Design of Global Robust Self-Tuning Controller for Multi-Input Uncertain Nonlinear Systems

LU Souyin, JING Yuanwei and LIN Xiaoping
(Department of Automatic Control, Northeastern University · Shenyang, 110006, PRC)

Abstract: In this paper we consider the robust stabilization problems of multi-input nonlinear systems with the time-varying uncertainties shich enter linearly, and give the robust stabilizing self-tuning controller.

Key words: robust stabilization: uncertain nonlinear systems; robust controller

1 Introduction

Consider uncertain nonlinear system:

$$\begin{cases} \Sigma_j^1(1,1): \dot z_j^1 = z_j^2 + \theta^{\mathrm T}(t)\phi_j^1(\xi_1^1), \\ \qquad\cdots \\ \Sigma_j^{t_1^1}(1,1): \dot z_j^{t_1^1} = z_j^{t_1^1+1} + \theta^{\mathrm T}(t)\phi_j^{t_1^1}(\xi_1^{t_1^1}), \end{cases}$$

$$\begin{cases} \Sigma_j^{t_1^1+1}(2,1): \dot z_j^{t_1^1+1} = z_j^{t_1^1+2} + \theta^{\mathrm T}(t)\phi_j^{t_1^1+1}(\xi_1^{t_1^1+1}, \xi_2^1), \\ \qquad\cdots \\ \Sigma_j^{t_1^2}(2,1): \dot z_j^{t_1^2} = z_j^{t_1^2+1} + \theta^{\mathrm T}(t)\phi_j^{t_1^2+1}(\xi^{t_1^2}, \xi_2^{t_2^2}), \end{cases}$$

$$\cdots,$$

$$\begin{cases} \Sigma_j^{t_1^{s-1}+1}(s,1): \dot z_j^{t_1^{s-1}+1} = z_j^{t_1^{s-1}+2} + \theta^{\mathrm T}(t)\phi_j^{t_1^{s-1}+1}(\xi_1^{t_1^{s-1}+1}, \cdots, \xi_{s-1}^{t_{s-1}^{s-1}+1}, \xi_s^1), \\ \qquad\cdots \\ \Sigma_j^{t_1^s}(s,1): \dot z_j^{t_1^s} = v_j + \theta^{\mathrm T}(t)\phi_j^{t_1^s+1}(\xi_1^{t_1^s}, \cdots, \xi_{s-1}^{t_{s-1}^s}, \xi_s^{t_s^s}), \end{cases}$$

$$j = 1, \cdots, r_1;$$

$$\begin{cases} \Sigma_j^1(2,2): \dot z_j^1 = z_j^2 + \theta^{\mathrm T}(t)\phi_j^1(\xi_1^{t_1^1+1}, \xi_2^1), \\ \qquad\cdots \\ \Sigma_j^{t_2^2}(2,2): \dot z_j^{t_2^2} = z_j^{t_2^2+1} + \theta^{\mathrm T}(t)\phi_j^{t_2^2}(\xi_1^{t_1^2}, \xi_2^{t_2^2}), \end{cases}$$

$$\begin{cases} \Sigma_j^{t_2^2+1}(3,2): \dot z_j^{t_2^2+1} = z_j^{t_2^2+2} + \theta^{\mathrm T}(t)\phi_j^{t_2^2+1}(\xi_1^{t_1^2+1}, \xi_2^{t_2^2+1}, z_{r_2+1}^1, \xi_3^1), \\ \qquad\cdots, \\ \Sigma_j^{t_2^3}(3,2): \dot z_j^{t_2^3} = z_j^{t_2^3+1} + \theta^{\mathrm T}(t)\phi_1^{t_2^3}(\xi_1^{t_1^3}, \xi_2^{t_2^3}, \xi_3^{t_3^3}), \end{cases}$$

Manuscript received Supt. 26, 1994, revised Apr. 30, 1996.

$$\cdots,$$

$$\begin{cases}\Sigma_j^{t_2^{s-1}+1}(s,2): \dot{z}_j^{t_2^{s-1}+1} = z_j^{t_2^{s-1}+2} + \theta^{\mathrm{T}}(t)\phi_j^{t_2^{s-1}+1}(\xi_1^{t_1^{s-1}+1}, \xi_2^{t_2^{s-1}+1}, \cdots, \xi_{s-1}^{t_{s-1}^{s-1}+1}, \xi_s^1), \\ \Sigma_j^{t_2^s}(s,2): \dot{z}_j^{t_2^s} = v_j + \theta^{\mathrm{T}}(t)\phi_j^{t_2^s}(\xi_1^{t_1^s}, \xi_2^{t_2^s}, \cdots, \xi_{s-1}^{t_{s-1}^s}, \xi_s^{t_s^s}),\end{cases}$$

$$j = r_1 + 1, \cdots, r_2;$$

$$\begin{cases}\Sigma_j^1(s,s): \dot{z}_j^1 = z_j^2 + \theta^{\mathrm{T}}(t)\phi_{1j}(\xi_1^{t_1^{s-1}+1}, \xi_2^{t_1^{s-1}+1}, \cdots, \xi_{s-1}^{t_{s-1}^{s-1}+1}, \xi_s^1), \\ \Sigma_j^{t_s^s}(s,s): \dot{z}_j^{t_s^s} = v_j + \theta^{\mathrm{T}}(t)\phi_j^{t_s^s}(\xi_1^{t_1^s}, \cdots, \xi_{s-1}^{t_{s-1}^s}, \xi_s^{t_s^s}),\end{cases}$$

$$j = r_{s-1} + 1, \cdots, r_s \tag{1}$$

where $\xi_j^1 = (z_{r_{j-1+1}}^1, \cdots, z_{r_j}^1)^{\mathrm{T}}$, $\xi_j^i = (z_{r_{j-1+1}}^1, \cdots, z_{r_j}^1, \cdots, z_{r_{j-1+1}}^i, \cdots, z_{r_j+1}^i)^{\mathrm{T}}$, $i = 2, 3, \cdots, t_j^1, t_j^1 + 1, \cdots, t_j^s$; $t_j^i = t_j^{i-1} + t_j^i$, $j \leqslant i = 1, \cdots, s$, $t_i^i = \rho_{r_i} - \rho_{r_{i+1}}$, $i = 1, \cdots, s-1$, $\rho_1 = \rho_2 = \cdots = \rho_{r_1} > \rho_{r_1+1} = \cdots = \rho_{r_2} > \cdots > \rho_{r_s-1+1} = \cdots = \rho_{r_s}$, $r_s = m$; $\rho_1 + \rho_2 + \cdots + \rho_{r_s} = n$. $r_0 = 0$, $t_j^i = 0$, $j > i = 1, \cdots, s$, $z = (\xi_1^{t_1^s}, \cdots, \xi_s^{t_s^s})^{\mathrm{T}} \in \mathbb{R}^n$, ϕ_j^i is smooth vector field, $\theta(t)$ is a vector of unknown, time-varying, piecewise continuous parameters or disturbances which take values in the compact set $\Omega \subset \mathbb{R}^p$.

Necesary and sufficient condtions for bringing a general uncertain nonlinear systems into the form (1) via a nonlinear coordinate change are given in [5], and which of the single-input case is given in [2].

Within the last few years, there has been an increasing interest in the problem of the robust control[1]. In the special case of time-varying disturbance or parameter, few results are available ([2]) for stabilizing the uncertain nonlinear systems. Marino and Tomei[2] studied the robust stabilization problem of feedback linearizable time-varying uncertain single-input nonlinear systems [$\dot{x} = f(x) + q(x, \theta(t)) + g(x)u$]. They give a local (global) robust stabilizing state feedback controller under three assumptions: 1) the nominal system (f, g) is local (global) feedback linearizable; 2) the uncertain vector $q(x, \theta(t))$ satisfies coordinate-free triangularity condition; 3) the unknown vector θ takes the values in a known compact set. But they only discussed the feedback linearizable uncertain nonlinear systems with signle-input.

Now the problem in question is to design a smooth state feedback controller, so that the closed-loop system is globally asymptotically stable at the origin for every $\theta(t)$, that is, the global robust stabilization problem.

2 Main Results

In this section we shall discuss the robust stabilization of the multi-input nonlinear system. First we give the following lemma:

Lemma 1 For the system $\Sigma_j^1(1,1), \cdots, \Sigma_j^{t_1^1}(1,1)$ (defined as (1), $j = 1, \cdots, r_1$ with inputs $v_j = z_j^{t_1^1+1}$. if Ω is a (unkown) compact set, then there exists a global state feedback self-tuning controller such that the closed loop system is globally asymptotically stable at the origin for any $\theta \in \Omega$.

Proof First let us construct the Lyapunov function of each subsystem $\Sigma_j^h(1,1)$.

Step 1 (1, 1) Let $\bar{z}_j^1 = z_j^1$ and define the Lyapunov function $V_j^1(1,1) = \frac{1}{2}(\bar{z}_j^1)^2 + \frac{1}{2}(\bar{\mu}_j^1)^2$, where $\bar{\mu}_j^1 = \mu_j^1 - \hat{\mu}_j^1$, μ_j^1 is a suitable constant, $\hat{\mu}_j^1$ is a function of $\bar{\xi}^1$ yet to be determined, $\bar{\xi}^1 = (\bar{z}_1^1, \cdots, \bar{z}_{r_1}^1)^{\mathrm{T}}$. Then the time derivative of the Lyapunov function is

$$\dot{V}_j^1(1,1) = z_j^1(z_j^2 - z_j^{2*}) + z_j^1(z_j^{2*} + \sum_{i=1}^{p} \theta_i \phi_{j,i}^1) - \dot{\hat{\mu}}_j^1(\mu_j^1 - \hat{\mu}_j^1).$$

Since $\phi_{j.i}^1$ is a smooth function of $\bar{\xi}^1$ and $\phi_{j.i}^1(0, \cdots, 0) = 0$, one can write ([3]):

$$\phi_{j.i}^1 = \sum_{k=1}^{r_1} \bar{z}_k^1 \psi_{j.i}^{1.k}(\bar{z}_1^1, \cdots, \bar{z}_{r_1}^1).$$

where $\psi_{j.i}^{1.k}$ is a continuous function, however, there exists a smooth function $\alpha_j^1(\bar{z}_1^1, \cdots, \bar{z}_{r_1}^1)$ such that

$$|\psi_{j.i}^{1.k}| \leqslant \frac{1}{r_1}\alpha_j^1.$$

In addition, since $\theta \in \Omega$, a compact set, there exists a suitable (may be unknow) positive real μ_1^1 such that

$$|\theta_i| \leqslant \sqrt{\mu_j^1}/p.$$

Therefore, one can obtain

$$\left|\sum_{i=1}^{p} \theta_i \phi_{j.i}^1\right| \leqslant \sum_{i=1}^{p} \theta_i \sum_{k=1}^{r_1} |\bar{z}_k^1| \, |\psi_{j.i}^{1.k}| \leqslant \|\bar{\xi}^1\| \alpha_j^1 \sqrt{\mu_j^1}.$$

By letting

$$\begin{aligned} &\bar{z}_j^2 = z_j^2 - z_j^{2*}, \quad z_j^{2*} = k_j^1 \bar{z}_j^1 - \frac{1}{4}\bar{z}_j^1(\alpha_j^1)^2 \hat{\mu}_j^1, \\ &\dot{\hat{\mu}}_j^1 = \frac{1}{4}(\bar{z}_j^1)^2(\alpha_j^1)^2, \quad j = 1, \cdots, r_1 \end{aligned} \tag{2}$$

one can get

$$\dot{V}_j^1(1,1) \leqslant \bar{z}_j^1 \bar{z}_j^2 - k_j^1(\bar{z}_j^1)^2 + \sum_{i=1}^{r_1} (\bar{z}_i^1)^2, \quad j = 1, \cdots, r_1. \tag{3}$$

Step 2(1, 1) Define the Lyapunov function $V_j^2(1,1) = \frac{1}{2}(\bar{z}_j^2)^2 + \frac{1}{2}(\bar{\mu}_j^2)^2$, where $\bar{\mu}_j^2 = \mu_j^2 - \hat{\mu}_j^2$, μ_j^2 is a suitable constant, $\hat{\mu}_j^2$ is a function of $\bar{\xi}^2$ yet to be determined, $\bar{\xi}^2 = (\bar{z}_1^1, \cdots, \bar{z}_{r_1}^1, \bar{z}_1^2, \cdots, \bar{z}_{r_1}^2)^{\mathrm{T}}$. Then the time derivative of the Lyapunov function is

$$\begin{aligned} \dot{V}_j^2(1,1) = {} & \bar{z}_j^2(z_j^3 - z_j^{3*}) + \bar{z}_j^2[\bar{z}_j^{3*} + \sum_{i=1}^{p} \theta_i \phi_{j.i}^1] - \sum_{j=1}^{1} \frac{\partial z_j^{2*}}{\partial \bar{z}_j^1} z_j^2 \\ & - \sum_{j=1}^{r_1} \frac{\partial z_j^{2*}}{\partial \bar{z}_j^1} \sum_{i=1}^{p} \theta_i \phi_{j.i}^1 + \sum_{j=1}^{r_1} \frac{\partial z_j^{2*}}{\partial \hat{\mu}_j^1} \dot{\hat{\mu}}_j^1] - \dot{\hat{\mu}}_j^2(\mu_j^2 - \hat{\mu}_j^2). \end{aligned}$$

Since $\phi_{j.i}^2$ is a smooth function of $\bar{\xi}^2$ and $\phi_{j.i}^2(0, \cdots, 0) = 0$ thus one can write ([3]):

$$\phi_{j.i}^2 = \sum_{j=1}^{r_1} \sum_{k=1}^{2} \bar{z}_j^k \psi_{j.i}^{2.k}(\bar{\xi}^2),$$

$$\sum_{i=1}^{p}\theta_i\phi_{j.i}^{2}-\sum_{j=1}^{r_1}\frac{\partial z_j^{2*}}{\partial \bar{z}_j^{1}}\sum_{i=1}^{p}\theta_i\phi_{j.i}^{1}=\sum_{i=1}^{p}\theta_i\sum_{j=1}^{r_1}\sum_{k=1}^{2}\bar{z}_j^{k}\psi_{j.i}^{2,k}(\bar{\xi}^2).$$

where $\psi_{j.i}^{2,k}$ is a continuous function, As $\theta\in\Omega$, there exist smooth function $\alpha_{j.i}^{2}(\bar{\xi}^2)$ such that

$$|\psi_{j.i}^{2,k}|\leqslant\frac{1}{2r_1}\alpha_j^2$$

and a suitable (unknown) constant μ_j^2 such that

$$|\theta_i|\leqslant\frac{\sqrt{\mu_j^2}}{p},\quad i=1,\cdots,p.$$

Therefore, one can obtain

$$\left|\sum_{i=1}^{p}\theta_i\phi_{j.i}^{2}-\sum_{j=1}^{r_1}\frac{\partial z_j^{2*}}{\partial \bar{z}_j^{1}}\sum_{i=1}^{p}\theta_i\phi_{j.i}^{1}\right|\leqslant\|\bar{\xi}^2\|\,\alpha_j^2\sqrt{\mu_j^2}.$$

By letting $\bar{z}_j^3=z_j^3-z_j^{3*}$,

$$z_j^{3*}=-\bar{z}_j^1-k_j^2\bar{z}_j^2-\frac{1}{4}\bar{z}_j^2(\alpha_j^2)^2\hat{\mu}_j^2+\sum_{j=1}^{r_1}\frac{\partial z_j^{2*}}{\partial\bar{\mu}_j^1}\dot{\hat{\mu}}_j^1+\sum_{j=1}^{r_1}\frac{\partial z_j^{2*}}{\partial\bar{z}_j^1}\bar{z}_j^2,$$

$$\dot{\hat{\mu}}_j^2=\frac{1}{4}(\bar{z}_j^2)^2(\alpha_j^2)^2,\quad j=1,\cdots,r_1 \tag{4}$$

one can get

$$\dot{V}_j^2(1,1)\leqslant\bar{z}_j^2\bar{z}_j^3-\bar{z}_j^1\bar{z}_j^2-k_j^2(\bar{z}_j^2)^2+\sum_{i=1}^{r_1}\sum_{k=1}^{2}(\bar{z}_i^k)^2,\quad j=1,\cdots,r_1. \tag{5}$$

Step $t_1^1(1,1)$ Similarly to Step 2(1, 1), assume that we have defined the Lyapunov functions $V_j^h(1,1)$, $h=1,\cdots,t_1^1-1$, $j=1,\cdots,r_1$, and proved that

$$\dot{V}_j^h(1,1)\leqslant\bar{z}_j^h\bar{z}_j^{h+1}-\bar{z}_j^{h-1}\bar{z}_j^h-k_j^h(\bar{z}_j^h)^2+\sum_{i=1}^{r_1}\sum_{k=1}^{h}(\bar{z}_i^k)^2,\ j=1,\cdots,\tau_1,\ h=1,\cdots,t_1^1-1 \tag{6}$$

where $\bar{z}_j^0=0$, $\bar{z}_j^1=z_j^1$, and

$$\bar{z}_j^{h+1}=z_j^{h+1}-z_j^{(h+1)*},$$

$$z_j^{(h+1)*}=-\bar{z}_j^{(h-1)}-k_j^h\bar{z}_j^h-\frac{1}{4}\bar{z}_j^h(\alpha_j^h)^2\hat{\mu}_j^h+\sum_{j=1}^{r_1}\sum_{\tau=1}^{h-1}\frac{\partial z_j^{h*}}{\partial\bar{\mu}_j^\tau}\dot{\hat{\mu}}_j^\tau+\sum_{j=1}^{r_1}\sum_{\tau=1}^{h-1}\frac{\partial z_j^{h*}}{\partial\bar{z}_j^\tau}\bar{z}_j^{\tau+1},$$

$$\dot{\hat{\mu}}_j^h=\frac{1}{4}(\bar{z}_j^h)^2(\alpha_j^h)^2,\quad j=1,\cdots,r_1,\quad h=1,\cdots,t_1^1-1. \tag{7}$$

Now define the Lyapunov function $V_j^{t_1^1}(1,1)=\frac{1}{2}(\bar{z}_j^{t_1^1})^2+\frac{1}{2}(\bar{\mu}_j^{t_1^1})^2$, where $\bar{\mu}_j^{t_1^1}=\mu_j^{t_1^1}-\hat{\mu}_j^{t_1^1}$, then the time derivative of the Lyapunov function

$$\dot{V}_j^{t_1^1}(1,1)=\bar{z}_j^{t_1^1}\dot{\bar{z}}_j^{t_1^1}-\dot{\hat{\mu}}_j^{t_1^1}(\mu_j^{t_1^1}-\hat{\mu}_j^{t_1^1})$$

$$=\bar{z}_j^{t_1^1}(z_j^{t_1^1+1}-z_j^{(t_1^1+1)*})+\bar{z}_j^{t_1^1}\Big[z_j^{(t_1^1+1)*}+\sum_{i=1}^{p}\theta_i\phi_{j.j}^{t_1^1}-\sum_{j=1}^{r_1}\sum_{\tau=1}^{t_1^1-1}\frac{\partial z_j^{t_1^1*}}{\partial\bar{z}_j^\tau}\sum_{i=1}^{p}\theta_i\phi_{j.i}^{t_1^1-1}$$

$$-\sum_{j=1}^{r_1}\sum_{\tau=1}^{t_1^1-1}\frac{\partial z_j^{t_1^1*}}{\partial\bar{\mu}_j^\tau}\bar{z}_j^{\tau+1}-\sum_{j=1}^{r_1}\sum_{\tau=1}^{t_1^1-1}\frac{\partial z_j^{t_1^1*}}{\partial\hat{\mu}_j^\tau}\dot{\hat{\mu}}_j^\tau\Big]-\dot{\mu}_j^{t_1^1}(\mu_j^{t_1^1}-\hat{\mu}_j^{t_1^1}).$$

Since $\phi_{j.i}^{t_1^1}$ is a smooth function, $\phi_{j.i}^{t_1^1}(0,\cdots,0)=0$, and (2), (4) and (7) hold, one can write

([3]):

$$\sum_{i=1}^{p}\theta_i\phi_{j,i}^{t_1^1} - \sum_{j=1}^{r_1}\sum_{\tau=1}^{t_1^1-1}\frac{\partial z_j^{t_1^1 *}}{\partial \bar{z}_j^{\tau}}\sum_{i=1}^{p}\theta_i\phi_{j,i}^{t_1^1-1} = \sum_{j=1}^{r_1}\sum_{k=1}^{t_1^1}\bar{z}_j^{k}\psi_{j,i}^{t_1^1,i}(\bar{\xi}^{t_1^1}),$$

where $\phi_{j,i}^{t_1^1,i}$ is a continuous function. Then there exists a smooth function $\alpha_j^{t_1^1}(\xi^{t_1^1})$ such that

$$|\psi_j^{k,i}| \leqslant \frac{1}{t_1^1\times r_1}\alpha_j^{t_1^1},$$

where $\bar{\xi}^{t_1^1} = (\bar{z}_1^1,\cdots,\bar{z}_{r_1}^1,\cdots,\bar{z}_1^{t_1^1},\cdots,\bar{z}_{r_1}^{t_1^1})^{\mathrm{T}}$. As $\theta\in\Omega$, one can get a suitable (unknown) constant $\mu_j^{t_1^1}$ such that

$$|\theta_i| \leqslant \sqrt{\mu_j^{t_1^1}/p}.$$

Therefore, one can obtain

$$\left|\sum_{i=1}^{p}\theta_i\phi_{j,i}^{t_1^1} - \sum_{j=1}^{r_1}\sum_{\tau=1}^{t_1^1-1}\frac{\partial z_j^{t_1^1 *}}{\partial \bar{z}_j^{\tau}}\sum_{i=1}^{p}\theta_i\phi_{j,i}^{t_1^1-1}\right| \leqslant \|\bar{\xi}^{t_1^1}\|\,\alpha_j^{t_1^1}\sqrt{\mu_j^{t_1^1}}.$$

By letting $\bar{z}_j^{t_1^1+1} = z_j^{t_1^1+1} - z_j^{(t_1^1+1)*}$,

$$z_j^{(t_1^1+1)*} = -\bar{z}_j^{t_1^1-1} - k_j^{t_1^1}\bar{z}_j^{t_1^1} - \frac{1}{4}\bar{z}_j^{t_1^1}(\alpha_j^{t_1^1})^2\hat{\mu}_j^{t_1^1} + \sum_{j=1}^{r_1}\sum_{\tau=1}^{t_1^1-1}\frac{\partial z_j^{t_1^1 *}}{\partial \bar{z}_j^{\tau}}\bar{z}_j^{\tau+1} + \sum_{j=1}^{r_1}\sum_{\tau=1}^{t_1^1-1}\frac{\partial z_j^{t_1^1 *}}{\partial \hat{\mu}_j^{\tau}}\dot{\hat{\mu}}_j^{\tau},$$

$$\dot{\hat{\mu}}_j^{t_1^1} = \frac{1}{4}(\bar{z}_j^{t_1^1}\alpha_j^{t_1^1})^2,\quad j=1,\cdots,r_1 \tag{8}$$

one can get

$$\dot{V}_j^{t_1^1} \leqslant \bar{z}_j^{t_1^1}\bar{z}_j^{t_1^1+1} - \bar{z}_j^{t_1^1-1}\bar{z}_j^{t_1^1} - k_j^{t_1^1}(\bar{z}_j^{t_1^1})^2 - \frac{1}{4}(\bar{z}_j^{t_1^1}\alpha_j^{t_1^1})^2\mu_j^{t_1^1} + |\bar{z}_j^{t_1^1}|\,\alpha_j^{t_1^1}\,\|\bar{\xi}^{t_1^1}\|\sqrt{\mu_j^{t_1^1}}$$

$$\leqslant \bar{z}_j^{t_1^1}\bar{z}_j^{t_1^1+1} - \bar{z}_j^{t_1^1-1}\bar{z}_j^{t_1^1} - k_j^{t_1^1}(\bar{z}_j^{t_1^1})^2 + \sum_{i=1}^{r_1}\sum_{k=1}^{t_1^1}(\bar{z}_i^{k})^2,\quad j=1,\cdots,r_1. \tag{9}$$

Now define the Lyapunov function as follows:

$$V_1 = \sum_{j=1}^{r_1}\sum_{h=1}^{t_1^1}V_j^h.$$

Then, it follows from (3), (5) and (9) that the time derivative of V_1 becomes

$$\dot{V}_1 \leqslant \sum_{j=1}^{r_1}\bar{z}_j^{t_1^1}\bar{z}_j^{t_1^1} - \sum_{j=1}^{r_1}\sum_{h=1}^{t_1^1}[k_j^h - r_1(t_1^1 - h + 1)](\bar{z}_j^h)^2.$$

Let $k_j^h - r_1(t_1^1 - h + 1) = \varepsilon$, i.e., $k_j^h = r_1(t_1^1 - h + 1) + \varepsilon$, and $\bar{z}_j^{t_1^1} = 0$, i.e. $z_j^{t_1^1+1} = z_j^{(t_1^1+1)*}$.
Then one can get

$$\dot{V} \leqslant -\varepsilon\|\bar{\xi}^{t_1^1}\|^2 = -\varepsilon\|\bar{z}\|^2,$$

with $\varepsilon > 0$.

As a result, the equilibrium point $z = 0$ is globally uniformly asymptotically stable ([4]). Since $\bar{z}$ is related to z by the transformations (2), (4), (7) and (8), it follows that $z = 0$ is globally uniformly asymptotically stable equilibrium puint.

Lemma 2 For the systems $\Sigma_j^1(1,1),\cdots,\Sigma_j^{t_1^1}(1,1),\cdots,\Sigma_j^{t_1^1+1}(2,1),\cdots,\Sigma_j^{t_1^2}(2,1), j=1,\cdots,$

r_1 and $\Sigma_j^1(2,2),\cdots,\Sigma_j^{t_2^2}(2,2), j = r_1+1,\cdots,r_2$ (defined as (1)) with inputs $\nu_j = z_j^{t_2^2+1}$. If Ω is a compact set, then there exists a global state feedback self-tuning controller such that the closed loop system is globally asymptotically stable at the origin for any $\theta \in \Omega$.

The proof is simillar to that of Lemm 1, and here it is ommitted.

Theorem 1 For the system (1), if Ω is a (unkown) compact set, then there exists a global state feedback atabilizing self-tuning controller such that the closed loop system is globally asymptotically stable at the origin for any $\theta \in \Omega$.

Proof by using the method of constructing the Lyapunov function given in Lemma 1, one can construct the Lyapunov function $V_j^h(q,\tau) = \frac{1}{2}(\bar{z}_j^h)^2 + \frac{1}{2}(\bar{\mu}_j^h)^2$, where $\bar{\mu}_j^h = \mu_j^h - \hat{\mu}_j^h$, μ_j^h is a suitable constant, $\hat{\mu}_j^h$ are self-tuning functions yet to be determined, for each subsystem $\Sigma_j^{t_\tau^{q-1}+h}(q,\tau)$ of the system(1). In addition, one can verify that the time derivative of such a function satisfies:

$$\dot{V}_j^{t_\tau^{q-1}+h}(q,\tau) \leqslant \bar{z}_j^{t_\tau^{q-1}+h}\bar{z}_j^{t_\tau^{q-1}+h+1} - \bar{z}_j^{t_\tau^{q-1}+h-1}\bar{z}_j^{t_\tau^{q-1}+h} - k_j^{t_\tau^{q-1}+h}(\bar{z}_j^{t_\tau^{q-1}+h})^2 + \sum_{p=1}^{q}\sum_{i=r_{p-1}+1}^{r_p}\sum_{k=1}^{t_p^{q-1}+h}(\bar{z}_i^k)^2,$$

$$j = r_{\tau-1}+1,\cdots,r_\tau;\quad h = 1,\cdots,t_q^q;\quad q = \tau,\cdots,s;\quad \tau = 1,\cdots,s$$

where

$$\bar{z}_j^{t_\tau^{q-1}+h+1} = z_j^{t_\tau^{q-1}+h+1} - z_j^{(t_\tau^{q-1}+h+1)*},$$

$$j = r_{\tau-1}+1,\cdots,r_\tau;\quad h = 1,\cdots,t_q^q;\quad q = \tau,\cdots,s;\quad \tau = 1,\cdots,s \tag{10}$$

with $\bar{z}_j^1 = z_j^1$, $\bar{z}_j^0 = 0$, $r_0 = 0$, $t_i^{i-1} = 0$, $i = 1,\cdots,s$.

Define the Lyapunov function of the systerm (1) as follows:

$$V = \sum_{\tau=1}^{s}\sum_{q=\tau}^{s}\sum_{j=r_{\tau-1}+1}^{r_\tau}\sum_{h=1}^{t_q^q} V_j^{t_\tau^{q-1}+h}(q,\tau).$$

then one can prove that the time derivative of the Lyapunov function V satisfies

$$\dot{V} \leqslant \sum_{\tau=1}^{s}\sum_{j=r_{\tau-1}+1}^{r_\tau} \bar{z}_j^{t}\bar{z}_j^{t+1} - \sum_{\tau=1}^{s}\sum_{q=\tau}^{s}\sum_{j=r_{\tau-1}+1}^{r_\tau}\sum_{h=1}^{t_q^q} k_j^{t_\tau^{q-1}+h}(\bar{z}_j^{t_\tau^{q-1}+h})^2 + \sum_{\tau=1}^{s}\sum_{q=\tau}^{s}\sum_{j=r_{\tau-1}+1}^{r_\tau}\sum_{h=1}^{t_q^q}\sum_{p=1}^{q}\sum_{j=r_{p-1}+1}^{r_p}\sum_{k=1}^{t_p^{q-1}+h}(-z_j^k)^2$$

one can obtain that

$$\dot{V} \leqslant \sum_{\tau=1}^{s}\sum_{j=r_{\tau-1}+1}^{r_\tau} \bar{z}_j^{t}\bar{z}_j^{t+1} - \sum_{q=1}^{s}\sum_{\tau=1}^{q}\sum_{j=r_{\tau-1}+1}^{r_\tau}\sum_{h=1}^{t_q^q}\{k_j^{t_\tau^{q-1}+h} - [r_q(t_q^q - h + 1) + \sum_{k=q+1}^{s} r_k t_k^k]\}(\bar{z}_j^{t_\tau^{q-1}+h})^2.$$

Choose $k_j^{t_\tau^{q-1}+h}$ such that

$$k_j^{t_\tau^{q-1}+h} - [r_q(t_q^q - h + 1) + \sum_{k=q+1}^{s} r_k t_k^k] = \varepsilon,$$

$$j = r_{\tau-1}+1,\cdots,r_\tau;\ h = 1,\cdots,t_q^q;\ \tau = 1,\cdots,q;\ q = 1,\cdots,s.$$

And let $\bar{z}_j^{t'_r+1} = 0$, i.e.

$$\bar{z}_j^{t'_r+1} = z_j^{(t'_r+1)*} = v_j, \quad j = r_\tau - 1, \cdots, r_\tau; \tau = 1, \cdots, s.$$

Then one can get

$$\dot{V} \leqslant -\varepsilon \| \bar{\xi}^{t'_1} \|^2 = -\varepsilon \| \bar{z} \|^2,$$

with $\varepsilon > 0$.

As a result, the equilibrium point $\bar{z} = 0$ is globally uniformly asymptotically stable ([7]). Since $\bar{z}$ is related to z by the transformations (10), it follows that $z = 0$ is globally uniformly asymptotically stable equilibrium point for the system(1).

3 Conclusion

This paper has discussed the state feedback robust stabilizing problem for multi-input time-varying uncertain nonlinear systems whose nominal system is globally feedback linearizable. A feedback controller which globally stabilizes each system in the family is constructed by a recursive algorithm.

References

1 Dorato, P.. A historical review of robust control. IEEE Control Systems Magazine, 1987, 7(2): 44 - 47

2 Marino, R, and Tomei, P.. Robust stabilization of feedback linearizable time-varying uncertain nonlinear systems. Automatica, 1993, 29(1): 181 - 189

3 Nijmeijer, H., A van der Schaft. Nonliear dynamical control systems. Springer-Verlag, Berlin, 1990

4 Hahn, W.. Stability of motion, Springer-Verlag, Berlin, 1967

5 Lu Souyin. Robust control of parameters uncertain nonlinear systems. M.S. Thesis, Northeast Universyty, 1995

多输入非线性不确定系统的全局鲁棒自校正控制器的设计

鲁守银 井元伟 刘晓平
(东北大学自动控制系·沈阳,110006)

摘要: 本文讨论了多输入非线性时变不确定系统的鲁棒镇定问题,其中的未知参变量是以线性形式出现的,同时给出了鲁棒可稳定化自校正控制器的设计.

关键词: 鲁棒镇定; 非线性不确定系统; 鲁棒自校正控制器

本文作者简介

鲁守银 1968 年生. 东北大学自动控制系博士生, 主要研究方向: 非线性系统, 机器人系统.

井元伟 1958 年生. 东北大学自动控制系副教授. 主要研究方向: 复杂大系统.

刘晓平 1962 年生. 东北大学自控系教授, 博士生导师. 主要研究方向: 非线性系统, 广义系统, 机器人系统.