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Elimination of Noise-Induced Biases in IIR Adaptive Filtering*

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Abstract: In order to eliminate the estimation bias induced by colored disturbances in adaptive filtering, a bias-eliminating least-squares adaptive filtering (BELSAF) algorithm is proposed in this paper. The convergence of the BELSAF is proved.

Key words: adaptive filtering; least-squares method; consistent estimation

1 Instroduction

In the adaptive filtering of infinite impulse response model the problem of how to eliminate the noise-induced biases has not yet been well solved when there is little a priori knowledge about the color noise[1]. To this end, a new kind of adaptive filtering algorithm called BELSAF algorithm is proposed in this paper. It has been shown that the BELSAF is convergent and it can yield consistent filtering results even without modelling the noise.

2 Problem Formulation

As well known the IIR adaptive filtering is equivalent to the parameter estimation of a system given by

$$y(k) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}u(k) + v(k) \tag{1}$$

where $A(q^{-1}) = 1 - a_1q^{-1} - \cdots - a_nq^{-n}$, $B(q^{-1}) = b_0 + b_1q^{-1} + \cdots + b_mq^{-m}$, and q^{-1} is the unit delay operator (i. e. $q^{-1}y(k)=y(k-1)$). $u(k)$ and $y(k)$ denote respectively the system input and observed noisy output. $v(k)$ is some disturbance of unspecified character and uncorrelated with $u(k)$.

Define the parameter vector $\theta^{\mathrm{T}} = [a^{\mathrm{T}}, b^{\mathrm{T}}] = [a_1, \cdots, a_n; b_0, b_1, \cdots, b_m]$. Then the problem of IIR adaptive filtering is to estimate the parameter vector θ from the available observed data $\{u(k), y(k)\}_1^N$. Of coures it is preferred that the estimates should converge to their true values as the sample size N tends to infinity.

3 The BELSAF Algorithm

By the same principle as in [2], a stable digital filter $1/F(q^{-1})$ is connected to the identified system at the input terminal, where $1/F(q^{-1})$ is defined as $1/F(q^{-1}) = 1 + f_1q^{-1} + \cdots + f_nq^{-n}$. The following augmented system is thus obtained

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$$y(k) = \frac{q^{-d}\bar{B}(q^{-1})}{A(q^{-1})}\bar{u}(k) + v(k), \tag{2}$$

where

$$\bar{B}(q^{-1}) = F(q^{-1})B(q^{-1}) = \bar{b}_0 + \bar{b}_1 q^{-1} + \cdots + \bar{b}_{\bar{m}} q^{-\bar{m}}, \bar{m} = m + n, \tag{3}$$

$$\bar{u}(k) = \frac{1}{F(q^{-1})}u(k). \tag{4}$$

Rewrite the system (2) in the signal-regressive form as

$$y(k) = \bar{\psi}_k^{\mathrm{T}}\bar{Q}_k + v(k) - v_k^{\mathrm{T}}a \tag{5}$$

where

$$\bar{\theta}^{\mathrm{T}} = [a^{\mathrm{T}}, \bar{b}^{\mathrm{T}}] = [a_1, a_2, \cdots, a_n; \bar{b}_0, \bar{b}_1, \cdots, \bar{b}_{\bar{m}}], \tag{6}$$

$$\bar{\psi}_k^{\mathrm{T}} = [y(k-1), \cdots, y(k-n); \bar{u}(k-d), \cdots, \bar{u}(k-d-m)]. \tag{7}$$

The LS estimate of $\bar{\theta}$ and it's asymptotical property are given by

$$\bar{\theta}_{\mathrm{LS}}(N) = \Big(\frac{1}{N}\sum_{k=1}^{N}\bar{\psi}_k\bar{\psi}_k^{\mathrm{T}}\Big)^{-1}\Big(\frac{1}{N}\sum_{k=1}^{N}\bar{\psi}_k y(k)\Big). \tag{8}$$

$$\lim_{N\to\infty}\bar{\theta}_{\mathrm{LS}}(N) = \bar{\theta} + R_{\bar{\psi}\bar{\psi}}^{-1}\Big(\bar{p}_v - \begin{bmatrix} R_{vv} \\ 0_{(\bar{m}+1)\times n}\end{bmatrix}a\Big), \tag{9}$$

where

$$\bar{p}_v^{\mathrm{T}} = [r_v^{\mathrm{T}}; 0] = [r_{vv}(1), r_{vv}(2), \cdots, r_{vv}(n); 0, \cdots, 0] \in R^{n+\bar{m}+1}, \tag{10}$$

$$R_{\bar{\psi}\bar{\psi}}^{-1} = \lim_{N\to\infty}\Big[\frac{1}{N}\sum_{k=1}^{N}\bar{\psi}_k\bar{\psi}_k^{\mathrm{T}}\Big]^{-1}. \tag{11}$$

Introducing a matrix H as

$$H^{\mathrm{T}} = \left[\begin{array}{c|cccc} & \lambda_1^{\bar{m}} & \cdots & \lambda_1 & 1 \\ 0 & \cdots & \cdots & \cdots & \cdots \\ & \lambda_n^{\bar{m}} & \cdots & \lambda_n & 1 \end{array}\right] \in \mathrm{R}^{n\times(n+\bar{m}+1)}, \tag{12}$$

where $\lambda_i (i=1,2,\cdots,n)$ denote the n zeros of $F(q^{-1})$. Then using Eq. (3) we can get

$$H^{\mathrm{T}}\bar{\theta} = 0. \tag{13}$$

Multiplying H^{T} on both sides of (9) gives

$$H^{\mathrm{T}}\lim_{N\to\infty}\bar{\theta}_{\mathrm{LS}}(N) = H^{\mathrm{T}}R_{\bar{\psi}\bar{\psi}}^{-1}\Big(\bar{p}_v - \begin{bmatrix} R_{vv} \\ 0_{(\bar{m}+1)\times n}\end{bmatrix}a\Big), \tag{14}$$

Next, let us examine $\bar{e}(k)$ which is defined as

$$\bar{e}(k) = y(k) - \bar{\psi}_k^{\mathrm{T}}\bar{\theta}_{\mathrm{LS}}(N). \tag{15}$$

From Eqns. (8) and (15), we get

$$\lim_{N\to\infty}\bar{J}_N(\bar{\theta}_{\mathrm{LS}}) \triangleq \lim_{N\to\infty}\frac{1}{N}\bar{e}(k)^2 = r_{vv}(0) - \lim_{N\to\infty}\bar{a}_{\mathrm{LS}}^{\mathrm{T}}(N)(r_v - R_{vv}a) - \bar{r}_v^{\mathrm{T}}a. \tag{16}$$

It is noticeable that Eqns. (14) and (16) provide the $(n+1)$ linear algebraic equations required for obtaining the $(n+1)$ unknowns $r_{vv}(i), i=0,1,\cdots,n$. Replacing the true value $\bar{\theta}$ with its BELS estimate $\hat{\theta}_{\mathrm{BELS}}(N)$ in Eqns. (14) and (16), we can establish the following recursive scheme for estimating $r_{vv}(i), i=0,1,2,\cdots,n$ at the time instant N.

$$H^{T}R_{\bar{\psi}\bar{\psi}}^{-1}(N)\left(\bar{p}_{v}(N)-\begin{bmatrix}\hat{R}_{vv}(N)\\ 0_{(\bar{m}+1)\times n}\end{bmatrix}\hat{a}_{\mathrm{BELS}}(N-1)\right)=H^{T}\bar{\theta}_{\mathrm{LS}}(N),\tag{17}$$

$$r_{vv}(0)(N)-\bar{\theta}_{\mathrm{LS}}^{T}(N)\left(\bar{p}_{v}(N)-\begin{bmatrix}\hat{R}_{vv}(N)\\ 0_{(\bar{m}+1)\times n}\end{bmatrix}\hat{a}_{\mathrm{BELS}}(N-1)\right)$$

$$-\bar{\theta}_{\mathrm{BELS}}^{T}(N-1)\bar{p}_{v}(N)=\dot{J}_{N}(\bar{\theta}_{\mathrm{LS}}),\tag{18}$$

where $r_{vv}(0)(N),\bar{p}_{v}(N),\hat{R}_{vv}(N),\hat{a}_{\mathrm{BELS}}(N)$ and $\bar{\theta}_{\mathrm{BELS}}(N)$ are the estimates of $r_{vv}(0),\bar{p}_{v}$, R_{vv},a and $\bar{\theta}$ respectively.

Thus we can get the following algorithm, which will be called the Bias-eliminated Least-squares Adaptive Filtering (BELSAF) algorithm.

Algorithm 1

step 1 Design a stable n-th order filter $F(q^{-1})$ (see [2]) and insert it into the system to obtain the corresponding augmented system;

step 2 Initialize the starting value of the BELS method by an arbitrary real vector $\bar{\theta}_{\mathrm{BELS}}(0)$.

step 3 Estimate the parameters of the augmented system via the ordinary LS metheod, which gives $\bar{\theta}_{\mathrm{LS}}(N)$ (see Eqn. (8)).

step 4 Solve the set of equations Eqns. (17) and (18) to determine the estimate $\bar{p}_{v}(N)$ and $\hat{R}_{vv}(N)$, whose elements are the estimated ACFs of the noise.

step 5 Calculate the estimate $\bar{\theta}_{\mathrm{BELS}}(N)$ by performing the bias correction as follows:

$$\bar{\theta}_{\mathrm{BELS}}(N)=\bar{\theta}_{\mathrm{LS}}(N)-R_{\bar{\psi}\bar{\psi}}^{-1}(N)\bar{p}_{v}(N)-\begin{bmatrix}\hat{R}_{vv}(N)\\ 0_{(\bar{m}+1)\times n}\end{bmatrix}\hat{a}_{\mathrm{BELS}}(N-1)).\tag{19}$$

step 6 Repeat step 3~5 until some convergence criterion is satisfied.

step 7 Compute the estimate $\hat{\theta}_{\mathrm{BELS}}(N)$ of the original system parameter vector θ from $\bar{\theta}_{\mathrm{BELS}}(N)$ (see Eqn. (3)).

4 Convergence Analysis

As discussed in [2], it will be sufficient to consider the consistency of $\bar{\theta}_{\mathrm{BELS}}(N)$. Here the technique of random contraction mapping [3] is employed to analyse the convergence property of the BELSAF algorithm. First of all, we will give two useful lemmas, which are easily proven to be true.

Lemma 1 P and D are two symmetric non-negative definite matrices such that $R=P+D$ is positive definite. Let $Q=R^{-1}D$. Then,

1. The eigenvalues of Q are real and all lie in the closed interval $[0,1]$;

2. Q has an eigenvalue equal to 1 if and only if P is singular.

Lemma 2 Suppose that $u(k)$ and $v(k)$ are ergodic random sequences with zero means, and are mutually independent statistically. If the input $u(k)$ is persistently exciting of order$(n+\bar{m}+1)$, then $\bar{\theta}$ can be determined uniquely by the algebraic equation(9). Besides,

$$\lambda_{\max}\left(R_{\bar{\psi}\bar{\psi}}^{-1}\left[\begin{array}{c:c}\hat{R}_{vv} & 0_{n\times(\bar{m}+1)}\\ \hdashline 0_{(\bar{m}+1)\times n} & 0_{(\bar{m}+1)\times(\bar{m}+1)}\end{array}\right]\right)<1, \tag{20}$$

where $\lambda_{\max}(\cdot)$ denotes the maximal eigenvalue of a matrix.

With the above lemmas we can obtain the following theorem to illustrate the convergence property of the BELSAF algorithm.

Theorem 1 When the size of sampled data approaches infinity, the estimated parameter vector $\bar{\theta}_{\mathrm{BELS}}(N)$ obtained from the BELSAF algtorithm is an asymptotically consistent estimate of $\bar{\theta}$ for any initial value $\dot{\bar{\theta}}_{\mathrm{BELS}}(0)$, namely,

$$\lim_{N\to\infty}\bar{\theta}_{\mathrm{BELS}}(0)=\bar{\theta}, \quad \text{w. p. 1.} \tag{21}$$

Proof The technique of random contraction mapping proposed in [3] is adopted here to verify the theorem.

Consider a mapping given by

$$T(x)=x-\left[\left(I-R_{\bar{\psi}\bar{\psi}}^{-1}\left[\begin{array}{c:c}R_{vv} & 0_{n\times(\bar{m}+1)}\\ \hdashline 0_{(\bar{m}+1)\times n} & 0_{(\bar{m}+1)\times(\bar{m}+1)}\end{array}\right]\right)x-\lim_{N\to\infty}\bar{\theta}_{\mathrm{LS}}(N)+R_{\bar{\psi}\bar{\psi}}^{-1}\bar{p}_v\right], \tag{22}$$

where x is an element in the space $\mathrm{R}^{n+\bar{m}+1}$.

Firstly, as pointed out in Lemma 2, Eqn. (9) has a unique solution $\bar{\theta}$. Thus, the mapping $T(x)$ has a unique fixed point, i. e. , there is unique x such that $T(x)=x$.

Secondly, since

$$\| T(x_2)-T(x_1)\| \leqslant R_{\bar{\psi}\bar{\psi}}^{-1}\left[\begin{array}{c:c}R_{vv} & 0_{n\times(\bar{m}+1)}\\ \hdashline 0_{(\bar{m}+1)\times n} & 0_{(\bar{m}+1)\times(\bar{m}+1)}\end{array}\right]\| \;\| x_2-x_1\|, \tag{23}$$

we know by Lemma 2 that the mapping T is contractive

Next it can be seen that the estimated ACFs of the noise computed from Eqn. (17) and (18) are consistent with their true values provided that $\bar{\theta}$ is known

Replacing $\bar{p}_v$, R_{vv}, $\lim_{N\to\infty}\bar{\theta}_{\mathrm{LS}}(N)$ and $R_{\bar{\psi}\bar{\psi}}^{-1}$ by their consistent estimates $\bar{p}_v(N,\bar{\theta})$, $R_{vv}(N,\bar{\theta})$, $\bar{\theta}_{\mathrm{LS}}(N)$ and $R_{\bar{\psi}\bar{\psi}}^{-1}(N)$ respectively, we will obtain a random contraction mapping as follows:

$$T_N(x)=x-\left[\left(I-R_{\bar{\psi}\bar{\psi}}^{-1}(N)\left[\begin{array}{c:c}\hat{R}_{vv}(N,x) & 0_{n\times(\bar{m}+1)}\\ \hdashline 0_{(\bar{m}+1)\times n} & 0_{(\bar{m}+1)\times(\bar{m}+1)}\end{array}\right]\right)x\right.$$
$$\left.-\dot{\bar{\theta}}_{\mathrm{LS}}(N)+R_{\bar{\psi}\bar{\psi}}^{-1}(N)\dot{\bar{p}}_v(N,x)\right]. \tag{24}$$

It has been demonstrated in [3] that the sequence of random vectors $\{\bar{\theta}_{\mathrm{BELS}}(N)\}$ generated by the algorithm

$$\dot{\bar{\theta}}_{\mathrm{BELS}}(N)=T_N(\dot{\bar{\theta}}_{\mathrm{BELS}}(N-1)), \tag{25}$$

with an arbitrary initial value $\bar{\theta}_{BELS}(0)$, converges to $\bar{\theta}$ in Eqn. (9) as N approaches infinity.

Substituting Eqn. (24) into Eqn. (25) lead to the scheme of Eqn. (19), then the conclusion of the theorem follows. □

5 Simulation Example

An example is given in this section to illustrate the performance of the proposed BELSAF algorithm.

Consider a fourth-order system where $A(q^{-1})=1+1.3q^{-1}-0.22q^{-2}-0.832q^{-3}-0.289q^{-4}$, and $B(q^{-1})=1$. Here the noise is assumed to be a colored noise simulated by

$$v(k)=\frac{1.0+1.5q^{-1}+0.75q^{-2}}{1.0-0.9q^{-1}+0.95q^{-2}}e(k).$$

$u(k)$ is a pseudorandom binary signal of unit magnitude. $e(k)$ is zero mean white noise with a variance such that the SNR is 10. The prefilter is designed to be $F(q^{-1})=(1-0.5q^{-1})(1-0.6q^{-1})(1-0.75q^{-1})(1-0.9q^{-1})$.

The BELSAF algorithm is applied ten times. For each test the sampled data size N changes from 200 to 500 and to 1500. ε is take to be 0.001. Table. 1 lists the mean values and standard deviations of the estimated parameters, from which we can see that the BELSAF algorithm has a good convergence rate and estimation accuracy.

Table 1 Simulation result

N	a_1	a_2	a_3	a_4	b_0
200	1.010±0.087	−0.247±0.043	−0.927±0.078	−0.248±0.047	1.073±0.032
500	1.109±0.044	−0.235±0.032	−0.892±0.065	−0.257±0.034	1.039±0.029
1500	1.211±0.027	−0.233±0.021	−0.874±0.33	−0.263±0.028	1.051±0.017
true value	1.3	−0.22	−0.83	−0.289	1.0

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消除噪声引起的自适应滤波偏差

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摘要:在自适应滤波中,如何消除有色噪声引起的估计偏差一直是人们关心的重要问题.为了解决此问题,本文提出了一种偏差补偿的自适应滤波算法(BELSAF).理论分析和仿真实验表明,本文所提出的算法是收敛的,并且它可在不对噪声建模的情况下去除噪声对滤波的影响.

关键词:自适应滤波;最小二乘法;一致估计

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