

A SVD-Based Extended Kalman Filter and Application to Flight State and Parameter Estimation of Aircraft

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Abstract: In this paper, a new robust extended Kalman filtering algorithm based on singular value decomposition (SVD) of covariance information matrix is presented with application to the flight state and parameter estimation of aircraft. The presented algorithm not only has a good numerical stability but also can handle correlated measurement noise without any additional transformation. The algorithm is formulated in the form of vector-matrix operations, so it is also useful for parallel computers. The applications to the flight state and parameter estimation by simulated and actual flight test data computation of two types of Chinese aircraft show that the new algorithm presented in this paper can give more accurate estimates of flight state and parameter than extended Kalman filter (EKF) for different initial values and noise statistics. Moreover, the new algorithm has less requirements for the maneuvering shapes, noise levels, data length and better convergency than those of EKF. The computational requirements of the new filtering algorithm have been reduced greatly by exploiting some special features of matrix computation and system model. It is proved that the new filtering algorithm can give good results even for low sample rate flight test data.

Key words: extended Kalman filter; singular value decomposition; state and parameter estimation; flight test

1 Introduction

Accurate estimation of aircraft motions from noisy or incomplete flight test measurements by optimal estimation theory is an important problem in the analysis of flight test experiments. The measurements may often contain significant errors which must be estimated before the flight test data are used in any performance calculations. These problems can be implemented as state and parameter estimation problem of nonlinear system. Solution to the problem can be obtained by EKF^[1,2] and maximum likelihood (ML)^[2] method. However, EKF may suffer from numerical ill-conditioning. To overcome this difficulty of Kalman filter, Potter^[3] introduced the idea of using a square root of the covariance matrix in the algorithmic implementation. Bierman^[4] have introduced strictly algorithm approaches to the square root filtering. Bierman also introduced the idea of using a UDU^T decomposition of the covariance matrix in place of the square root decomposition.

* The project is supported by National Natural Science Foundation of China and Science Foundation of Aeronautics of China.

Manuscript received Jun. 16, 1994, revised Nov. 14, 1994.

Both the square root and UDU^T decompositions may result in numerically stable filter algorithms. But these formulations can only be used if one has single dimension measurements with uncorrelated measurements noise. Generally one does not have this in practice. To handle correlated measurements noise, additional transformations have to be used which increase the computation cost. Moreover, these formulations can not be effectively implement on vector processors because their designs are virtually serial in structure.

In recent papers by Oshman^[5,6], the author gives a V -Lambada filter based on the spectral decomposition of covariance or information matrix in a $V\Lambda V^T$ form. But, this filtering algorithm is complex, and the computational requirements is large. Wang et al.^[7] also gives a Kalman filter algorithm based on SVD, but the authors just give the linear system filter formulations.

In this paper, we will present a SVD-based extended Kalman filtering algorithm which not only has the UDU^T formulation as in the Bierman's method, but also is suitable for parallel computer, it has a higher numerical stability and computational efficiency than EKF and other previous algorithms. The applications in the flight state and parameter estimation by the simulated and actual flight state and parameter estimation by the simulated and actual flight test data computation of two types of Chinese aircraft show that the new algorithm can not only improve the numerical robustness and accuracy of flight state and parameter estimation but also make the computation efficient by exploiting some special features of matrix computation and system model of flight state estimation.

2 Singular Value Decomposition and Its Computing

The singular value decomposition of an m -by- n matrix A ($m > n$), is a factorization of A into product of three matrices. That is, there exist orthogonal matrices $U \in R^{m \times m}$ and $V \in R^{n \times n}$ such that

$$A = UAV^T, \quad \Lambda = \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix} \quad (1)$$

where $\Lambda \in R^{m \times m}$ and $S = \text{diag}(\sigma_1, \dots, \sigma_r)$ with

$$\sigma_1 > \dots > \sigma_r > 0.$$

The numbers $\sigma_1, \dots, \sigma_r$ together with $\sigma_{r+1} = 0, \dots, \sigma_n = 0$ are called the singular values of A and they are the positive square roots of the eigenvalues of $A^T A$. The columns of U are called the left singular vectors of A (the orthonormal eigenvectors of AA^T) while the columns of V are called the right singular vectors of A (the orthonormal eigenvectors of $A^T A$).

In practice, if $A^T A$ is positive definite, then (1) can be reduced to

$$A = U \begin{bmatrix} S \\ 0 \end{bmatrix} V^T. \quad (2)$$

where S is an n -by- n diagonal matrix. Especially, if A itself is symmetric positive definite then we will have a symmetric singular value decomposition

$$A = USU^T = UD^2U^T. \quad (3)$$

In our filter algorithm derivation, (2) and (3) will be of particularly real interest.

3 Extended Kalman Filter Formulation

Consider a nonlinear system described by the discrete-time state space equations:

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k) + \mathbf{g}(\mathbf{x}_k)\eta_k, \quad (4)$$

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \zeta_k. \quad (5)$$

where $\mathbf{x}_k \in \mathbb{R}^n$ is the state vector, $\mathbf{z}_k \in \mathbb{R}^m$ is the measurement vector, $\mathbf{f}(\cdot)$, $\mathbf{g}(\cdot)$ and $\mathbf{h}(\cdot)$ are n , l and m dimension nonlinear vector functions, respectively, $\eta_k \in \mathbb{R}^l$ is the disturbance input vector, $\zeta_k \in \mathbb{R}^m$ is the measurement noise vector. The sequences $\{\eta_k\}$ and $\{\zeta_k\}$ are assumed to be zero mean Gaussian white noise sequences with symmetric positive definite covariance matrices Q_k and R_k , respectively. Initial state \mathbf{x}_0 is Gaussian random variable with $N(\bar{\mathbf{x}}_0, P_0)$, and the sequences $\{\eta_k\}$, $\{\zeta_k\}$ and \mathbf{x}_0 are assumed to be mutually independent.

The extended Kalman filter formulation in covariance/information mode is then described by the following recursive equations under assumptions that the nonlinear functions $\mathbf{f}(\mathbf{x}_k)$, $\mathbf{g}(\mathbf{x}_k)$ and $\mathbf{h}(\mathbf{x}_k)$ can be expanded in Taylor series about the conditional means $\hat{\mathbf{x}}_{k|k}$ and $\hat{\mathbf{x}}_{k+1|k}$ with neglecting higher order terms.

Time update (covariance mode):

$$\hat{\mathbf{x}}_{k+1|k} = \mathbf{f}(\hat{\mathbf{x}}_{k|k}), \quad (6)$$

$$P_{k+1|k} = \Phi_{k+1,k} P_{k|k} \Phi_{k+1,k}^T + G_k Q_k G_k^T. \quad (7)$$

Measurement update (information mode):

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + K_k [\mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1})], \quad (8)$$

$$P_{k|k}^{-1} = P_{k|k-1}^{-1} + H_k^T R_k^{-1} H_k, \quad (9)$$

$$K_k = P_{k|k} H_k^T R_k^{-1}, \quad (10)$$

where

$$\Phi_{k+1,k} = \left. \frac{\partial \mathbf{f}(\mathbf{x}_k)}{\partial \mathbf{x}_k} \right|_{\mathbf{x}_k = \hat{\mathbf{x}}_{k|k}}, \quad G_k = \mathbf{g}(\hat{\mathbf{x}}_{k|k}),$$

$$H_k = \left. \frac{\partial \mathbf{h}(\mathbf{x}_k)}{\partial \mathbf{x}_k} \right|_{\mathbf{x}_k = \hat{\mathbf{x}}_{k|k-1}}$$

where P is the covariance matrix of estimation error, and K_k is the Kalman gain matrix. The initial condition given by $\hat{\mathbf{x}}_{0|0} = \bar{\mathbf{x}}_0$, $P_{0|0} = P_0$.

4 New SVD-Based Extended Kalman Filter Formulation

4.1 Time Update Formulation

In the covariance equation (7) of the extended Kalman filter, assume that the singular value decomposition of $P_{k|k}$ is available for all k and has been propagated and updated by the filter algorithm. Thus, we have

$$P_{k|k} = U_{k|k} D_{k|k}^2 U_{k|k}^T. \quad (11)$$

Eq. (7) can therefore be written as

$$P_{k+1|k} = \Phi_{k+1,k} U_{k|k} D_{k|k}^2 U_{k|k}^T \Phi_{k+1,k}^T + G_k Q_k G_k^T. \quad (12)$$

Our goal is to find the factors $U_{k+1|k}$ and $D_{k+1|k}$ from Eq. (12) such that $P_{k+1|k} = U_{k+1|k} D_{k+1|k}^2 U_{k+1|k}^T$, where U factors are orthogonal and D factors are diagonal. Provided that there is no danger of numerical accuracy deterioration, one could, in a brute force fashion, compute $P_{k+1|k}$ and then apply the singular value decomposition of symmetric positive definite matrix which given by Eq. (3). However, it has been shown that this is not a good numerical exercise^[9]. Instead we define the following $(l+n)$ -by- n matrix

$$\begin{bmatrix} D_{k|k} U_{k|k}^T \Phi_{k+1,k}^T \\ \sqrt{Q_k^T} G_k^T \end{bmatrix} \quad (13)$$

and compute its singular value decomposition

$$\begin{bmatrix} D_{k|k} U_{k|k}^T \Phi_{k+1,k}^T \\ \sqrt{Q_k^T} G_k^T \end{bmatrix} = U_k \begin{bmatrix} D'_k \\ 0 \end{bmatrix} V_k^T. \quad (14)$$

Multiplying each side on the left by its transpose, we have

$$\Phi_{k+1,k} U_{k|k} D_{k|k}^2 U_{k|k}^T \Phi_{k+1,k}^T + G_k \sqrt{Q_k} \sqrt{Q_k^T} G_k^T = V_k' [D_k'^T \ 0] U_k'^T U_k' \begin{bmatrix} D'_k \\ 0 \end{bmatrix} V_k'^T.$$

That is

$$\Phi_{k+1,k} U_{k|k} D_{k|k}^2 U_{k|k}^T \Phi_{k+1,k}^T + G_k Q_k G_k^T = V_k' D_k'^2 V_k'^T. \quad (15)$$

Comparing the result with (12), we find that V_k' and D_k' are just the $U_{k+1,k}$ and $D_{k+1,k}$ we are looking for. Here we want to point out that the $(l+n)$ -by- $(l+n)$ orthogonal matrix U_k' and its transpose $U_k'^T$ are not needed directly in our algorithm and it is not necessary to store or compute them explicitly.

4.2 Measurement Update Formulation

In the information mode covariance equation (9) of extended Kalman filter, applying the singular value decomposition of symmetric positive definite matrix to $P_{k|k}$ and $P_{k|k-1}$ respectively, we may get

$$\begin{aligned} (U_{k|k} D_{k|k}^2 U_{k|k}^T)^{-1} &= (U_{k|k-1} D_{k|k-1}^2 U_{k|k-1}^T)^{-1} + H_k^T R_k^{-1} H_k, \\ &= (U_{k|k-1}^T)^{-1} D_{k|k-1}^{-2} U_{k|k-1}^{-1} + (U_{k|k-1}^T)^{-1} U_{k|k-1}^T H_k^T R_k^{-1} H_k U_{k|k-1} U_{k|k-1}^{-1}, \\ &= (U_{k|k-1}^T)^{-1} (D_{k|k-1}^{-2} + U_{k|k-1}^T H_k^T R_k^{-1} H_k U_{k|k-1}) U_{k|k-1}^{-1}. \end{aligned} \quad (16)$$

In Eq. (16) let

$$L_k L_k^T = R_k^{-1} \quad (17)$$

be the Cholesky decomposition of the inverse of the covariance matrix.

Now considering the $(m+n)$ -by- n matrix

$$\begin{bmatrix} L_k^T H_k U_{k|k-1} \\ D_{k|k-1}^{-1} \end{bmatrix}, \quad (18)$$

and computing its singular value decomposition, we have

$$\begin{bmatrix} L_k^T H_k U_{k|k-1} \\ D_{k|k-1}^{-1} \end{bmatrix} = U_k'' \begin{bmatrix} D_k'' \\ 0 \end{bmatrix} V_k''^T. \quad (19)$$

Multiplying each side on the left by its transpose yields

$$D_{k|k-1}^{-2} + U_{k|k-1}^T H_k^T L_k L_k^T H_k U_{k|k-1} = V_k^* D_k^{*2} V_k^{*T}, \quad (20)$$

then Eq. (16) can be written as

$$\begin{aligned} (U_{k|k}^T)^{-1} (D_{k|k})^{-2} (U_{k|k})^{-1} &= (U_{k|k-1}^T)^{-1} V_k^* D_k^{*2} V_k^{*T} U_{k|k-1}^{-1}, \\ &= [(U_{k|k-1} V_k^*)^T]^{-1} D_k^{*2} [U_{k|k-1} V_k^*]^{-1}. \end{aligned} \quad (21)$$

Comparing two sides of Eq. (21) we get

$$U_{k|k} = U_{k|k-1} V_k^*, \quad (22)$$

$$D_{k|k} = (D_k^*)^{-1}. \quad (23)$$

In this manner, a new measurement update formulation has been obtained.

For the Kalman gain, an alternative expression may also be get by equations (10), (11) and (17)

$$K_k = U_{k|k} D_{k|k}^2 U_{k|k}^T H_k^T L_k L_k^T. \quad (24)$$

There is no need to obtain a formula for the singular value decomposition of K_k .

The state vector measurement update is given by

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k [z_k - h(\hat{x}_{k|k-1})]. \quad (25)$$

Together with the time update (6), (14) and (15) described in the above section and the measurement update of the covariance matrix and the state vector (19) ~ (25) described here, a SVD-based new extended Kalman filter algorithm is formulated.

Note 1 In order to reduce the computational requirement, only the right SVD factor V of matrices (13) and (18) are need computed in our filter formulation.

Note 2 Computations of K_k from (24) and $\hat{x}_{k|k}$ from (25) are straightforward, the essential calculation for K_k and $\hat{x}_{k|k}$ is just the matrix-matrix and matrix-vector multiplications. Notice that L_k^T is triangular and $H_k^T L_k = (L_k^T H_k)^T$ can be obtained from the previous computation, the computational requirement can be reduced by exploiting this feature.

5 Applications to Flight State and Parameter Estimation

5.1 Nonlinear Model for Flight State Estimation

The six degrees-of-freedom nonlinear model for flight state estimation is presented as follows:

$$\begin{cases} \dot{u} = -qw + rv - g\sin\theta + a_x, \\ \dot{v} = -ru + pw + g\cos\theta\sin\varphi + a_y, \\ \dot{w} = -pv + qu + g\cos\theta\cos\varphi + a_z, \\ \dot{\psi} = (q\sin\varphi + r\cos\varphi)/\cos\theta, \\ \dot{\theta} = q\cos\varphi - r\sin\varphi, \\ \dot{\varphi} = p + q\sin\varphi\tan\theta + r\cos\varphi\tan\theta, \\ \dot{h} = u\sin\theta - v\cos\theta\sin\varphi - w\cos\theta\cos\varphi \end{cases} \quad (26)$$

where, a_x, a_y, a_z are linear acceleration components of the aircraft; u, v, w are components

of the linear velocity in body axes; ψ, θ, φ (yaw, pitch and roll) are Euler angles; p, q, r are components of angular velocity, h is the altitude and g denotes the gravitational constant.

In general, the measurements of input variables, $u = [a_x, a_y, a_z, p, q, r]^T$ and observation variables $z = [V, \beta, \alpha, \theta, \varphi, h]^T$ are corrupted by scale factor errors, biases and random noises, where

$$\begin{cases} a_x = a_{x_m} + b_x + \eta_x, \\ a_y = a_{y_m} + b_y + \eta_y, \\ a_z = a_{z_m} + b_z + \eta_z, \\ p = p_m + b_p + \eta_p, \\ q = q_m + b_q + \eta_q, \\ r = r_m + b_r + \eta_r. \end{cases} \quad (27)$$

The observation equation of aircraft can be described as follows:

$$\begin{cases} V_m = (1 + \lambda_v) \sqrt{u^2 + v^2 + w^2} + b_v + \zeta_v, \\ \beta_m = (1 + \lambda_\beta) \tan^{-1}[(v + rx_\beta - pz_\beta)/u] + b_\beta + \zeta_\beta, \\ \alpha_m = (1 + \lambda_\alpha) \tan^{-1}[(w - qx_\alpha + py_\alpha)/u] + b_\alpha + \zeta_\alpha, \\ \theta_m = \theta + b_\theta + \zeta_\theta, \\ \varphi_m = \varphi + b_\varphi + \zeta_\varphi, \\ h_m = (1 + \lambda_h)h + b_h + \zeta_h. \end{cases} \quad (28)$$

The system described by Eqs. (26)~(28) forms a set of nonlinear dynamic equations of the form:

$$\dot{x}(t) = f(x(t), u_m(t), b, \eta(t)), \quad (29)$$

$$z_m(t) = h(x(t), u_m(t), b) + \zeta(t), \quad (30)$$

where

$$x = [u, v, w, \psi, \varphi, \theta, h]^T,$$

$$\eta = [\eta_x, \eta_y, \eta_z, \eta_p, \eta_q, \eta_r]^T,$$

$$\zeta = [\zeta_v, \zeta_\beta, \zeta_\alpha, \zeta_\theta, \zeta_\varphi, \zeta_h]^T,$$

$$b = [b_x, b_y, b_z, b_p, b_q, b_r, b_v, b_\beta, b_\alpha, b_\theta, b_\varphi, b_h, \lambda_v, \lambda_\beta, \lambda_\alpha, \lambda_h]^T.$$

Thus, our problem can be stated as follows: given the nonlinear model (29), (30) and a set of noisy input and output measurements, estimate the system state x and parameter b .

In order to estimate the state and unknown parameters, in general, one can form an augmented state model, that is, set

$$x^a = \begin{bmatrix} x \\ b \end{bmatrix}. \quad (31)$$

Usually, the nonlinear equations (29), (30) are first linearized and then transformed into the discrete-time version, thus the larger modelling errors may often appear when the aircraft is maneuvering. In order to decrease the modelling errors, a direct and exact

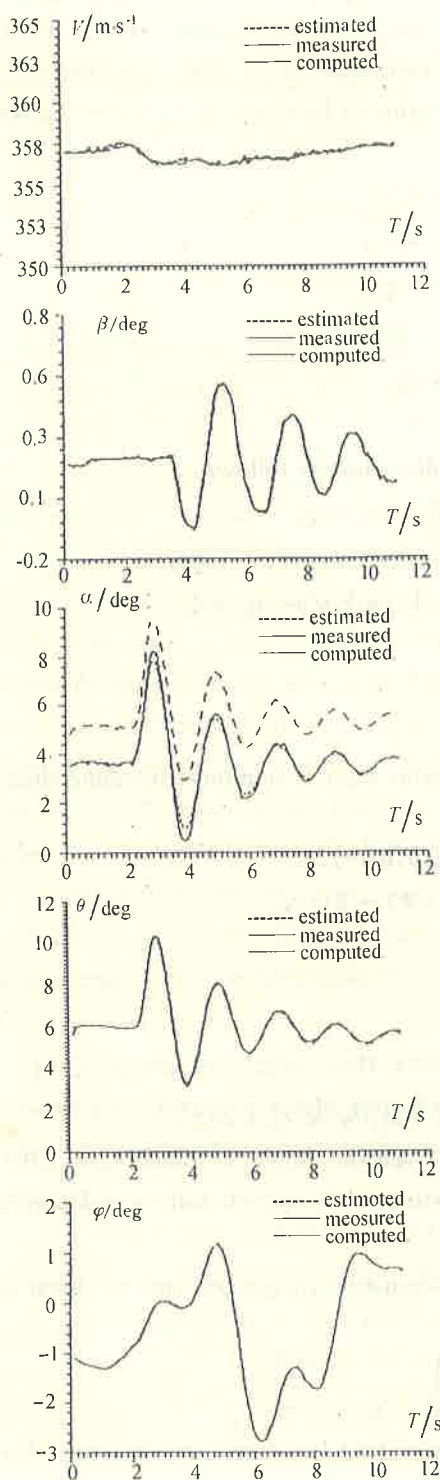


Fig. 1 Results of state estimate(1/10s)

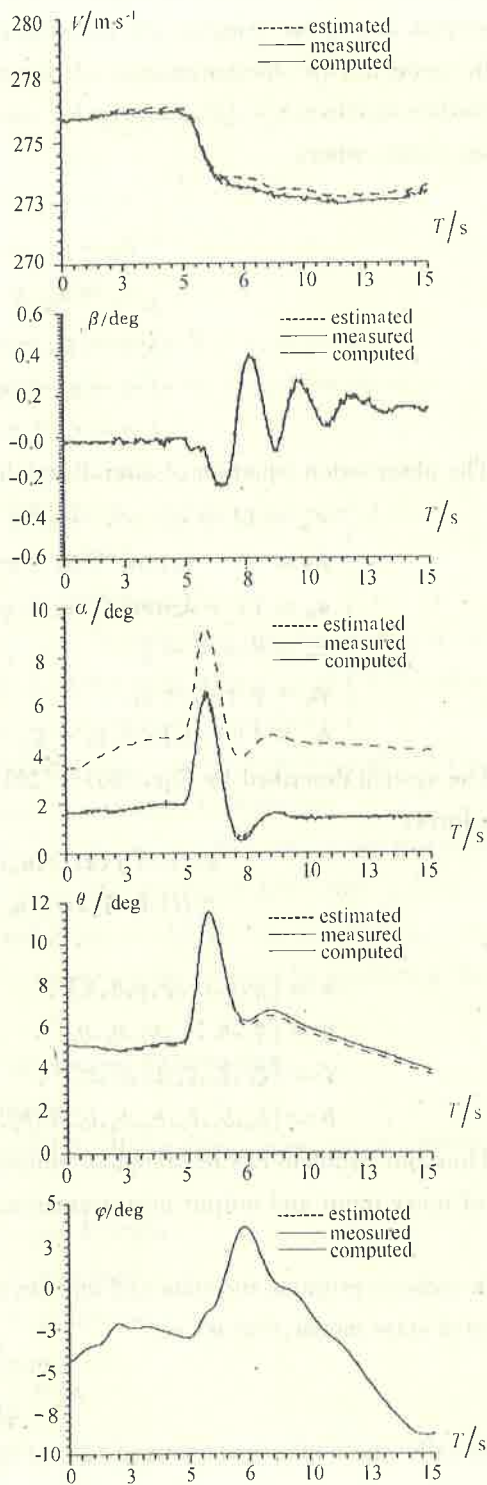


Fig. 2 Results of state estimate(1/32s)

argumented state discrete-time model for flight state and parameter estimation is given as follows:

$$\mathbf{x}_{k+1}^a = \mathbf{f}(\mathbf{x}_k^a) + \mathbf{G}(\mathbf{x}_k^a)\eta_k. \quad (32)$$

The discrete form of output equation is given by:

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k^a) + \zeta_k. \quad (33)$$

where, the process noise $\{\eta_k\}$ and measurement noise $\{\zeta_k\}$ sequences are assumed to be independent, zero mean and Gaussian with covariance \mathbf{Q} and \mathbf{R} , respectively.

5.2 Results of Simulation and Application Computations

The new algorithm presented in this paper has been applied to the flight state and parameter estimation with simulated and actual flight test data. The results of simulation and application to two types of Chinese aircraft show that the new algorithm can give accurate estimate and is much more stable and accurate than EKF. The results of estimated true state value, and the fits of computed responses with measurement responses are shown in Fig. 1 and Fig. 2 with the sample of 1/10 and 1/32 second, respectively. It is obvious that the fits of computed responses with measurement responses are accurate and satisfactory both for lower and higher sampling rates.

6 Conclusions

In this paper, a new extended Kalman filter algorithm, which is based on the well-known singular value decomposition, is proposed. The new filter formulation have the highest accuracy and stability characteristics in all existing filter algorithms, and the new filter formulation can handle correlated measurement noise without any additional transformations. Third, the present algorithm is also suitable for parallel computer because it is formulated in the form of matrix-matrix and matrix-vector operations. The applications of presented new algorithm to the flight state and parameter estimation of aircraft show that the new algorithm is much more numerically stable and accurate than EKF.

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基于 SVD 的推广卡尔曼滤波及其 在飞行状态和参数估计中的应用

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摘要: 基于矩阵的奇异值分解(SVD)技术, 本文提出一种鲁棒推广卡尔曼滤波新算法, 并将该算法应用于飞行状态和参数估计中. 该算法不仅具有很好的数值稳定性, 而且无需任何变换即可处理相关噪声, 且适于并行计算. 两种不同型号飞机飞行状态和参数估计的仿真及实际试飞数据计算结果表明: 与 EKF 相比, 本文算法对不同初始值和不同噪声均可获得更准确的估计结果, 并且对飞机机动形式、噪声水平、数据长度要求不高, 收敛性好. 利用系统和量测模型的一些特点及对奇异值分解算法的改进, 使算法计算量大大减少.

关键词: 推广卡尔曼滤波; 奇异值分解; 状态和参数估计; 飞行试验

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