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Robust Stability of Uncertain Large-Scale Dynamical System with Delay*

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Abstract: In this paper, the problem of robust stability for large-scale time-delay system made of several nonlinear perturbed subsystems is considered. By making use of the Lyapunov stability criterion, the method of the diagonally dominant matrix and combining with the matrix Riccati equation, the results of the exponential stability for uncertain large-scale time-delay dynamical system are derived. The conditions obtained in this paper improve and extend the main result of [4,6].

Key words: robust stability; large-scale system; delay

1 Introduction

The robust stability problem of dynamical systems in the presence of uncertainty has been receiving considerable attention, because in many practical control problems, uncertainty often occurs in dynamical systems due to modelling errors, measurement errors, linearization approximations, and so on. Many design techniques for uncertain dynamical systems have been developed to guarantee a required degree of robustness. Since time-delay is frequently a source of instability and encountered in various engineering systems such as chemical processes, long transmission lines in pneumatic systems, etc., the problem of the stability of the time-delay systems has been explored over the last decade.

Many methods to check the stability of time-delay systems were proposed by [1~4]. Additionally, the problem of the stabilization of uncertain time-delay systems has also been explored over the last year. Many approaches to solving the problem have been proposed by [4~6]. Hence, in this paper, it is intended to derive the conditions of the stabilizability independent of delay of a class of nonlinear large-scale system with delay. Following the idea of [1,3], the results of exponential stability of delay differential systems are derived. The conditions obtained in this paper improve and extend the main result of [4,6].

Nomenclature

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R^n real vector space of dimension n; A^T transpose of matrix A; $R^{n\times m}$ real matrix space of dimension $n\times m$; $\lambda_m(A)$ minimal eigenvalue of matrix A; $\lambda_M(A)$ maximal eigenvalue of matrix A; $\|x\|$ Euclidean norm of vector $x = [x_1, x_2, \cdots, x_n]^T$, i. e., $\|x\| = \sqrt{\left(\sum_{i=1}^{n} x_i^2\right)^{1/2}}$; $\|A\|$ norm of matrix $A \in R^{n\times m}$, which is compatible with the vector Euclidean norm (i. e. $\|Ax\| \leqslant \|A\| \cdot \|x\|$), may be one of the following forms:

$$\|A\| = \sqrt{\lambda_M(A^TA)}, \qquad \|A\| = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m} a_{ij}^2}.$$

2 System Description and Preliminaries

Let S be the uncertain large-scale time-delay system composed of N interconnected subsystems $S_i, i = 1, 2, \cdots, N$. Each S_i is described by the following differential-difference equations

$$S_i: \quad \dot{x}_i(t) = A_i x_i(t) + B_i x_i(t-\tau_i) + f_i(x(t), t) + g_i(x(t-\tau), t),$$
$$t > 0, \quad i = 1, 2, \cdots, N, \tag{1}$$
$$x_i(t) = \psi_i(t), \quad t \in [-\tau_i, 0],$$

where $x_i \in R^{n_i}$ is the state vector, matrices A_i and $B_i \in R^{n_i\times n_i}$, $\tau_i \geqslant 0$ is a constant, $x(t) = [x_1^T(t), x_2^T(t), \cdots, x_N^T(t)]^T \in R^n$, $x(t-\tau) = [x_1^T(t-\tau_1), x_2^T(t-\tau_2), \cdots, x_N^T(t-\tau_N)]^T \in R^n$. Let $h = \max_i\{\tau_i\}$, $\psi_i(t)$ is a continuous vector-valued initial function, $\|\psi\| = \max_i\{\sup_{-\tau_i\leqslant\theta\leqslant 0}\{\|\psi_i(\theta)\|\}\}$. The uncertainties f_i and g_i are unknown and represent the nonlinear parameter perturbations with respect to the current state $x(t)$ and delayed state $x(t-\tau)$ of the subsystem S_i, respectively. In general, it is assumed that $\|f_i(x(t), t)\|$ and $\|g_i(x(t-\tau), t)\|$ are bounded, i. e.

$$\|f_i(x(t), t)\| \leqslant \sum_{j=1}^{N} \alpha_{ij}\|x_j(t)\|, \quad i = 1, 2, \cdots, N, \tag{2}$$

$$\|g_i(x(t-\tau), t)\| \leqslant \sum_{j=1}^{N} \beta_{ij}\|x_j(t-\tau_j)\|,$$

where $\alpha_{ij} \geqslant 0, \beta_{ij} \geqslant 0$ are given.

A system is said to be robustly stable if it is allowed to change in certain specific bounds of perturbation. Here, let us derive a sufficient condition such that system S is robustly stable with a prescribed stability degree. In the following, we first state definitions and summarize those results which we will require.

Definition 1[5] The system S is said to have a stability degree $\beta > 0$ if there exists $k > 0$ (depending on initial conditions) such that any solution $x(t)$ of S satisfies.

$$\|x(t_2)\| \leqslant k\|x(t_1)\| \exp[-\beta(t_2 - t_1)] \tag{3}$$

for all $t_1, t_2 \in R^+$ and $t_2 \geqslant t_1$.

Lemma 1[1] If

$$z(t) = e^{\beta t}x(t). \tag{4}$$

where $x(t)$ is the solution of system S and $z(t) = 0$ is asymptotically stable, then the system S is robustly stable with a stability degree β.

Utilize (4) to transform (1) into

$$\hat{S}_i:\ \dot{z}(t) = (A_i + \beta I_i)z_i(t) + e^{\beta\tau_i}B_i z_i(t-\tau_i) + \tilde{f}_i(z(t),t) + \tilde{g}_i(z(t-\tau),t). \quad (5)$$

By using (2), we have

$$\| \tilde{f}_i(z(t),t) \| = \| e^{\beta t} f_i(x(t),t) \| \leqslant \sum_{j=1}^{N} \alpha_{ij} e^{\beta t} \| x_j \| = \sum_{j=1}^{N} \alpha_{ij} \| z_j \|, \quad (6)$$

$$\| \tilde{g}_i(z(t-\tau),\tau) \| = \| e^{\beta t} g_i(x(t-\tau),t) \| \leqslant \sum_{j=1}^{N} \beta_{ij} e^{\beta t} \| x_j(t-\tau_j) \|$$

$$= \sum_{j=1}^{N} \beta_{ij} e^{\beta\tau_j} \| z_j(t-\tau_j) \|. \quad (7)$$

3 Main Results

Theorem 1 If we can select a set of parameters (α_i, β_i) and symmetric positive definite matrices Q_i such that

$$\alpha_i \lambda_m(P_i) + \beta_i \lambda_m(Q_i) > \frac{1}{2} \sum_{j=1}^{N} [\alpha_{ij} \| P_i \| + \alpha_{ji} \| P_j \| + \beta_{ji} + \delta_{ji} \| B_j \|$$
$$+ (\beta_{ij} + \delta_{ij} \| B_i \|) \| P_i \|^2], \quad i = 1,2,\cdots,N. \quad (8)$$

Then the large-scale time-delay dynamical system S is exponentially stable, where symmetric positive definite matrices P_i is the solution of

$$(A_i + \alpha_i I_i)^{\mathrm{T}} P_i + P_i(A_i + \alpha_i I_i) + 2\beta_i Q_i = 0 \quad (9)$$

and
$$\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases}$$

Proof Let

$$C_i(\beta) \triangleq (\alpha_i - \beta)\lambda_m(P_i) + \beta_i \lambda_m(Q_i) - \frac{1}{2} \sum_{j=1}^{N} [\alpha_{ij} \| P_i \| + \alpha_{ji} \| P_{ji} \| + \beta_{ji}$$
$$+ \delta_{ji} \| B_j \| + e^{2\beta h}(\beta_{ij} + \delta_{ij} \| B_i \|) \| P_i \|^2], \quad i = 1,2,\cdots,N. \quad (10)$$

It is obvious that $C_i(\beta)$ $(i = 1,2,\cdots,N)$ are continuous functions with respect to β, and by using (8), we have

$$C_i(0) > 0.$$

So there exists $\beta > 0$ enough small, such that

$$C_i(\beta) > 0, \quad i = 1,2,\cdots,N. \quad (11)$$

Let us take the positive definite function as follows:

$$V_i(t) = z_i^{\mathrm{T}} P_i z_i, \quad i = 1,2,\cdots,N, \quad (12)$$

where P_i is a solution of (9).

By (6), (7), and (9), we obtain the derivative of $V_i(t)$ along the trajectory of the system (5) as follows.

$$\frac{\mathrm{d}V_i}{\mathrm{d}t} = z_i^{\mathrm{T}}[(A_i + \beta I_i)^{\mathrm{T}} P_i + P_i(A_i + \beta I_i)]z_i + 2z_i^{\mathrm{T}} P_i e^{\beta\tau_i} B_i z_i(t-\tau_i)$$

$$+ 2z_i^{\mathrm{T}}P_i\tilde{f}_i(z(t),t) + 2z_i^{\mathrm{T}}P_i\tilde{g}_i(z(t-\tau),t)$$
$$\leqslant z_i^{\mathrm{T}}[-2(\alpha_i-\beta)P_i - 2\beta_iQ_i]z_i + 2e^{\beta h}\|P_i\|\ \|B_i\|\ \|z_i\|\ \|z_i(t-\tau_i)\|$$
$$+ \sum_{j=1}^{N}2\alpha_{ij}\|P_i\|\ \|z_i\|\ \|z_j\| + \sum_{j=1}^{N}2e^{\beta h}\beta_{ij}\|P_i\|\ \|z_i\|\ \|z_j(t-\tau_j)\|$$
$$\leqslant \sum_{j=1}^{N}\Big\{-\Big[2(\alpha_i-\beta)\lambda_m(P_i) + \beta_i\lambda_m(Q_i) - (\|B_i\| + \sum_{l=1}^{N}\beta_{il})\|P_i\|^2)\delta_{ij}$$
$$-2\alpha_{ij}\|P_i\|\Big]\|z_i\|\ \|z_j\| + e^{2\beta h}(\|B_i\|\delta_{ij} + \beta_{ij})\|z_j(t-\tau_j)\|^2\Big\}. \tag{13}$$

By using the Theorem 2 in [2] and (11), we get that system (5) is asymptotically stable. By Lemma 1, this prove that the large-scale time-delay system S is exponentially stable.

Remark 1 When $\beta_{ij}=0$, $\|B_i\|=0$, $(i,j=1,2,\cdots,N)$, condition (8) becomes

$$\alpha_i\lambda_m(P_i) + \beta_i\lambda_m(Q_i) > \frac{1}{2}\sum_{j=1}^{N}[\alpha_{ij}\|P_i\| + \alpha_{ji}\|P_j\|],\quad i=1,2,\cdots,N \tag{14}$$

which is a sufficient condition for exponential stability of the uncertain lareg-scale dynamical system in the absence of time-delay.

Remark 2 When $N=1$, condition (14) becomes

$$\alpha_1\lambda_m(P_1) + \beta_1\lambda_m(Q_1) > \alpha_{11}\|P_1\|. \tag{15}$$

We can select
$$A_1 = 1 - \frac{k}{2}BB^{\mathrm{T}}P_1,$$
$$f_1(x,t) = \left(\Delta A - \frac{k}{2}\Delta BB^{\mathrm{T}}P_1\right)x(t),$$

then
$$\|f_1(x,t)\| \leqslant \left[l_1 + \frac{1}{2}kl_2\|B\|\ \|P_1\|\right]\|x\| = \alpha_{11}\|x\|,$$

condition (15) becomes

$$\alpha_1\lambda_m(P_1) + \beta_1\lambda_m(Q_1) > \left[l_1 + \frac{1}{2}kl_2\|B\|\ \|P_1\|\right]\|P_1\|, \tag{16}$$

condition (16) is a sufficient conditon for robust stabilization of the single system discussed by [6]. However the condition given by [6] is

$$\alpha_1\lambda_m(P_1) - \beta_1\lambda_m(Q_1) > \left(l_1 + \frac{1}{2}kl_2\|B\|\ \|P_1\|\right)\|P_1\|,$$

which is obviously a special case of (16).

Theorem 2 If we can select a set of parameters (α_i,β_i) and symmetric positive definite matrices Q_i such that

$$(\alpha_i-\beta)\lambda_m(P_i) + \beta_i\lambda_m(Q_i) > \frac{1}{2}\sum_{j=1}^{N}[\alpha_{ij}\|P_i\| + \alpha_{ji}\|P_j\| + (\beta_{ij}\|P_i\| + \delta_{ij}\|P_iB_i\|$$
$$+ \beta_{ji}\|P_j\| + \delta_{ji}\|P_jB_j\|)e^{\beta h}],\quad i=1,2,\cdots,N, \tag{17}$$

then the large-scale time-delay dynamical system S is robustly stable with a stability degree β. In (17) symmetric positive definite matrices P_i is the solution of (9).

The proof of theorem 2 is similar to the proof of Theorem 1.

Remark 3 When $N=1$, condition (17) becomes

$$(\alpha_1-\beta)\lambda_m(P_1)+\beta_1\lambda_m(Q_1)>\alpha_{11}\|P_1\|+(\beta_{11}\|P_1\|+\|P_1B_1\|)e^{\beta h}. \tag{18}$$

Condition (18) is a sufficient condition that the single system discussed by [4] is robustly stable with the stability degree β. However, the condition given by [4] is

$$\beta_1\lambda_m(Q_1)>\alpha_{11}\|P_1\|+\alpha e^{\beta h}(\beta_{11}\|P_1\|+\|P_1B_1\|), \tag{19}$$

where $\alpha=\sqrt{\dfrac{\lambda_M(P_1)}{\lambda_m(P_1)}}\geqslant 1$, which is obviously a special case of (18).

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时滞大系统的 Robust 稳定性

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摘要：在本文中考虑了由非线性扰动子系统组成的时滞大系统的鲁棒稳定性问题,利用 Lyapunov 稳定性准则,对角占优矩阵的方法和结合矩阵 Riccati 方程,获得了系统指数稳定的充分判据,本文的结果改进并推广了文[4,6]的结果.

关键词：鲁棒稳定性；大系统；时滞

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