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Elimination of Impulsive Modes by Output Feedback in Descriptor Systems

WANG Dianhui
(Research Center of Automation, Northeastern University · Shenyang, 110006, PRC)
XIE Xukai
(Department of Mathematics, Northeastern University · Shenyang, 110006, PRC)

Abstract: In this paper, a problem of eliminating the impulsive modes by output feedback in descriptor systems is considered. First, we redefine the impulsive controllable subspace R_{IC}, unobservable subspace $R_{I\bar{O}}$ of the descriptor systems from a new point of view. Secondly, on the basis of these subspaces defined above, an interesting and meaningful mathematical identity is established as follows:

$$\dim(R_{IC})-\dim(R_{IC}\cap R_{I\bar{O}})+r=\max_{K\in R[m\times l]}\deg\det(sE-A+BKC). \qquad (*)$$

As a corollary of (*), a significant conclusion, i.e., there exist p linearly independent impulsive modes which can be eliminated by an output feedback law if and only if the system possesses p linearly independent impulsive modes which are controllable and observable. It is also shown that the equality (*) holds for almost any gain K.

Key words: desriptor systems; impulsive modes; output feedback

1 Introduction

In recent years, there has been considerable interest in descriptor systems of the form

$$E\dot{x}(t)=Ax(t)+Bu(t), \qquad (1a)$$

$$y(t)=Cx(t), \qquad (1b)$$

where E, A are $n\times n$ real constant matrices with matrix E singular, B and C are $n\times m$ and $l\times n$, respectively. Because of the extensive applications of such systems in areas which include large-scale systems, singularly perturbed systems, circuit theory and economic models[1~3], its theory has been developed rapidly.

For the structure properties and the impulse behavior of the system (1), many authors have studied them from various viewpoints[4~14] and got some valuable and significant results. It is worth noticing that these previous work mainly focused on the complete infinite frequency behavior of the system, for instance, the impulse controllability, observability and elimination problems of all impulsive modes by various feedback applied to the system[4~8]. It has been shown[11,12] that all impulsive modes of the system can be eliminated by almost any constant output feedback when the system is both impulse controllable and observable. However, the problems such as which impulsive modes is impulse controllable and/or observable, calculating the

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numbers of the impulsive modes which can be eliminated by various feedback laws have not been discussed.

Our goal in this paper is to investigate which impulsive modes is controllable and/or unobservable and how many impulsive modes can be eliminated by a constant output feedback applied to the system.

It is assumed here that the polynomial det $(sE-A)$ is not identically zero which guarantee uniqueness for the solutions of (1).

2 Preliminaries

Let $\delta(E,A)=\{s:\det(sE-A)=0\}$, $r=\deg\det(sE-A)$, $E_\lambda=(\lambda E-A)^{-1}E$ and $B_\lambda=(\lambda E-A)^{-1}B$ with $\lambda\overline{\in}\delta(E,A)$, $R[m\times l]$ denotes the set of all real $m\times l$ matrix.

Let R_1, R_2 denote eigensubspaces associated with non-zero eigenvalues and zero eigenvalues of the mapping E_λ, respectively; J_1, J_2 denote the restrictions of the mapping E_λ to the subspace R_1 and R_2, respectively; $P(Q)$ denotes a natural projection along subspace $R_2(R_1)$ on subspace $R_1(R_2)$; $G_1=PB_\lambda, G_2=QB_\lambda$; C_1, C_2 denote the restrictions of the mapping C to the subspace R_1 and R_2, respectively. For a real number $\lambda\overline{\in}\delta(E,A)$, with the left multiplication of $(\lambda E-A)^{-1}$ to both sides of (1a) and a siutable coordinate transformation, say $x=T(x_1^t, x_2^t)^t$, the "T" denotes a linear transformation and the "t" denotes transpose of a matrix, thus the system (1) can be decomposed into the following two subsystems:

$$\dot{x}_1(t)=A_1x_1(t)+B_1u(t), \tag{2a}$$

$$y_1(t)=C_1x_1(t), \tag{2b}$$

and

$$A_2\dot{x}_2(t)=x_2(t)+B_2u(t), \tag{3a}$$

$$y_2(t)=C_2x_2(t), \tag{3b}$$

where $x_1\in \mathrm{R}_1, x_2\in \mathrm{R}_2, y_1+y_2=y$, $A_1=\lambda I_1-J_1^{-1}$, $A_2=(\lambda J_2-I_2)^{-1}J_2, B_1=J_1^{-1}G_1, B_2=(\lambda J_2-I_2)^{-1}G_2, J_2$ is an nilpotent matrix.

Definition 1 An initial state $x_2(0_-)$ is said to be impulsive controllable if there exists a control input $u(t)\in C_p^q$(here $q=\mathrm{Ind}\ (A_2)$) such that

$$\mathrm{Imp}\ (x_2(t))=0, \tag{4}$$

where Imp (*) denotes the impulse part of (*). If R_{2IC} denotes the subspace in R_2 which consists of all the initial states $x_2(0_-)$ that are impulse controllable, then we call R_{2IC} the impulse controllable subspace of the system (3).

Definition 2 An initial state $x_2(0_-)$ is said to be impulse unobservable if the observation output $y_2(t)=0$ with $u(t)=0$. If $R_{2I\bar{O}}$ denotes the subspace in R_2 which consists of all the initial states $x_2(0_-)$ that are impulse unobservable, then we call $R_{2I\bar{O}}$ the impulse unobservable subspace and R_{2IO} the impulse observable subspace of the system (3), where R_{2IO} denotes the orthogonal complement of the subspace $R_{2I\bar{O}}$ in R_2.

Similarly, definitions on the impulse controllable subspace, the impulse unobservable subspace and the observable subspace for the system (1) can be made out in a same way, and they are denoted by R_{IC}, $B_{I\bar{O}}$ and R_{IO}, respectively.

Lemma 1 $$R_{2IC}=\sum_{k=0}^{q-1}\mathrm{Im}(A_2^k B_2)+\mathrm{Ker}(A_2),$$

Lemma 2 $$R_{IC}=\sum_{k=0}^{n-1}\mathrm{Im}(E_\lambda^k B_\lambda)+\mathrm{Ker}(E_\lambda)+\mathrm{Im}(E_\lambda^D E_\lambda),$$

Lemma 3 $$R_{2I\bar{O}}=\bigcap_{k=1}^{q-1}\mathrm{Ker}(C_2 A_2^k),$$

Lemma 4 $$R_{I\bar{O}}=\bigcap_{k=1}^{n-1}\mathrm{Ker}(CE_\lambda^k)+\mathrm{Im}(E_\lambda^D E_\lambda)$$

where E_λ^D stands for the Drazin inverse of the matrix E_λ.

The proofs of the lemmas mentioned above can be found in [13]and[14].

3 Main Results

In this section, we are interested in the effects of applying the linear feedback law

$$u(t)=Ky(t)+w(t) \tag{5}$$

to the system (1) where K is an $m\times l$ real constant matrix.

Although some conditions on eliminating all impulsive modes by feedback (5) have been established in [11] and [12], it is still a meaningful task to answer how many linearly independent impulsive modes of the system can be eliminated by the feedback.

Theorem 1[14] Let N_{CO} be the numbers of controllable and observable impulsive modes which are linearly independent. Then we have

$$N_{CO}=\dim(R_{IC})-\dim(R_{IC}\cap R_{I\bar{O}}). \tag{6}$$

Theorem 2 $$\dim(R_{IC})-\dim(R_{IC}\cap R_{I\bar{O}})+r=\max_{K\in\mathrm{R}[m\times l]}\deg\det(sE-A+BKC). \tag{7}$$

Proof It has been shown in [4] that a system matrix of the form

$$P(s)=\begin{bmatrix} sE-A & -B \\ C & D\end{bmatrix}, \tag{8}$$

can always be brought by allowed transformations to the form

$$\left[\begin{array}{cccc|c} sE_{\bar{O}C}-A_{\bar{O}C} & * & * & * & -B_{\bar{O}C} \\ 0 & sE_{\bar{O}\bar{C}}-A_{\bar{O}\bar{C}} & 0 & 0 & 0 \\ 0 & 0 & sE_{OC}-A_{OC} & * & -B_{OC} \\ 0 & 0 & 0 & sE_{O\bar{C}}-A_{O\bar{C}} & 0 \\ \hline 0 & 0 & C_{OC} & C_{O\bar{C}} & D \end{array}\right], \tag{9}$$

where the $*$ denotes constant matrices, and the subscripts $O,\bar{O},C$ and $\bar{C}$ denote strongly observable, unobservable, strongly controllable, uncontrollable subsystems, respectively.

Notice that adding or eliminating nondynamic variables does not affect the dynamical order assignment of the system (1). Without loss of generality, we can assume that the system (1) has no nondynamic variables. Then, it is not difficult to see that there are two nonsingular matrices M and N such that

$$M(sE-A+BKC)N=\begin{bmatrix} sE_{\bar{o}c}-A_{\bar{o}c} & * & * & * \\ 0 & sE_{\bar{o}\bar{c}}-A_{\bar{o}\bar{c}} & 0 & * \\ 0 & 0 & sE_{oc}-A_{oc}+B_{oc}KC_{oc} & * \\ 0 & 0 & 0 & sE_{o\bar{c}}-A_{o\bar{c}} \end{bmatrix}. \tag{10}$$

Thus we have

$$\max_{K\in R[m\times l]} \deg\det(sE-A+BKC)=\max_{K\in R[m\times l]} \deg\det(sE_{oc}-A_{oc}+B_{oc}KC_{oc}) + \deg\det\begin{bmatrix} sE_{\bar{o}c}-A_{\bar{o}c} & * & * \\ 0 & sE_{\bar{o}\bar{c}}-A_{\bar{o}\bar{c}} & * \\ 0 & 0 & sE_{o\bar{c}}-A_{o\bar{c}} \end{bmatrix}. \tag{11}$$

From Theorem 1 above, it is easy to show that

$$\max_{K\in R[m\times l]} \deg\det(sE_{oc}-A_{oc}+B_{oc}KC_{oc}) =\dim(R_{2IC})-\dim(R_{2IC}\cap R_{2I\bar{O}})+\deg\det(sE_{oc}-A_{oc}), \tag{12}$$

also,

$$\dim(R_{IC})=\dim(R_{2IC})+r, \tag{13}$$

$$\dim(R_{IC}\cap R_{I\bar{O}})=\dim(R_{2IC}\cap R_{2I\bar{O}})+r. \tag{14}$$

Finally, from (10)~(14), Theorem 2 follows immediately. The proof is over.

Corollary 1 The system (1) has p linearly independent impulsive modes which can be eliminated by a feedback (5) if and only if there exists p linearly independent controllable and observable impulsive modes in the system.

Remark 1 The conclusion in Corollary1 contains a well-known result, i.e., all impulsive modes of the system can be eliminated by the feedback (5) if and only if the system is impulse controllable and impulse observable[11,12].

Corollary 2 Let $C=I$ and $B=I$ in Theorem 2, respectively. Then we can easily get the following formulas

$$\dim(R_{IC})=\max_{K\in R[m\times l]} \deg\det(sE-A+BK)+\dim(\mathrm{Ker}E), \tag{15}$$

$$\dim(R_{I\bar{O}})=n+r-\max_{K\in R[m\times l]} \deg\det(sE-A+KC). \tag{16}$$

Remark 2 The equality (15) implies that the system (1) possesses p linearly independent impulsive modes which can be eliminated by a state feedback $u(t)=Kx(t)+w(t)$ if and only if the system exists p linearly independent impulsive modes which are controllable. If $R_{IC}=R^n$, equation (15) is just a condition on impulse controllability of the system (1), which has been first obtained by Cobb in [7]. Also, from Corollary 2, we can obtain that

$$N_O=\dim(R_{IC})-\dim(\mathrm{Ker}E)-r, \tag{17}$$

$$N_O=n-\dim(R_{I\bar{O}}), \tag{18}$$

where N_c, N_o denote the numbers of the linearly independent controllable impulsive modes and observable impulsive modes of the system (1), respectively.

From Theorem 2 above and the results obtained in [11] and [12], it is very easy to get the following result.

Theorem 3 Let $N_P=\dim(R_{IC})-\dim(R_{IC}\cap R_{I\bar{o}})+r$. Then the set $F=\{K:\deg\det(sE-A+BKC)<N_P\}$ is either empty or a hypersurface in $R[m\times l]$.

Corollary 3 The closed-loop system generates rank $(E)-\dim(R_{IC})+\dim(R_{IC}\cap R_{I\bar{o}})-r$ impulsive modes for almost any $K\in R[m\times l]$.

By the way, it should be pointed out that the equalities (15) and (16) hold for almost any $K\in R[m\times l]$.

4 Conclusion

This paper redefines an impulse controllable subspace and an unobservable subspace of the descriptor system. On the basis of these subspaces, some important and meaningful mathematical identities are established. As a direct corollary of one of the identities, a significant conclusion given in [7],[8], [11] and [12] are generalized.

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广义系统在输出反馈下的脉冲行为

王殿辉 谢绪恺

(东北大学自动化研究中心·沈阳,110006) (东北大学数学系·沈阳,110006)

摘要:基于作者在文[14]中给出的脉冲观控性子空间,建立了如下数学关系式:

$$\max_{K\in R[m\times l]} \deg|sE - A + BKC| = \dim(R_{IC}) - \dim(R_{IC} \cap R_{I\bar{O}}) + r$$

作为上述公式的一个直接推论,得到以下结论:广义系统在输出反馈下可以消出 P 个脉冲模式的充要条件是,系统存在 P 个线性独立的能控能观的脉冲模式.

关键词:广义系统;输出反馈;脉冲模式

本文作者简介

王殿辉 1962年生.1984年毕业于辽宁大学数学系计算数学专业,获理学学士学位;1991年底毕业于东北大学应用数学系,获理学硕士学位;1995年初毕业于东北大学自动控制系,获工学博士学位.主要研究兴趣为广义系统、人工神经网络在非线性系统建模及控制中的应用.目前研究领域为基于人工神经网络模型的非线性系统自适应控制.

谢绪恺 1925年生.1947年毕业于南京中央大学电机系,1950年至1952年大连工学院讲师;1952年至今在东北大学工作,现为该校理学院院长,应用数学系教授.主要研究兴趣为线性系统稳定性,广义系统,分散大系统等.目前研究领域为广义分散控制系统.