to the input terminal so as to add some known zeros to the system. The bias caused by the measurement noise will be determined in the same way as in [4].

A digital filter  $F^{-1}(q^{-1})$  is connected to the system at the input terminal, where  $F(q^{-1})$  is defined as

$$F(q^{-1}) = 1 + f_1 q^{-1} + \dots + f_n q^{-n}. \quad (26)$$

The digital filter  $F^{-1}(q^{-1})$  is designed to be stable, namely, the polynomial  $F^*(q) = q^n F(q^{-1})$  has all its zeros located strictly inside the unit disc.

The augmented system thus obtained is expressed by the model

$$A(q^{-1})y_k = \bar{B}(q^{-1})(1 + \Delta(q^{-1})W(q^{-1}))\bar{u}_k + v_k, \quad (27)$$

where

$$\begin{aligned} \bar{B}(q^{-1}) &= F(q^{-1})B(q^{-1}) \\ &= \bar{b}_0 + \bar{b}_1 q^{-1} + \bar{b}_2 q^{-2} + \dots + \bar{b}_{n+m} q^{-(n+m)}, \end{aligned} \quad (28)$$

$$\bar{u}_k = \frac{1}{F(q^{-1})} u_k. \quad (29)$$

Performing the same procedures as in the proof of Theorem 2, we can get the set-membership identification results of the augmented system (27) as follows:

$$\bar{\Theta}_N = \{\bar{\theta}; (\bar{\theta} - \bar{\Theta}_N)^T \bar{I}_N (\bar{\theta} - \bar{\Theta}_N) \leq \bar{V}_N\}, \quad (30)$$

where

$$\begin{aligned} \bar{\theta} &= [a_1, a_2, \dots, a_n; \bar{b}_0, \bar{b}_1, \bar{b}_2, \dots, \bar{b}_{n+m}]^T, \\ \bar{\Phi} &= [-y(k-1), \dots, -y(k-n); \bar{u}(k), \dots, \bar{u}(k-m-n)]^T \\ &= [\bar{\Phi}_y^T; \bar{\Phi}_u^T]^T, \end{aligned} \quad (32)$$

$$\bar{I}_N = \varepsilon_N(\bar{\Phi}\bar{\Phi}^T) = \begin{bmatrix} 0 & 0 \\ 0 & \varepsilon_N((W\bar{\Phi}_u)(W\bar{\Phi}_u)^T) \end{bmatrix}, \quad (33)$$

$$\bar{\beta}_N = \varepsilon_N(\bar{\Phi}y) - \varepsilon_N(\bar{\Phi}v) = \varepsilon_N(\bar{\Phi}y) - [R_{yv}^T | 0]^T, \quad (34)$$

$$\bar{\alpha}_N = \varepsilon_N(y^2) - \varepsilon_N(v^2), \quad (35)$$

$$\bar{V}_N = \beta_N^T \bar{I}_N^{-1} \bar{\beta}_N - \bar{\alpha}_N, \quad (36)$$

$$\bar{\Theta}_N = \bar{I}_N^{-1} \bar{\beta}_N = \bar{I}_N^{-1} \varepsilon_N(\bar{\Phi}y) - \bar{I}_N^{-1} [R_{yv}^T | 0]^T, \quad (37)$$

$$R_{yv}^T = [r_{yv}(1), r_{yv}(2), \dots, r_{yv}(n)]. \quad (38)$$

Among Eqs. (34), (35) and (37), there are  $n+1$  unknowns to be estimated, including  $r_{yv}(i)$  ( $i=1, 2, \dots, n$ ) and  $\varepsilon(v^2) = r_{yv}(0)$ . The following subsections give the method to determine  $r_{yv}(i)$  ( $i=1, 2, \dots, n$ ) and  $\varepsilon(v^2) = r_{yv}(0)$ , respectively.

#### 4.1 Estimation of $R_{yv}$

Let  $\lambda_i$  ( $i=1, 2, \dots, n$ ) be the zeros of  $F^*(q)$ . This implies that the following equations hold

$$\begin{aligned} \bar{B}^*(\lambda_i) &= F^*(\lambda_i)B^*(\lambda_i) \\ &= \bar{b}_0 \lambda_i^{n+m} + \bar{b}_1 \lambda_i^{n+m-1} + \dots + \bar{b}_{n+m} = 0, \quad i=1, 2, \dots, n, \end{aligned} \quad (39)$$

where

$$B^*(q) = q^m B(q^{-1}).$$

It is obvious that

$$H^T \bar{\theta} = 0, \quad \forall \bar{\theta} \in \bar{\Theta}_N, \quad (40)$$

where

$$H^T = \begin{bmatrix} 0 & \dots & 0 & \lambda_1^{n+m} & \dots & \lambda_1 & 1 \\ \vdots & \dots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & \dots & 0 & \lambda_n^{n+m} & \dots & \lambda_n & 1 \end{bmatrix} \in \mathbb{R}^{n \times (2n+m+1)}. \quad (41)$$

Simple mathematical manipulations on Eq. (37) give the following set of  $n$  linear algebraic equations for calculating  $R_{yv}$ ,

$$GR_{yv} = H^T \bar{\theta}_N, \quad (42)$$

where

$$G = H^T \bar{T}_N^{-1} Q \in R^{n \times n}, \quad Q^T = [I_n | 0] \in R^{n \times (2n+m+1)}, \quad (43)$$

and  $I_n$  is an identity matrix of order  $n$ . Hence the estimate of  $R_{yv}$  is given by

$$R_{yv} = (H^T \bar{T}_N^{-1} Q)^{-1} H^T \bar{T}_N^{-1} \varepsilon_N(\bar{\Phi} y). \quad (44)$$

#### 4.2 Estimation of $\varepsilon_N(v^2)$

To estimate  $\varepsilon_N(v^2)$ , i.e.  $r_{yv}(0)$ , let us rewrite the augmented system (27) as

$$a_n y(t-n) - \bar{\Phi}^T \bar{\theta} = \bar{\theta}^T \begin{bmatrix} 0 \\ \Delta W \bar{\Phi}_u \end{bmatrix}, \quad (45)$$

where

$$\bar{\Phi}^T = [-y(t), -y(t-1), \dots, -y(t-n+1), u(k), \dots, u(k-m-n)], \quad (46)$$

$$\bar{\theta}^T = [1, a_1, a_2, \dots, a_{n-1}, \bar{b}_0, \dots, \bar{b}_{n+m}]. \quad (47)$$

Like Theorem 2 the set-membership identification results of the system (45) will be

$$\tilde{\theta}_N = \{\bar{\theta}; (\bar{\theta} - \tilde{\theta}_N)^T \tilde{T}(\bar{\theta} - \tilde{\theta}_N) \leq \tilde{V}_N\}, \quad (48)$$

$$\tilde{\beta}_N = \varepsilon_N(\bar{\Phi} y) - \varepsilon_N(\bar{\Phi} v) = \varepsilon_N(\bar{\Phi} y) - [R_{yv}^T; 0]^T, \quad (49)$$

$$\tilde{\alpha}_N = \varepsilon_N(y^2) - \varepsilon_N(v^2), \quad (50)$$

$$\tilde{V}_N = \bar{\beta}_N^T \bar{T}_N^{-1} \bar{\beta}_N - \tilde{\alpha}_N, \quad (51)$$

$$\tilde{\theta}_N = \tilde{T}_N^{-1} \tilde{\beta}_N = \bar{T}_N^{-1} \varepsilon_N(\bar{\Phi} y) - \tilde{T}_N^{-1} [R_{yv}^T; 0]^T, \quad (52)$$

$$\tilde{T}_N = \varepsilon_N(\bar{\Phi} \bar{\Phi}^T) - \begin{bmatrix} 0 & 0 \\ 0 & \varepsilon_N((W \bar{\Phi}_u)(W \bar{\Phi}_u)^T) \end{bmatrix}, \quad (53)$$

$$\bar{R}_{yv}^T = [r_{yv}(0), r_{yv}(1), \dots, r_{yv}(n-1)]. \quad (54)$$

In the new parameter space described by (47), it is also valid that

$$H^T \bar{\theta} = 0, \quad \forall \bar{\theta} \in \tilde{\theta}_N. \quad (55)$$

Therefore, like the result given by Eq. (44), the value of  $R_{yv}$  is obtained as

$$\bar{R}_{yv} = (H^T \tilde{T}_N^{-1} Q)^{-1} H^T \tilde{T}_N^{-1} \varepsilon_N(\bar{\Phi} y(t-n)) a_n. \quad (56)$$

$a_n$  in (56) is also unknown. However, Eq. (44) gives the value of  $r_{yv}(i)$ ,  $i=1, 2, \dots, n$

1. Substituting one of  $r_{yv}(i)$  into (56) the values of  $a_n$  and  $r_{yv}(0)$  can be obtained.

So far,  $R_{yv}$  and  $\varepsilon_N(v^2)$  are determined by using the known zeros of  $F^*(q)$ . With these values Theorem 2 gives a complete set-membership identification method for the system (1).

## 5 Simulation

Suppose that the true transfer function is

$$G(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})} (1 + \Delta(q^{-1})W(q^{-1}))$$

$$= \text{ZOH} \left\{ \left[ \frac{10}{s+1} \right] \left[ \frac{100}{s^2 + 0.1s + 100} \right] \right\}. \quad (57)$$

The sampling frequency is chosen to be  $20\pi$  rad/s or 10 Hz. The nominal part is selected as

$$A(q^{-1}) = 1 + aq^{-1} \quad \text{and} \quad B(q^{-1}) = bq^{-1}. \quad (58)$$

The weights  $W(q^{-1})$  is taken as

$$W(q^{-1}) = 65 \left\{ \text{ZOH} \left[ \frac{s+1}{s+5} \right]^4 \right\}. \quad (59)$$

and the noise sequence  $v_k$  is generated by

$$v_k = e_k - 1.0e_{k-1} + 0.2e_{k-2}, \quad (60)$$

where  $e_k$  is a white noise with zero mean and the variance 1.0.

The above method is applied to the simulated data with a sample size of 200, 500 and 1000, respectively. The results are depicted in Fig. 1, in which the estimates and the true model set  $\Theta^*$  are given. It can be seen that the proposed method can obtain a satisfactory estimate of the model set.

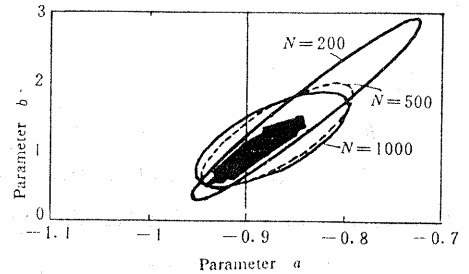


Fig 1  $\Theta_*$  for different  $N$  ( $\delta=1.0$ )

## 6 Conclusion

The problem of the set-membership identification for a linear system is studied in this paper. We show that the method proposed in [4] and [5] can be used to eliminate the biases and inaccuracies in the estimated parameter set induced by the additive noise. In [4] and [5] a bias-eliminating method for the identification of a linear system with an accurately known structure was presented. Here, in this paper, we have shown that this method can also be applied to eliminate the biases caused by noise, even if there is unmodeled dynamics. Since the biases caused by noise can be eliminated, then the problem of set-membership identification turns to be a problem of a model fitting, the same as for a deterministic linear system. If we increase the degree of the nominal part of the system and at the same time weaken the unmodeled dynamics, then the estimated bound of the parameters set will be reduced, and a more accurate estimation of the system can be reached. It is worthy to be noticed that the method presented here needs no priori knowledge of the noise model.

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## 具有非参数化不确定系统的无偏辨识

张 颖 冯纯伯

(东南大学自动化研究所·南京, 210018)

**摘要:** 本文研究了具有参数和非参数不确定性系统的集员辨识问题: 分析表明利用我们在文[5]中提出的 BELS 方法可以消除集员辨识中观测噪声引起的偏差. 文中通过对系统输入数据的预滤波将已知零点嵌入系统, 利用这些零点提供的信息将观测噪声引起的辨识偏差予以消除.

**关键词:** 系统辨识; 集员辨识; 最小二乘法; 不确定性

### 本文作者简介

张 颖 1967 年生. 分别于 1989 年和 1992 年在东南大学获学士和硕士学位. 目前在东南大学自动化研究所攻读博士学位. 主要研究方向是系统辨识、信号处理等.

冯纯伯 1928 年生. 1950 年毕业于浙江大学电机系, 1953 年毕业于哈尔滨工业大学研究生班, 1958 年获苏联技术科学副博士学位. 现任东南大学研究生院副院长, 教授, 中国自动化学会常务理事, 俄罗斯联邦自然科学院外籍院士. 目前主要从事系统建模、自适应、鲁棒及智能化控制理论及应用、机器人控制等方面的研究.