Sliding Mode Control for Discrete-Time Systems and Its Application to the Control of a Servo-Control System

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Abstract: A sliding mode control approach for discrete-time systems with bounded parameter and disturbance uncertainty is developed by virtue of Lyapunov stability theory, and is applied to the high precision servo-control of a flight simulaor. Good simulation and experimental results are got.

Key words: sliding mode control; discrete-time systems; servo control

1 Introduction

The continuous time sliding mode control system (SMCS) is robust to parameter uncertainty and external disturbances. However, the robustness usually cannot be guaranteed when the sliding mode control scheme is implemented on a digital computer because the changing frequency is limited by the finite sampling rate[1]. So it is neccessary to design a discrete time controller based on sliding mode for a digitally controlled system.

This paper is concerned with the sliding mode control for the computer control of a continuous time system with bounded parameter and disturbance uncertainty. A novel sliding mode control approach for discrete time systems is developed by virtue of Lyapunov stability theory. Here, concepts of the sliding boundary layer, control bandwidth[2,4] and perturbations compensation[4,5] are employed in the developed control approach for reducing the upper bound of perturbation, the elimilation of chattering, the fast reaching of the system quasisliding motion. The validity of the proposed approach is verified through digital simulations and experiments of a servo-control system.

2 The Sliding Mode Control for Discrete Time Systems

Consider the following discrete time state equation related to the SISO system $x^{(n)} = f(X,t)+b(X,t)u(t)+d(t)$, considered in [2]

$$X(k+1)=AX(k)+Bu(k)+W(X,u,d), \tag{2.1}$$

where $X=(x,\dot{x},\cdots,x^{n-1})^{\mathrm{T}}$ is the state and u is the control input, $W(X,u,d)$ is the lumped system parameter and disturbance uncertainty.

A sliding surface in discrete state space is defined as

$$S(k)=C(X(k)-X_d(k))=0, \tag{2.2}$$

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where $X_d(k)=(x_d(k),\dot{x}_d(k),\cdots,x_d^{n-1})^T$, $x_d(k)$ is the desired trajectory to be tracked, and $C=(c_1,c_2,\cdots,c_n)$ is the n-dimensional row vector.

Similar to [2], if the desired control bandwidth of system is λ, then a matching relation must be satisfied as

$$c_i=C_{i-1}^{n-1}\lambda^{n-i},\quad i=1,2,\cdots,n. \tag{2.3}$$

Consider the following control law

$$\begin{aligned}u(k)&=u_e(k)+u_n(k)\\&=-(CB)^{-1}\{CA(X(k)-X(k-1))-(CB)u(k-1)+C(X_d(k)-X_d(k+1))\}\\&\quad-\rho(k)\ \mathrm{sgn}\ (S(k)),\end{aligned} \tag{2.4}$$

where

$$u_e(k)=-(CB)^{-1}\{CA(X(k)-X(k-1))-(CB)u(k-1)+C(X_d(k)-X_d(k+1))\}, \tag{2.5}$$

and

$$u_n(k)=-\rho(k)\ \mathrm{sgn}\ (S(k)), \tag{2.6}$$

represent the equivalent control component and nonlinear control component respectively.

Using (2.1) and (2.2), together with (2.4), $S(k+1)$ satisfies

$$\begin{aligned}S(k+1)&=S(k)+C(W(k)-W(k-1))-\rho(k)\ \mathrm{sgn}\ (S(k))\\&=S(k)+E(k)-\rho(k)\ \mathrm{sgn}\ (S(k)),\end{aligned} \tag{2.7}$$

where $W(k)$ is short for $W(X(k),u(k),d(k))$, and $E(k)=C(W(k)-W(k-1))$. (2.7) shows that the equivalent control component can guarantee the realization of the quasisliding motion when the system perturbation is constant or no system perturbation exists. At the same time, slowly varying parameter and disturbance uncertainty can be compensated automatically under the control of (2.4), which reduces the upper bound of the perturbation and improves the tracking precision of controlled systems. And the nonlinear control component $u_n(k)$ is used to compensate for the fast varying parameter and disturbance uncertainty.

In the sliding mode control for discrete time systems, the sliding mode motion occurs on the open neighbourhood of the switching surface instead of on the switching surface because of the finite switching rate[1,3]. So in this paper, a sliding boundary layer is defined as follows

$$\Phi(k)=\{X(k):|S(k)|\leqslant\varphi(E(k)),k=0,1,\cdots\}, \tag{2.8}$$

where $\varphi(E(k))$, which will be defined afterwards, is a function of the system parameter and disturbance uncertainty $E(k)$. If the system perturbation is constant or the function of system state, the sliding boundary layer will becomes the sliding sector in [3].

The purpose of control is to drive the system switching function to the sliding boundary layer. Once the switching function enters the sliding boundary layer, the quasisliding mode motion occurs. In order to guarantee a good design performence of the system, the following problems should the solved:

i) how to guarantee a good performence of the system when the switching function is in the reaching phase?

ii) how to keep the switching function inside the sliding boundary layer once the switching function enters it?

iii) how to define the sliding boundary layer?

Above-mentioned problems will be solved by the following three theorems.

Theorem 2. 1 For syetem (2. 1), the following control law

$$u(k)=-(CB)^{-1}\{CA(X(k)-X(k-1))-(CB)u(k-1)$$
$$+C(X_d(k)-X_d(k+1))\}-\rho(k)\ \mathrm{sgn}\ (S(k)),\quad X(k)\in\Phi^c(k),\tag{2.9}$$

can ensure that the system switching function converges to the sliding boundary layer exponentially fast. Where $\Phi^c(k)$ is the complement of the sliding boundary layer, and the control gain $\rho(k)$ can be determined as

$$(1-e^{-a})|S(k)|+D(k)\leqslant\rho(k)\leqslant(1+e^{-a})|S(k)|-D(k)\tag{2.10}$$

with $a>0$ the convergence coefficient, and $D(k)=|E(k)|$.

Proof Let the Lyapunov function is defined by

$$V(k)=\frac{1}{2}S^2(k)\ ,$$

and the system switching function satisfies

$$|S(k+1)|\leqslant e^{-a}|S(k)|.\tag{2.11}$$

Thus, the forward difference $\Delta V(k)$ of the Lyapunov function is obtained as

$$\Delta V(k)=V(k+1)-V(k)\leqslant(e^{-a}-1)S^2(k)<0.$$

which shows the system has stable sliding motion. Substituting (2. 7) into (2. 11) gives (2. 10) as required.

Remark 2. 1 From (2. 10), the solution of $\rho(k)$ exists if

$$|S(k)|\geqslant e^{a}D(k).\tag{2.12}$$

Then, the region of attraction $\Phi^c(k)$ satisfies

$$\varphi(k)\geqslant e^{a}D(k).\tag{2.13}$$

Theorem 2. 2 For system (2. 1), the following control law

$$u(k)=-(CB)^{-1}\{CA(X(k)-X(k-1))-(CB)u(k-1)$$
$$+C(X_d(k)-X_d(k+1))\}-\rho(k)S(k)/\varphi(k),\quad X(k)\in\Phi(k),\tag{2.14}$$

can keep the switching function inside the sliding boundary layer indefinitely for all remaining ks once the switching function enters it, and the unmodelled system dynamics is not excited. The control gain can be determined by

$$D(k)\leqslant\rho(k)\leqslant r_1\varphi(k),\tag{2.15}$$

where
$$r_1=\cos(\lambda T)+\sqrt{(2-\cos(\lambda T))^2-1}-1,\tag{2.16}$$

with T the sampling interval.

Proof Following the work given in [4].

Theorem 2. 3 The sliding boundary layer dynamics can be determined by

$$\Delta\varphi(k+1)=\rho_d(k)-r_1\varphi(k),\tag{2.17}$$

where
$$\rho_d(k)=e^{a}|E(k)|_{X(k)=X_d(K),X(k-1)=X_d(k-1)}.\tag{2.18}$$

Proof If the sliding boundary layer can vary with the system parameter and disturbance uncertainty, the switching function must satisfy

$$-\varphi(k+1)\leqslant S(k+1)\leqslant\varphi(k+1).\tag{2.19}$$

Suppose the system state is in the neibourhood of the desired trajectory, then the switching function dynamics of the system can be rewritten as

$$S(k+1)=S(k)+E_d(k)-\hat{\rho}_d(k)\ \mathrm{sgn}\ (S(k)), \tag{2.20}$$

where $E_d(k)\cong C(W(k)-W(k-1))\,|_{X(k)=X_d(k),X(k-1)=X_d(k-1)}$.

For convenience, let $S(k)=\varphi(k)$, the same result can be got by letting $S(k)=-\varphi(k)$. Substituting $S(k)=\varphi(k)$ into (2.20) and satisfying (2.19) yield

$$-\varphi(k+1)\leqslant S(k+1)=\varphi(k)+E_d(k)-\hat{\rho}_d(k)\ \mathrm{sgn}\ (S(k))\leqslant\varphi(k+1),$$

which reusults in

$$E_d(k)-\Delta\varphi(k+1)\leqslant\hat{\rho}_d(k)\leqslant\Delta\varphi(k+1)+2\varphi(k)+E_d(k), \tag{2.21}$$

where $\Delta\varphi(k+1)=\varphi(k+1)-\varphi(k)$. Let $D_d(k)=|E_d(k)|$, then (2.21) is satisfied if

$$|\hat{\rho}_d(k)-\varphi(k)|_{\max}=\Delta\varphi(k+1)+\varphi(k)-D_d(k). \tag{2.22}$$

If $\Delta\varphi(k+1)=0$, i.e. the sliding boundary layer is invariant, then (2.22) gives

$$|\rho_d(k)-\varphi(k)|_{\max}=\varphi(k)-D_d(k) \tag{2.23}$$

where $\rho_d(k)$ represents the control gain corresponding to the case $\Delta\varphi(k+1)=0$. Substituting (2.23) into (2.22) yields

$$|\hat{\rho}_d(k)-\varphi(k)|_{\max}=\Delta\varphi(k+1)+|\rho_d(k)-\varphi(k)|_{\max}. \tag{2.24}$$

(2.15) shows $\hat{\rho}_d(k)-\varphi(k)<0;\ \rho_d(k)-\varphi(k)<0$ (2.25)

with (2.25), (2.24) gives

$$(\hat{\rho}_d(k))_{\min}=(\rho_d(k))_{\min}-\Delta\varphi(k+1). \tag{2.26}$$

(2.26) shows that $\rho_d(k)$ must be revised according to (2.26) for the satisfaction of (2.19) as the variation of the sliding boundary layer is considered. Substituting (2.26) into the right side of (2.15) gives

$$\Delta\varphi(k+1)=(\rho_d(k))_{\min}-r_1\varphi(k). \tag{2.27}$$

Considering (2.23) and (2.13), we got

$$(\rho_d(k))_{\min}=e^a D_d(k). \tag{2.28}$$

Remark 2. 2 The system control bandwidth λ and sampling interval T are important design parameters, their selections can refer to reference [2,4] for details.

3 Application Example

The above developed control strategy will be tested on the real time control of a flight simulator, which has a high requirement for the tracking property of its servo-system, i.e. high precision, fast tracking response and high reliability. Fig. 1 is the photograph of the servo flight simulator. Its hardware detail is shown in Fig. 2, which is mainly composed of DC motor, PWM power amplifier, DSP－C25 digital signal processor and induction synchro-encoder. Fig. 3 shows the equivalent block diagram of the system.

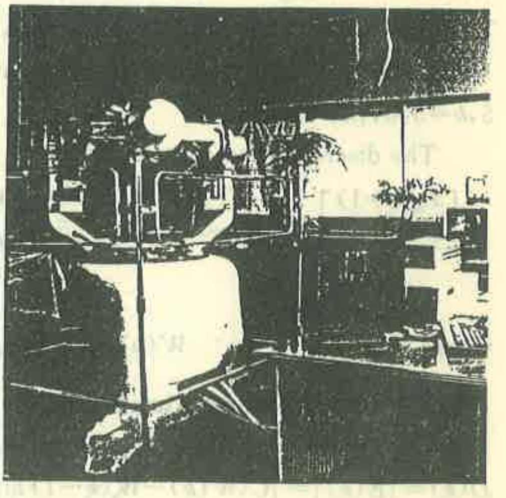

Fig. 1 Servo-control system for flight simulator

Mathematical Model of the Servo-Control System

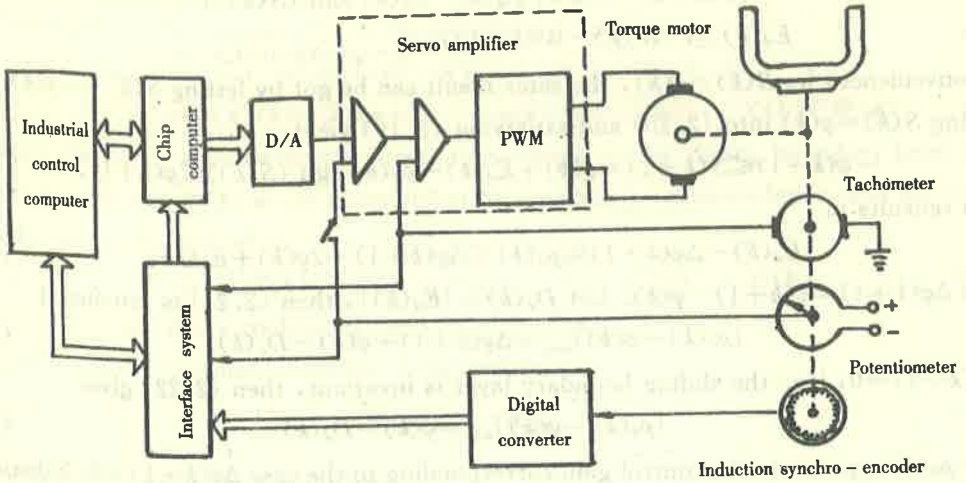

Fig. 2 Hardware details of servo system

From Fig. 3, the state space representation of the system can be written as follows:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & -1/T_M\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ K_M/T_M\end{bmatrix}u+\begin{bmatrix}0\\ K_v\end{bmatrix}M. \tag{3.1}$$

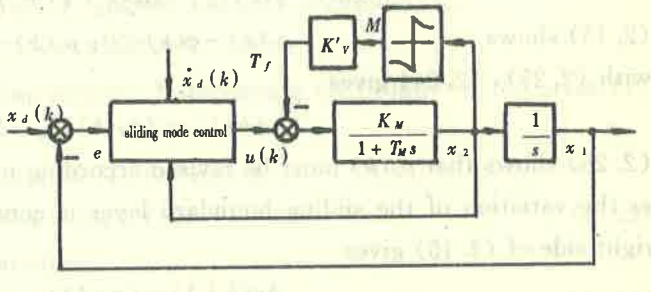

Fig. 3 Block diagram of the system

where, Mechanical time constant: $T_M=26\mathrm{s}$; Motor transfer coefficient with null speed: $K_M=13.351\mathrm{deg}\cdot\mathrm{s}^{-1}/\mathrm{V}$; Coefficient: $K_v=487.05\mathrm{deg}\cdot\mathrm{s}^{-1}/\mathrm{kg}\cdot\mathrm{m}$; Friction moment: $M=M_H(1+3\mathrm{e}^{-bx_2})\mathrm{kg}\cdot\mathrm{m}$; among them, $M_H=0.5$, $b=500$, $|u|\leqslant 43\mathrm{V}$.

The discretization form of the state equation (3.1) can be written as

$$\begin{bmatrix}x_1(k+1)\\ x_2(k+1)\end{bmatrix}=\begin{bmatrix}1 & T_M(1-\mathrm{e}^{-T/T_M})\\ 0 & \mathrm{e}^{-T/T_M}\end{bmatrix}\begin{bmatrix}x_1(k)\\ x_2(k)\end{bmatrix}+\begin{bmatrix}(T-T_M(1-\mathrm{e}^{-T/T_M}))K_M\\ K_M(1-\mathrm{e}^{-T/T_M})\end{bmatrix}u(k)+W(k). \tag{3.2}$$

where

$$W(k)\approx T_M M(t_k)K_v\begin{bmatrix}T-T_M(1-\mathrm{e}^{-T/T_M})\\ 1-\mathrm{e}^{-T/T_M}\end{bmatrix}. \tag{3.3}$$

The Upper Bound Estimation of the Perturbation $E(k)$

The perturbation $E(k)$ can be estimated as follows:

$$\begin{aligned}D(k)&=|E(k)|=|C(W(k)-W(k-1))|\\ &\approx\left|[\lambda\quad 1]T_M K_v\begin{bmatrix}T-T_M(1-\mathrm{e}^{-T/T_M})\\ 1-\mathrm{e}^{-T/T_M}\end{bmatrix}(M(t_k)-M(t_{k-1}))\right|\\ &=T_M K_v|\{\lambda(T-T_M)+1+(\lambda T_M-1)\mathrm{e}^{-T/T_M}\}|\cdot|\{M_H(1+0.3\mathrm{e}^{-b|x_2|})-M_H(1+0.3\mathrm{e}^{-b|x_2|})\}|\\ &\leqslant 0.3M_H bTT_M K_v|\lambda(T-T_M)+1+(\lambda T_M-1)\mathrm{e}^{-T/T_M}|\max|\mathrm{e}^{-b|x_2(\zeta)|}\dot{x}_2(\zeta)|,\ (t_{k-1}\leqslant\zeta\leqslant t_k).\end{aligned} \tag{3.4}$$

Simulation and Experiment Results of the Servo-Control System

The system is controlled to track two desired trajectories——sine curve with high angular frequency and slope input signal with extremely low speed, for evaluating the fast tracking response, extremely-low-speed tracking feature and high tracking precision of the designed system. The desired trajectories are

a) Sine input signal $x_d(t)=\sin(16\pi t)(\mathrm{deg})$, $t\in[0,0.5]\mathrm{s}$. (3.5)

b) Slope input signal $x_d(t)=0.001t(\mathrm{deg})$, $t\in[0,1]\mathrm{s}$. (3.6)

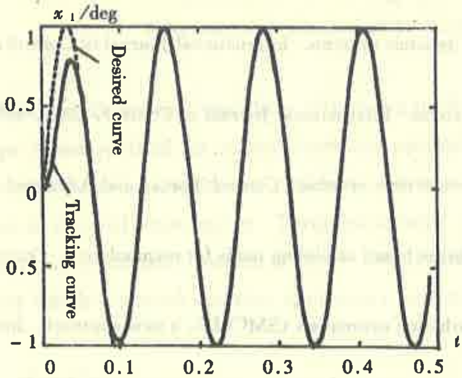

Fig. 4(a) Tracking sine signal through simulation

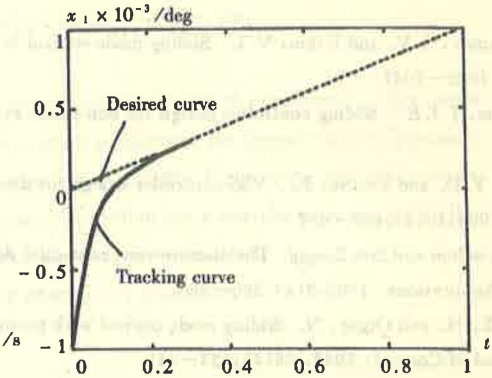

Fig. 4(b) Tracking slope signal through simulation

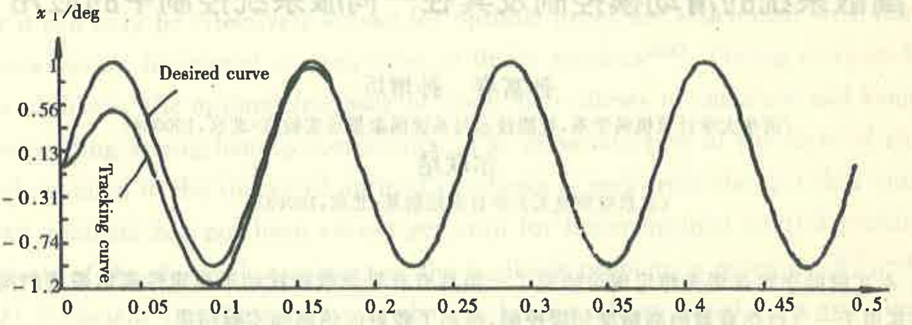

Fig. 5(a) Tracking sine signal through experiment

Fig. 4(a)(b) show the simulation results tracking two desired trajectories. Fig. 5(a)(b) are the corresponding experimental ones. The initial conditions for simulations and experiments are $(x_1,x_2)=(0,0)$ but Fig. 5(a) with $(x_1,x_2)=(0.001,0)$, the sampling interval of controller is 0.01 second and $\lambda=20$. It can be seen that the experimental results are very similar to the simulation outcomes if considering the position sensor resolution as well as unmeasurable noises existing in the actual system. The validity of the proposed control strategy is verified by digital simulations and experiments.

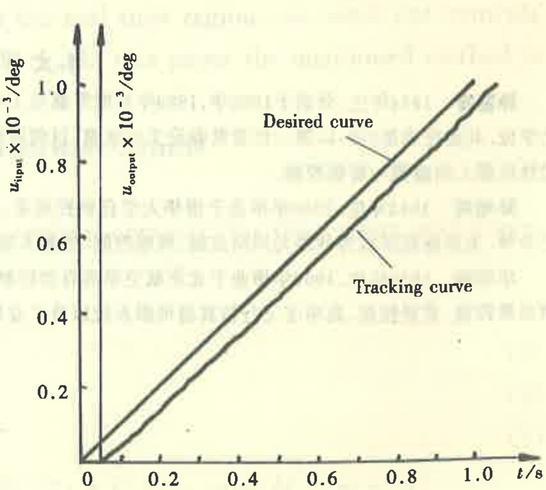

Fig. 5(b) Tracking slope signal through experiment

4 Conclusion

The sliding mode control scheme for discrete time systems is presented. Its validity and good control performence are confirmed through digital simulations and experiments.

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离散系统的滑动模控制及其在一伺服系统控制中的应用

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摘要：本文借助李雅普诺夫稳定理论研究了一类具有有界参数和扰动不确定性离散系统的滑动模控制方法,并将其用于一飞行仿真器的高精度伺服控制,得到了较好的仿真和实验结果.

关键词：滑动模控制；离散时间系统；伺服控制

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孙富春　1964年生.分别于1986年、1989年在海军航空工程学院获学士学位和硕士学位.现在清华大学计算机系攻读博士学位.共鉴定成果5项,以第一作者发表论文30余篇.目前研究兴趣为模糊逻辑和神经网络控制、离散系统的滑动模控制和柔性机器人的建模与智能控制.

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