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Decentralized H_∞-Control for Composite Systems

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Abstract: This paper is concerned with the problem of decentralized H_∞-control design via state feedback for linear composite systems. A decentralized state feedback control design which stabilizes a given composite system and guarantees an H_∞-norm bound constraint on disturbance attenuation is presented in terms of a positive semi-definite solution to Riccati-like equation. For the composite systems with symmetric circulant structure, a simple decentralized control design procedure is obtained. As an application, a sufficient condition for solvability of simultaneous H_∞-control problem for a collection of linear systems is derived.

Key Words: composite systems; decentralized control; Riccati equations; symmetric circulant structure; simultaneous H_∞-controller

1 Introduction

In the last decade, a great deal of attention has been paid to the H_∞-control problem for centralized control systems, and some important advances have been achieved, see [1~3] and the references therein. For the H_∞-control problem of decentralized control systems with multiple control channels, Veillette et al. [4] presented a decentralized control design procedure by using the Riccati-like equation approach, which is a new approach in the area of decentralized control. Shor et al. [5] extended the result to the case of discrete-time decentralized control systems. However, little attention has been paid to designing decentralized H_∞-controller for composite systems so far.

This paper will study the problem of decentralized H_∞-control design via state feedback for composite systems. The paper is organized as follows. In Section 2, a decentralized state feedback control design which stabilizes a given composite system and guarantees an H_∞-norm bound constraint on disturbance attenuation is presented by means of the Riccati-like equation approach. Section 3 discusses a class of composite systems composed of several similar subsystems interconnected in a symmetrical fashion, and the system matrices for these systems are block symmetric circulant. Recently, there has been a great interest in the class of systems, see [6~9] and the references therein. For the class of composite systems, a simple decentralized H_∞-control design procedure is given by utilizing the

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structural properties of these systems. In Section 4, the relation between composite systems with symmetric circulant structure and a collection of linear systems is discussed, and a sufficient condition for solvability of simultaneous H_∞-control problem for a collection of linear systems is derived.

2 Decentralized H_∞-Control Design

Consider a linear composite system Σ composed of N subsystems described by

$$\dot{x}_i = A_{ii}x_i + \sum_{j=1, j\neq i}^{N} A_{ij}x_j + \sum_{j=1}^{N} G_{ij}w_j + \sum_{j=1}^{N} B_{ij}u_j, \tag{1}$$

$$z_i = \begin{bmatrix} \sum_{j=1}^{N} H_{ij}x_j \\ u_i \end{bmatrix}, \quad i = 1, \dots, N \tag{2}$$

where x_i and u_i are the state and the input of the ith subsystem, respectively, the z_i's are outputs to be regulated, and the w_i's are square-integrable disturbances.

Denote

$$A = [A_{ij}], \quad G = [G_{ij}], \quad B = [B_{ij}], \quad H = [H_{ij}]. \tag{3}$$

The problem under consideration is to design a decentralized state feedback control law

$$u_i = K_i x_i, \quad i = 1, \dots, N \tag{4}$$

such that the resulting closed-loop system is asymptotically stable, and the closed-loop transfer matrix

$$T(s) = H_c(sI - A_c)^{-1}G, \tag{5}$$

satisfies $\| T \|_\infty \leq \alpha$ for some prescribed $\alpha > 0$, where $H_c = \begin{bmatrix} H \\ K \end{bmatrix}$, and $A_c = A + BK$ with $K = \operatorname{diag}[K_1\ K_2 \dots K_N]$.

The following result presents a procedure of designing the decentralized state feedback gains $K_i, i = 1, \dots, N$.

Theorem 1 Let (A, H) be a detectable pair and α be a positive constant. Suppose that

$$K_i = - B_{ii}^{\mathrm{T}} X_{ii}, \quad i = 1, \dots, N \tag{6}$$

where $X \geq 0$ satisfies the following Riccati-like algebraic equation

$$A^{\mathrm{T}}X + XA + \frac{1}{\alpha^2}XGG^{\mathrm{T}}X - XBB^{\mathrm{T}}X + H^{\mathrm{T}}H + (XB - X_D B_D)(B^{\mathrm{T}}X - B_D^{\mathrm{T}}X_D) = 0 \tag{7}$$

with X, X_D and B_D defined by

$$X = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1N} \\ X_{21} & X_{22} & \cdots & X_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ X_{N1} & X_{N2} & \cdots & X_{NN} \end{bmatrix}, \quad X_D = \operatorname{diag}[X_{11} \quad X_{22} \quad \cdots \quad X_{NN}], \tag{8}$$

$$B_D = \mathrm{diag}[B_{11} \quad B_{22} \quad \cdots \quad B_{NN}]. \tag{9}$$

Then the decentralized state feedback control law (4) stabilizes the system Σ of (1) and (2), and the closed-loop transfer matrix $T(s)=H_c(sI-A_c)^{-1}G$ satisfies $\|T\|_\infty \leq \alpha$.

The proof of the theorem is based on the following result from Veillette et al[4].

Lemma 1[4] Let $T_0(s)=H_0(sI-F_0)^{-1}G_0$, with (F_0, H_0) a detectable pair. If there exists a real matrix $X\geq 0$ and a positive constant α such that

$$F_0^{\mathrm{T}}X + XF_0 + \frac{1}{\alpha^2}XG_0G_0^{\mathrm{T}}X + H_0^{\mathrm{T}}H_0 \leqslant 0, \tag{10}$$

then F_0 is Hurwitz, and $T_0(s)$ satisfies $\|T_0\|_\infty \leq \alpha$.

Proof of Theorem 1: Let $K=-B_D^{\mathrm{T}}X_D$. Then, from equation (7),

$$\begin{aligned} &A_c^{\mathrm{T}}X + XA_c + \frac{1}{\alpha^2}XGG^{\mathrm{T}}X + H_c^{\mathrm{T}}H_c \\ &= A^{\mathrm{T}}X + XA + \frac{1}{\alpha^2}XGG^{\mathrm{T}}X - XBB^{\mathrm{T}}X + H^{\mathrm{T}}H \\ &\quad + (XB + K^{\mathrm{T}})(B^{\mathrm{T}}X + K) = 0. \end{aligned} \tag{11}$$

By the detectability of (A, H), it is easy to see that (A_c, H_c) is a detectable pair. Thus, the proof is complete from Lemma 1. Q.E.D.

Remark 1 The matrix equation with the form of (7) was first introduced by Veillette et al. in [4]. The cause generating the class of matrix equations comes from the restriction of the decentralized control structure. For example, it is impossible in general to pick $K=-B^{\mathrm{T}}X$ and make equation (11) into a simple algebraic Riccati equation. A natural method that can be chosen is to set K equal to $-B_D^{\mathrm{T}}X_D$, which is the main-diagonal blocks of $-B^{\mathrm{T}}X$ when B is block-diagonal. The chosen method of K generates the design equation (7).

Remark 2 In the earlier developed methods of decentralized control for composite systems, the decentralized state feedback controllers are often designed by using the information of subsystems and the conservative estimates for interconnection matrices (see [10]). The design procedure given in Theorem 1 is derived by using the Riccati-like equation approach, which utilizes the information of overall systems. However, it may be difficult to solve the Riccati-like equations for the composite systems with high dimensions. In next section, we shall present a simple design procedure for a class of composite systems with special structure.

3 Composite Systems with Symmetric Circulant Structure

The following preliminaries will be used in the development to follow.

Definition 1[6,7] A matrix $C\in \mathrm{R}^{Nm\times Np}$ is called block circulant if C has the following structure

$$C = \begin{bmatrix} C_1 & C_2 & C_3 & \cdots & C_N \\ C_N & C_1 & C_2 & \cdots & C_{N-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_2 & C_3 & C_4 & \cdots & C_1 \end{bmatrix} \tag{12}$$

where $C_i \in \mathrm{R}^{m\times p}(i = 1,\cdots,N)$. If $C_i = C_{N-i+2}(i = 2,\cdots,N)$, then the matrix C is called block symmetric circulant, and denoted by $\mathrm{scl}[C_1 C_2 \cdots C_N]$.

Denote

$$m_j = [1 \quad v_j \quad v_j^2 \quad \cdots \quad v_j^{N-1}]^{\mathrm{T}}, \quad j = 1,2,\cdots,N \tag{13}$$

where $v_j = \exp(2\pi(j-1)\sqrt{-1}/N)$, $j = 1,2,\cdots,N$, i.e., v_j is a root of the equation $v^N = 1$.

Let

$$R_N = \frac{1}{\sqrt{N}}[r_1 \quad r_2 \quad \cdots \quad r_N], \tag{14}$$

with $r_1 = m_1 = [1 \quad 1 \quad \cdots \quad 1]^{\mathrm{T}}$, $r_{N/2+1} = m_{N/2+1}$ if N is an even number, $r_p = \frac{1}{\sqrt{2}}(m_p + m_{N+2-p})$, $r_{N+2-p} = \frac{\sqrt{-1}}{\sqrt{2}}(m_p - m_{N+2-p})(p = 2,3,\cdots,t)$, where $t = \frac{N+1}{2}$ if N is odd and $t = \frac{N}{2}$ if N is even.

Then R_N is a real orthogonal matrix, and the following result holds.

Lemma 2[7] Let $C = \mathrm{scl}[C_1 \quad C_2 \quad \cdots \quad C_N]$ with $C_i \in \mathrm{R}^{m\times p}(i = 1,\cdots,N)$. Then $C_d = (R_N \otimes I_m)^{\mathrm{T}} C (R_N \otimes I_p) = \mathrm{diag}[C_{d1} \quad C_{d2} \quad \cdots \quad C_{dN}]$ is a block diagonal matrix, and $C_{di} = C_{d(N+2-i)}(i = 2,\cdots,t)$, where $\otimes$ denotes the Kronecker product, and I_q denotes a $q\times q$ identity matrix.

Consider a composite system Σ_s composed of N similar subsystems described by the following composite equations:

$$\dot{x}_s = A_s x_s + B_s u_s + G_s w_s, \tag{15}$$

$$z_s = \begin{bmatrix} H_s x_s \\ u_s \end{bmatrix}, \tag{16}$$

where $x_s = [x_{s1}^{\mathrm{T}} \quad x_{s2}^{\mathrm{T}} \quad \cdots \quad x_{sN}^{\mathrm{T}}]^{\mathrm{T}}$, $u_s = [u_{s1}^{\mathrm{T}} \quad u_{s2}^{\mathrm{T}} \quad \cdots \quad u_{sN}^{\mathrm{T}}]^{\mathrm{T}}$, $x_{si} \in \mathrm{R}^n$ and $u_{si} \in \mathrm{R}^m$ are the state and the input of the ith subsystem, respectively, $w_s \in \mathrm{R}^k$ and $z_s \in \mathrm{R}^{p+m}$ are square-integrable disturbance and an output to be regulated. The composite matrices A_s, B_s, G_s and H_s are block symmetric circulant, and given by

$$A_s = \mathrm{scl}[A_{s1} \quad A_{s2} \quad \cdots \quad A_{sN}], \quad B_s = \mathrm{scl}[B_{s1} \quad B_{s2} \quad \cdots \quad B_{sN}], \tag{17}$$

$$G_s = \mathrm{scl}[G_{s1} \quad G_{s2} \quad \cdots \quad G_{sN}], \quad H_s = \mathrm{scl}[H_{s1} \quad H_{s2} \quad \cdots \quad H_{sN}] \tag{18}$$

with $A_{si} \in \mathrm{R}^{n\times n}$, $B_{si} \in \mathrm{R}^{n\times m}$, $G_{si} \in \mathrm{R}^{n\times k}$, and $H_{si} \in \mathrm{R}^{p\times n}$, $(i = 1,\cdots,N)$.

We shall hereafter refer to this system as a composite system with symmetric circulant structure.

Remark 3 The composite systems with symmetric circulant structure constitute an important class of large-scale systems. Some examples and references are referred to [6,7].

Denote

$$A_{sd} = (R_N \otimes I_n)^T A_s (R_N \otimes I_n) = \mathrm{diag}[A_{sd1} \quad A_{sd2} \quad \cdots \quad A_{sdN}], \tag{19}$$

$$B_{sd} = (R_N \otimes I_n)^T B_s (R_N \otimes I_m) = \mathrm{diag}[B_{sd1} \quad B_{sd2} \quad \cdots \quad B_{sdN}], \tag{20}$$

$$G_{sd} = (R_N \otimes I_n)^T G_s (R_N \otimes I_k) = \mathrm{diag}[G_{sd1} \quad G_{sd2} \quad \cdots \quad G_{sdN}], \tag{21}$$

$$H_{sd} = (R_N \otimes I_p)^T H_s (R_N \otimes I_n) = \mathrm{diag}[H_{sd1} \quad H_{sd2} \quad \cdots \quad H_{sdN}]. \tag{22}$$

Then, we have

Theorem 2 Let $(A_{sdi}, H_{sdi})(i = 1, \cdots, t)$ be detectable, α be a positive constant. If the following Riccati-like algebraic equation

$$A_{sdi}^T P_i + P_i A_{sdi} + \frac{1}{\alpha^2} P_i G_{sdi} G_{sdi}^T P_i - P_i B_{sdi} B_{sdi}^T P_i + H_{sdi}^T H_{sdi}$$
$$+ (P_i B_{sdi} - P_0 B_1)(B_{sdi}^T P_i - B_1^T P_0) = 0, \quad i = 1, \cdots, t \tag{23}$$

have positive semidefinite solutions $P_i (i = 1, \cdots, t)$, where $P_0 = (P_1 + 2P_2 + \cdots + 2P_{\frac{N+1}{2}})/N$ if N is odd, $P_0 = (P_1 + 2P_2 + \cdots + 2P_{\frac{N}{2}} + P_{\frac{N}{2}+1})/N$ if N is even, Then, the decentralized state feedback control law

$$u_{si} = - B_1^T P_0 x_{si}, \quad i = 1, \cdots, N \tag{24}$$

stabilizes the system Σ_s of (15) and (16), and the closed-loop transfer function matrix

$$T_s(s) = \begin{bmatrix} H_s \\ K_s \end{bmatrix} (sI - A - BK_s)^{-1} G_s \text{ satisfies } \| T_s \|_\infty \leqslant \alpha,$$

where $\quad K_s = \mathrm{diag}[-B_1^T P_0 \quad -B_1^T P_0 \quad \cdots \quad -B_1^T P_0].$

The following lemma is required in the proof of Theorem 2.

Lemma 3 The pair (A_s, H_s) is detectable if and only if the pairs $(A_{sdi}, H_{sdi})(i=1, \cdots, t)$ are detectable.

Proof From the results in [11], the pair (A_s, H_s) is detectable if and only if the set

$$F_s = \{\lambda : \mathrm{rank}[\lambda I - A_s^T \quad H_s^T] < Nn, \quad \lambda \text{ is a complex number } \} \subset C_-.$$

By equations (19) and (20)

$$(R_N \otimes I_n)^T [\lambda I - A_s^T \quad H_s^T] \mathrm{diag}[R_N \otimes I_n \quad R_N \otimes I_p]$$
$$= [(R_N \otimes I_n)^T (\lambda I - A_s^T)(R_N \otimes I_n) \quad (R_N \otimes I_n)^T H_s^T (R_N \otimes I_p)]$$
$$= \mathrm{diag}[V_{d1}(\lambda) \quad V_{d2}(\lambda) \quad \cdots \quad V_{dN}(\lambda)],$$

where $V_{di}(\lambda) = [\lambda I_n - A_{sdi}^T \quad H_{sdi}^T], i = 1, \cdots, N$. From Lemma 2,

$$F_s = F_{s1} \cup F_{s2} \cup \cdots \cup F_{sN} = F_{s1} \cup F_{s2} \cup \cdots \cup F_{st},$$

where $F_{si} = \{\lambda : \mathrm{rank}\ V_{di}(\lambda) < n, \lambda \text{ is a complex number}\}$, $i = 1, \cdots, N$. It further follows that $F_\lambda \subset C^-$ if and only if $F_{si} \subset C^-$ $(i = 1, \cdots, t)$. Thus, the proof of the lemma is complete.

Proof of Theorem 2 Let

$$P_{N+2-i} = P_i, \quad i = 1, \cdots, t, \tag{25}$$

$$P = (R_N \otimes I_n) \mathrm{diag}[P_1 \quad P_2 \quad \cdots \quad P_N](R_N \otimes I_n)^T = \mathrm{scl}[P_{01} \quad P_{02} \quad \cdots \quad P_{0N}]. \tag{26}$$

By computing directly, we have

$$P_{01} = P_0. \tag{27}$$

Denote

$$P_D = \mathrm{diag}[P_{01} \ \cdots \ P_{01}], \quad B_{sD} = \mathrm{diag}[B_1 \cdots B_1], \tag{28}$$

$$\begin{aligned} Y(A_s,P) = A_s^{\mathrm{T}}P + PA_s + \frac{1}{\alpha^2}PG_sG_s^{\mathrm{T}}P - PB_sB_s^{\mathrm{T}}P + H_s^{\mathrm{T}}H_s \\ + (PB_s - P_DB_{sD})(B_s^{\mathrm{T}}P - B_{sD}^{\mathrm{T}}P_D). \end{aligned} \tag{29}$$

By Theorem 1, it is sufficient to show that $Y(A_s,P)=0$. Since the matrix $R_N \otimes I_n$ is an orthogonal matrix, it follows from equations (25), (26), (27) and (28) that

$$\begin{aligned} R_{N_n}^{\mathrm{T}}Y(A_s,P)R_{N_n} &= R_{N_n}^{\mathrm{T}}A_s^{\mathrm{T}}R_{N_n}R_{N_n}^{\mathrm{T}}PR_{N_n} + R_{N_n}^{\mathrm{T}}PR_{N_n}R_{N_n}^{\mathrm{T}}A_sR_{N_n} \\ &+ \frac{1}{\alpha^2}R_{N_n}^{\mathrm{T}}PR_{N_n}R_{N_n}^{\mathrm{T}}G_sR_{N_k}R_{N_k}^{\mathrm{T}}G_s^{\mathrm{T}}R_{N_n}R_{N_n}^{\mathrm{T}}PR_{N_n} \\ &- R_{N_n}^{\mathrm{T}}PR_{N_n}R_{N_n}^{\mathrm{T}}B_sR_{N_m}R_{N_m}^{\mathrm{T}}B_s^{\mathrm{T}}R_{N_n}R_{N_n}^{\mathrm{T}}PR_{N_n} + R_{N_n}^{\mathrm{T}}H_s^{\mathrm{T}}R_{N_p}R_{NP}H_sR_{N_n} \\ &+ (R_{N_n}^{\mathrm{T}}PB_sR_{N_n} - P_DB_{sD})(R_{N_n}^{\mathrm{T}}B_s^{\mathrm{T}}PR_{N_n} - B_{sD}^{\mathrm{T}}P_D) \\ &= \mathrm{diag}[\Delta_1 \ \ \Delta_2 \ \ \cdots \ \ \Delta_N] \end{aligned} \tag{30}$$

where R_{N_q} denotes the matrix $R_N \otimes I_q$,

$$\begin{aligned} \Delta_i = A_{sdi}^{\mathrm{T}}P_i + P_iA_{sdi} + \frac{1}{\alpha^2}P_iG_{sdi}G_{sdi}^{\mathrm{T}}P_i - P_iB_{sdi}B_{sdi}^{\mathrm{T}}P_i \\ + H_{sdi}^{\mathrm{T}}H_{sdi} + (P_iB_{sdi} - P_0B_1)(B_{sdi}^{\mathrm{T}}P_i - B_1^{\mathrm{T}}P_0) = 0, \quad i = 1,\cdots,N. \end{aligned}$$

By Lemma 2 and equation (25), we know that $\Delta_i = \Delta_{N+2-i} (i = 2,\cdots,t)$. Thus, from equations (23), (27) and (30), it follows that $Y(A_s,P) = 0$. Q. E. D.

Remark 4 From Theorem 2, it is easy to see that a decentralized state feedback control law for the system Σ_s of (15) and (16) can be conducted by the solutions of the matrix equations (23) with a simple form. If the system Σ_s of (15) and (16) is a symmetric composite system discussed in [8], i. e., $A_{si} = A_{s2}, B_{si} = 0, G_{si} = 0, H_{si} = 0 (i = 2,\cdots,N)$ in equations (17), (18), then the matrix equations (23) will be with a simpler form, the details are omitted.

4 Simultaneous H_∞-Control for a Collection of Systems

Consider a collection S_c of linear systems described by the state equations:

$$\dot{x}_c = A_ix_c + B_iu_c + G_iw_c, \tag{31}$$

$$z_c = \begin{bmatrix} H_ix_c \\ u_c \end{bmatrix}, \quad i = 1,\cdots,q, \tag{32}$$

where $x_c \in \mathrm{R}^n$ and $u_c \in \mathrm{R}^m$ are the state and the input, respectively, $w_c \in \mathrm{R}^k$ and $z_c \in \mathrm{R}^{p+m}$ are square-integrable disturbance and an output to be regulated.

The goal is to design a state feedback control law $u_c = K_cx_c$ such that the resulting closed-loop system matrices $A_i + B_iK_c (i = 1,\cdots,q)$ are Hurwitz, and the closed-loop transfer function matrices $T_{ci} = \begin{bmatrix} H_i \\ K_c \end{bmatrix}(sI - A_i - B_iK_c)^{-1}G_i (i = 1,\cdots,q)$, satisfies $\| T_{ci} \|_\infty \leqslant \alpha$ for some prescribed $\alpha > 0$.

Denote

$$A_c = \mathrm{diag}[A_1 \ \cdots \ A_q \ \ A_1 \ \cdots \ A_q],$$

$$B_c = \operatorname{diag}[B_1 \ \cdots \ B_q \ B_1 \ \cdots \ B_q], \tag{33}$$
$$G_c = \operatorname{diag}[G_1 \ \cdots \ G_q \ G_1 \ \cdots \ G_q],$$
$$H_c = \operatorname{diag}[H_1 \ \cdots \ H_q \ H_1 \ \cdots \ H_q]. \tag{34}$$

Let a composite system Σ_{cs} be defined as follows:

$$\dot{x}_{cs} = A_{cs}x_{cs} + B_{cs}u_{cs} + G_{cs}w_{cs}, \tag{35}$$
$$z_{cs} = \begin{bmatrix} H_{cs}x_{cs} \\ u_{cs} \end{bmatrix}, \tag{36}$$

where $x_{cs} = [x_{c1}^{\mathrm{T}} \cdots x_{c2q}^{\mathrm{T}}]^{\mathrm{T}}$, $u_{cs} = [u_{c1}^{\mathrm{T}} \cdots u_{c2q}^{\mathrm{T}}]^{\mathrm{T}}$, and the composite matrices $A_{cs}, B_{cs}, G_{cs}, H_{cs}$ given by

$$A_{cs} = (R_{2q} \otimes I_n)A_c(R_{2q} \otimes I_n)^{\mathrm{T}}, \quad B_{cs} = (R_{2q} \otimes I_n)B_c(R_{2q} \otimes I_m)^{\mathrm{T}}, \tag{37}$$
$$G_{cs} = (R_{2q} \otimes I_n)G_c(R_{2q} \otimes I_n)^{\mathrm{T}}, \quad H_{cs} = (R_{2q} \otimes I_p)H_c(R_{2q} \otimes I_n)^{\mathrm{T}}. \tag{38}$$

Then, we have

Theorem 3 Let $\alpha > 0$ be a constant. Then there exists a state feedback control law $u_c = K_c x_c$ for the collection S_c of systems such that $A_i + B_i K_c$ is Hurwitz, and $\| T_{ci} \|_\infty \leqslant \alpha$ for $i = 1, \cdots, q$, if and only if there exists a decentralized state feedback control law for the composite system Σ_{cs} with the form

$$u_{ci} = K_c x_{ci}, \quad i = 1, \cdots, 2q, \tag{39}$$

such that the resulting closed-loop system is asymptotically stable, and the closed-loop transfer function matrix T_{cs}, from w_{cs} to z_{cs}, satisfies $\| T_{cs} \|_\infty \leqslant \alpha$.

Proof Since the matrix $R_N \otimes I_n$ is orthogonal for any positive integers N and n, and the H∞-norm of a transfer function matrix is invariant under an orthogonal transformation, it follows easily from the definition of the system Σ_{cs} that the above conclusion is true.

Q. E. D.

Remark 5 By Lemma 2, it is easy to see that the system Σ_{cs} is a composite system with symmetric circulant structure. Theorem 3 presents a relation between the collection S_c of linear systems and the composite system Σ_{cs} with symmetric circulant structure.

Combining Theorem 2 and Theorem 3, a design procedure of simultaneous H∞-controllers for the collection S_c of linear systems can be obtained as follows.

Theorem 4 Let $(A_i, H_i)(i = 1, \cdots q)$ be detectable, α be a positive constant. If the following Riccati-like algebraic equation

$$A_i^{\mathrm{T}}P_i + P_iA_i + \frac{1}{\alpha^2}P_iG_iG_i^{\mathrm{T}}P_i - P_iB_iB_i^{\mathrm{T}}P_i + H_i^{\mathrm{T}}H_i$$
$$+ (P_iB_i - P_0B_0)(B_i^{\mathrm{T}}P_i - B_0^{\mathrm{T}}P_0) = 0, \quad i = 1, \cdots, q, \tag{40}$$

have positive semidefinite solutions $P_i (i = 1, \cdots, q)$ where $P_0 = (P_1 + P_2 + \cdots + P_q)/q$ and $B_0 = (B_1 + B_2 + \cdots + B_q)/q$. Then, the state feedback control law

$$u_c = -B_0^{\mathrm{T}}P_0x_c, \tag{41}$$

such that the resulting close-loop system matrices $A_i - B_iB_0^{\mathrm{T}}P_0 (i = 1, \cdots, q)$ are Hurwitz, and $\| T_{ci} \|_\infty \leqslant \alpha (i = 1, \cdots, q)$.

Proof This is immediate from Theorem 2 and Theorem 3, the details are omitted here. Q.E.D.

Remark 6 When the H_∞ performance criterion is not considered, the simultaneous stabilization problem for a collection of linear systems has been treated by several authors, see [12,13]and the references therein. Theorem 4 presents a new design procedure for a collection of linear systems by using the composite system method.

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关于组合系统的分散 H_∞ 控制

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摘要:本文考虑线性组合系统的分散状态反馈 H_∞ 控制问题. 以 Riccati-like 方程半正定解的术语给出了一个使得给定的组合系统稳定并且对干扰抑制保证一个 H_∞ 界限制的分散状态反馈控制设计. 对具有对称循环结构的组合系统,得到一个简单的分散控制设计程序. 作为一个应用,对一族线性系统的同时 H_∞ 控制问题的可解性导出了一个充分条件.

关键词：组合系统；分散控制；Riccati 方程；对称循环结构；同时 H_∞控制器

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