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Synthesis of Optimal Feedback Controls for Piecewise Linear System

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Abstruct: Based on algorithms elaborated in Minsk for optimal control problems[1], this paper develops a new method for solving feedback optimal control problems. By introducing a dynamical statement for optimal synthesis problem, we construct an algorithm for solving optimal control problem in the real time regime. Terminal optimal control problem for a system with piecewise linear inputs under constratints on controls and terminal states is inverstigated.

Key words: control system; synthesis; defining equations; controller

1 Introduction

The feedback optimal control problem[2] remains a central one in the control theory. For the present it can only be effectively solved for optimal problems which deal with minimization of convex square functional on trajectries of linear systems[4,5]. Owing to special structure of this problem, the optimal feedback is linear that allows to calculate and keep effectively corresponding strengthening coefficients. The dissatisfaction of the slate of the feedback control problem in the theory of optimal synthesis is seen from the fact that untill now the synthesis problem has not been sloved yet even for linear optimal control problems. A critical analysis of the classical statement of the feedback problem is given in [6] and a new (dynamical) statement and the solution of the synthesis problem based on it are suggested. Optimal controllers functioning in real time model for linear systems are discribed. These controllers for every concrete control process in the real time regime can work out controls circulating in the system closed by optimal feedback. In this paper the mentioned method is developed for piecewise linear control problem.

2 Synthesis of Optimal Systems with Piecewise Inputs

2.1 Statement of the Problem

Consider the following terminal problem of optimal control[1] for the system with piecewise input:

$$J(u) = c'x(t^*) \to \max, \tag{1}$$

$$\dot{x} = Ax + b(u), \quad x(0) = x_0, \tag{2}$$

$$Hx(t^*) = g, \tag{3}$$

$$|u(t)| \leqslant 1, \quad t \in T = [0, t^*], \tag{4}$$

$$(x \in \mathbb{R}^n,\ u \in \mathbb{R},\ A \in \mathbb{R}^{n \times n},\ H \in \mathbb{R}^{m \times n},\ g \in \mathbb{R}^m,\ \text{rank } H = m < n)$$

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where

$$b(u)=\begin{cases} b_1u+v_1, & u\geqslant u_0,\\ b_2u+v_2, & u\leqslant u_0,\end{cases}$$

b_1,b_2,v_1,v_2, are n-vectors, $|u_0|\leqslant 1, b_1u_0+v_1=b_2u_0+v_2$.

As usual the piecewise function $u(t),t\in T$, is said to be the admissible control of the problem (1)~(4), if it satisfies the inequality constraint (4) and gives the trajectory $x(t)$, $t\in T$, which at the terminal time t^* gets into the terminal set $X^*=\{x\in\mathbb{R}^n: Hx=g\}$. The admissible control of the problem (1)~(4) will be called optimal if along it the criterion (1) attains maximum value.

Embed the problem (1)~(4) in to the family of the problems

$$c'x(t^*)\to\max, \tag{5}$$

$$\dot{x}=Ax+b(u),\quad x(\tau)=z, \tag{6}$$

$$Hx(t^*)=g, \tag{7}$$

$$|u(t)|\leqslant 1,\quad t\in T_\tau=[\tau,t^*], \tag{8}$$

which depend on a number τ and an n-vector z.

A pair $s=\{\tau,z\}$ is said to be an admissible position if for which there exists the soultion of the problem (5)~(8).

Let S be the set of admissible positions, $u^\circ(t|s),x^\circ(t|s),t\in T\tau$, be the optimal control and the trajectory of the probles (5)~(8) for the admissible position $s\in S$.

Definition 1 The piecewise continuous function $u^\circ(s)$, $s\in S$, is said to be an optimal feedback control of the problem (1)~(4), if it satisfies the inequality constraint (4) and gives the trajectry $x(t|s),t\in T\tau$, of $\dot{x}=Ax+b(u^\circ(t,x)),x(\tau)=z$, which for all positions $s\in S$ coincides with $X^\circ(t|s),t\in T\tau$.

Suppose that in real conditions the system (2) experiences the action of an unknown piecewise continuous perturbance $w(t)$, $t\in[0,t^\circ]$; $w(t)=0,t\in(t^\circ,t^*],t^\circ<t^*$. Due to the action of perturbances, the trajectory of closed-loop system is discribed not by (2), but by the equation

$$\dot{x}=Ax+b(u^\circ(t,x))+w(t),\quad x(0)=x_0. \tag{9}$$

Assume that in the concrete control process the perturbance $w^*(t)$, $t\in T$, is relized. Let $x^*(t),t\in T$, be the corresponding trajectory of the equation (9). Our goal is to contruct controller which for every concrete process in real time regime produces the control $u^*(t)=u^\circ(t,x^*(t)),t\in T$, using exact information about the current state of the system in the inverstigated process.

2.2 Optimality Criterion for Program Controls:

Applying the similar methods in [2], we can prove that the optimal control of the problem (5)~(8) belongs to the class of piecewise constant functions which take only three values 1, u_0, -1 if the following condition is satisfied:

$$\operatorname{rank}\left(\begin{pmatrix}H\\c'\end{pmatrix}b_i,\begin{pmatrix}H\\c'\end{pmatrix}Ab_i,\cdots\begin{pmatrix}H\\c'\end{pmatrix}A^{n-1}b_i\right)=m+1,\quad i=1,2.$$

Let U be the set of all piecewise constant functions which take only three values 1, u_0,

-1.

Suppose that $u(\cdot)=(u(t),t\in T\tau)\in U$ is an admissible control of the problem (5)~(8). $t_*(\tau)=\{t_j(\tau),j\in J\},|J|=p$ is the set of switching points of the control $u(t),t\in T$. Break $t_*(\tau)$ in to the subsets:

$$\begin{cases} t^{+-}(\tau)=\{t\in T\tau:u(t-0)=1,u(t+0)=-1\}=\{t_j(\tau),j\in J^{+-}\}, \\ t^{-+}(\tau)=\{t\in T\tau:u(t-0)=-1,u(t+0)=1\}=\{t_j(\tau),j\in J^{-+}\}, \\ t^{+0}(\tau)=\{t\in T\tau:u(t-0)=1,u(t+0)=u_0\}=\{t_j(\tau),j\in J^{+0}\}, \\ t^{0+}(\tau)=\{t\in T\tau:u(t-0)=u_0,u(t+0)=1\}=\{t_j(\tau),j\in J^{0+}\}, \\ t^{0-}(\tau)=\{t\in T\tau:u(t-0)=u_0,u(t+0)=-1\}=\{t_j(\tau),j\in J^{0-}\}, \\ t^{-0}(\tau)=\{t\in T\tau:u(t-0)=-1,u(t+0)=u_0\}=\{t_j(\tau),j\in J^{-0}\}. \end{cases} \tag{10}$$

Let $t_{sp}=\{t_1,t_2,\cdots,t_m\}$ be a set of m switching moments of the admissible control $u(t),t\in T\tau$. $t_{sp}^{+-}=t_{sp}\cap t^{+-},t_{sp}^{-+}=t_{sp}\cap t^{-+},t_{sp}^{0+}=t_{sp}\cap t^{0+},t_{sp}^{+0}=t_{sp}\cap t^{+0},t_{sp}^{0-}=t_{sp}\cap t^{0-},t_{sp}^{-0}=t_{sp}\cap t^{-0}$.

$$H_1(t)=HF(t^*,t)b_1,H_2(t)=HF(t^*,t)b_2,$$
$$H_3(t)=HF(t^*,t)[b_1(1-u_0)+b_2(1+u_0)],\ t\in T\tau.$$

$F(t,\tau),\tau\in[0,t]$ is fundamental matrix of the solution for the homogeneous part $\dot{x}=Ax$ of the dynamical system (6).

Definition 2 A set $t_{sp}=\{t_1,t_2,\cdots,t_m\}$ is said to be a $u(\cdot)$ support of the problem (5)~(8), it the matrix

$p=[H_1(t),t\in t_{sp}^{+0}\cup t_{sp}^{0+}\,|\,H_2(t),t\in t_{sp}^{-0}\cup t_{sp}^{0-}\,|\,H_3(t),t\in t_{sp}^{+-}\cup t_{sp}^{-+}]$is nonsingular.

The pair $(u(\cdot),t_{sp})$ from the admissible control and the $u(\cdot)$-support is called the support control.

Let

$$C_1(t)=c'F(t^*,t)b_1,C_2(t)=c'F(t^*,t)b_2,$$
$$C_3(t)=c'F(t^*,t)[b_1(1-u_0)+b_2(1+u_0)],t\in T\tau.$$
$$C_{sp}=[C_1(t),\ t\in t_{sp}^{+0}\cup t_{sp}^{0+}\,|C_2(t),t\in t_{sp}^{-0}\cup t_{sp}^{0-}\,|C_3(t),t_{sp}^{+-}\cup t_{sp}^{-+}].$$

Calculae the potential vector

$$y(\tau)=C_{sp}'P^{-1}$$

and construct the functions

$$\psi'(t)=(c'-y'H)F(t^*,t),\quad \Delta_1(t)=\psi'(t)b_1,\quad \Delta_2(t)=\psi'(t)b_2,$$
$$\Delta_3(t)=\psi'(t)[b_1(1-u_0)+b_2(1+u_0)],\quad t\in T\tau.$$

The function $\psi(t),t\in T\tau$, is the solution of the conjugate system

$$\dot{\psi}=-A'\psi,\quad \psi(t^*)=c-H'y. \tag{11}$$

Theorem 1 (Optimality criterion). For optimality of the support control $(u(\cdot),t_{sp})$ it is nessary and sufficient that the relations

$$\begin{cases} \Delta_1(t)\geqslant 0,\quad \Delta_3(t)\geqslant 0,\quad \text{if } u(t)=1, \\ \Delta_2(t)\leqslant 0,\quad \Delta_3(t)\leqslant 0,\quad \text{if } u(t)=-1, \\ \Delta_1(t)\leqslant 0,\quad \Delta_2(t)\geqslant 0,\quad \text{if } u(t)=u_0,\quad t\in T\tau, \end{cases} \tag{12}$$

are satisfied.

The proof the theorem 1 is not given due to the capacity limit of the paper.

2.3 Defining equations of the optimal controller

From now on suppose that the functions $\Delta_1(t)$, $\Delta_2(t)$, $\Delta_3(t)$, $t\in T\tau$ do not turn in to zero at the same moment.

By (12) in terms of the functions $\Delta_1(t)$, $\Delta_2(t)$, $\Delta_3(t)$, $t\in T\tau$ the sets (10) can be determined as follows:

$$t^{+-}(\tau)=\{t\in T\tau:\Delta_3(t)=0,\dot{\Delta}_3(t)<0,\Delta_1(t)>0,\Delta_2(t)<0\},$$
$$t^{-+}(\tau)=\{t\in T\tau:\Delta_3(t)=0,\dot{\Delta}_3(t)>0,\Delta_1(t)>0,\Delta_2(t)<0\},$$
$$t^{+0}(\tau)=\{t\in T\tau:\Delta_1(t)=0,\dot{\Delta}_1(t)<0,\Delta_2(t)>0\},$$
$$t^{0+}(\tau)=\{t\in T\tau:\Delta_1(t)=0,\dot{\Delta}_1(t)>0,\Delta_2(t)>0\},$$
$$t^{0-}(\tau)=\{t\in T\tau:\Delta_2(t)=0,\dot{\Delta}_2(t)<0,\Delta_1(t)<0\},$$
$$t^{-0}(\tau)=\{t\in T\tau:\Delta_2(t)=0,\dot{\Delta}_2(t)>0,\Delta_1(t)<0\}.$$

Obviously the optimal control $u^\circ(t|s),t\in T\tau$ of the problem (5)~(8) is completely determined by

$$t_*(\tau),y(\tau), \tag{13}$$

which consists of p switching points and the m-potential vector which satisfy the equations:

$$f(\tau,t_1(\tau),\ i=\overline{1,p};\ y(\tau),z)=0, \tag{14}$$
$$q_j(t_i(\tau),\ i=\overline{1,p};\ y(\tau))=0,\quad j=\overline{1,p}. \tag{15}$$

where

$$\begin{aligned}&f(\tau,t_i(\tau),i=\overline{1,p};y(\tau),z)\\&=\sum_{i\in K^+(\tau)}\int_{t_i}^{t_{i+1}}HF(t^*,t)(b_1+v_1)\mathrm{d}t+\sum_{i\in K^-(\tau)}\int_{t_i}^{t_{i+1}}HF(t^*,t)(-b_2+v_2)\mathrm{d}t\\&\quad-\sum_{i\in K^0(\tau)}\int_{t_0}^{t_{i+1}}HF(t^*,t)(b_1u_0+v_1)\mathrm{d}t+HF(t^*,\tau)z-g,\end{aligned}$$

$$q_j(t_i(\tau),i=\overline{1,p};\ y(\tau))=\begin{cases}\Delta_1(t_j(\tau)),\ j\in J^{0+}\cup J^{+0},\\ \Delta_2(t_j(\tau)),\ j\in J^{0-}\cup J^{-0},\\ \Delta_3(t_j(\tau)),\ j\in J^{+-}\cup J^{-+},\end{cases}$$

$$K(\tau)=\{0,1,\cdots,p\},$$
$$K^+(\tau)=\{i\in K(\tau):u^\circ(t)=1,t\in(t_i(\tau),t_{i+1}(\tau))\},$$
$$K^-(\tau)=\{i\in K(\tau):u^\circ(t)=-1,t\in(t_i(\tau),t_{i+1}(\tau))\},$$
$$K^\circ(\tau)=\{i\in K(\tau):u^\circ(t)=u_0,t\in(t_i(\tau),t_{i+1}(\tau))\},$$
$$t_0(\tau)=\tau,\quad t_p(\tau)=t^*.$$

Introduce numbers $k_i,i\in K(\tau):k_i=1$ if $i\in K^+(\tau)$; $k_i=-1$, if $i\in K^-(\tau)$; $k_i=u_0$ if $i\in K^0(\tau)$.

The Jacobi matrix for the system (14),(15) gets the form:

$$G(t_i,\ (\tau),i=\overline{1,p};\ y(\tau))=\left[\begin{array}{ccc:c}G_1&G_2&G_3&0\\\hdashline G_7&0&0&-G_4\\C&G_8&0&G_5'\\0&0&G_9&-G_6'\end{array}\right]$$

where

$$G_1 = [k_{j-1}HF(t^*,t_j)\,(b_1(1-u_0)+b_2(1+u_0)),j \in J^{+-}\cup J^{-+}],$$
$$G_2 = [(k_{j-1}-k_j)HF(t^*,t_j)b_1,j \in J^{0+}\cup J^{+0}],$$
$$G_3 = [(k_{j-1}-k_j)HF(t^*,t_j)b_2,j \in J^{-0}\cup J^{0-}],$$
$$G_4 = [HF(t^*,t_j)(b_1(1-u_0)+b_2(1+u_0)),j \in J^{-0}\cup J^{0-}],$$
$$G_5 = [HF(t^*,t_j)b_1,j \in J^{0+}\cup J^{+-}],$$
$$G_6 = [HF(t^*,t_j)b_2,j \in J^{-0}\cup J^{0-}],$$
$$G_7 = \operatorname*{diag}_{j\in J^{+-}\cup J^{-0}} (y'H-c')F(t^*,t_j)A(b_1(1-u_0)+b_2(1+u_0)),$$
$$G_8 = \operatorname*{diag}_{j\in J^{0+}\cup J^{+0}} (y'H-c')F(t^*,t_j)Ab_1,$$
$$G_9 = \operatorname*{diag}_{j\in J^{0-}\cup J^{-0}} (y'H-c')F(t^*,t_j)Ab_2.$$

Definition 3 The problem (5)~(8) is called regular if its Jacobi matrix $G(t_i(\tau),i=1,p,y(\tau))$ is nonsingular.

It can be proved that the problem (5)~(8) is regular if

1)
$$\left.\frac{\partial\Delta_1(t|s)}{\partial t}\right|_{t=t_j(\tau)} \neq 0,\quad j\in J^{0+}\cup J^{+0},$$
$$\left.\frac{\partial\Delta_2(t|s)}{\partial t}\right|_{t=t_j(\tau)} \neq 0,\quad j\in J^{0-}\cup J^{-0},$$
$$\left.\frac{\partial\Delta_3(t|s)}{\partial t}\right|_{t=t_j(\tau)} \neq 0,\quad j\in J^{+-}\cup J^{-+};$$

2) rank $(G_4|G_5|G_6)=m$.

Regularity of the problem (5)~(8) means that for any $\tau\in T^*(\tau)$ ($T^*(\tau)$ is some right-side neighbourhood of the proint τ) the system (14), (15) has only one solution (13) which determines the optimal control $u^0(t)=u^0(t|s), t\in T\tau$, and it will be used to construct the optimal controller. Therefore the equations (14), (15) are called the defining equations of the optimal controller. The family $S(\tau)=\{K^+(\tau),K^-(\tau),K^0(\tau),p(\tau)\}$ is called the structure of the optimal controller.

2.4 Numerical Method for Solving the Defining Equations

Now let us describe an algorithm for sloving the functional equations (14), (15).

Assume that for $\tau=\tau_*\in T$

$$\tau < t_1(\tau), \tag{16}$$
$$t_p(\tau) < t^*. \tag{17}$$

Suppose that we have the start values $t_i(\tau_*), i=\overline{1,p}; y(\tau_*), x^*(t_*)$ such that satisfy the following equations:

$$f(\tau_*,t_i(\tau_*),\ i=\overline{1,p};\quad y(\tau_*),x^*(\tau_*))=0,$$
$$q_j(t_i(\tau_*),i=\overline{1,p};\quad y(\tau_*))=0,\quad j=\overline{1,p}.$$

As for numerical solution of ordinary diffierential equations, approximate solution of the equations (14), (15) will be structructed at the net:

$$T^h(\tau_*)=[\tau_*,\tau_*+h,\cdots,t^*-h,t^*],\quad h=t^*/N,\quad N<\infty.$$

Assume that the sequence $t_i(\tau_l+sh), s=\overline{0,k-1}$ has been constructed which corresponds to $\{\tau_*+sh, x^*(\tau_*+sh)\}, s=\overline{0,k-1}$.

For calculating the moments $t_i(\tau_*+kh), i=\overline{1,p}$, and the potential vector $y(\tau_*+kh)$ we construct the vectors:

$$Z^l = (t_j^l, i = 1,p;\ y^l), l = \overline{1,l_0}:$$

$$Z^1 = (t_i^1 = t_i(\tau_* + (k-1)h), i = \overline{1,p}; y^1 = y(\tau_* + (k-1)h)), \tag{18}$$

$$Z^l = Z^{l-1} - G^{-1}(Z^{l-1})[f'(\tau_* + kh, t_i^{l-1}, i = \overline{1,p}, y^{l-1}, \dot{x}(\tau_* + kh)),$$

$$q_j(t_i^{l-1}, i = \overline{1,p};\ y^{l-1}), j = \overline{1,p}], l = \overline{2,l_0}. \tag{19}$$

Let $$t_i(\tau_*+kh)=t_i^{l_0}, i=\overline{1,p};\ y(\tau_*+kh)=y^{l_0}.$$

Assume that for some $\tau=\bar{\tau}$ the condition (16) is violated. For constructing the collection

$$t_i(\bar{\tau}+h), i=1,\ p;\ y(\bar{\tau}+h), \tag{20}$$

corresponding to the position $\{\bar{\tau}+h, x^*(\bar{\tau}+h)\}$, the vector Z^1 is given not by (18), but as follows:

$$Z^1 = (t_i^1 = \bar{\tau} + h,\ t_i^1 = t_i(\bar{\tau}),\ i = \overline{2,p};\ y^1 = y(\bar{\tau} = y(\bar{\tau}))).$$

Subsequent operations conincide with the written above.

Now suppose for some $\tau=\bar{\tau}$ the condition (17) is violated. For constructing (20) vector Z^1 is calculated not by (18) but as follows:

$$Z^1 = (t_i^1 = t_i(\bar{\tau}),\ i = \overline{1,p-1},\ t_p^1 = t_p(\bar{\tau}) - h;\quad y^1 = y(\bar{\tau})).$$

Subsequent operations conincide with the written above.

2.5 Algorithm for Acting of the Optimal Controller

Set up the parameter $\nu>0$ which determines the maximum switching frenquency of controls produced by the controller.

The controller begins to function from the moment $\tau=0$ with $t_*(0), y(0)$, where $t_*(0) = \{t_i(0), i=\overline{1,p}\}$ is the set of switching points of the optimal control $u^0(t|0,x_0), t\in T$, for the problem (5)~(8). $y(0)$ is the optimal optential vector of this problem for the $u(\cdot)$-support t_{sp}. This information can be calculaed before starting of the controller.

Suppose that controller had worked in the period $[0,\tau), 0<\tau<t^0$, and produced the control $u^*(t), t\in[0,\tau)$. Let $x^*(\tau)$ be the state at the moment τ given by the system (2) under action of the control $u^*(t), t\in[0,\tau)$ and the perturbance $w^*(t), t\in[0,\tau)$.

Denote the last discontinuious point. (the nearst from the left to τ) of the control $u^*(t), t\in[0,\tau)$by $t_-(\tau)$. (if $\tau=0, t_-(\tau)=-\infty$).

By the information $\{\tau, x^*, (\tau)\}$ the processor responsible for the solution of the defining equations (14) and (15) supplies $\{t_*(\tau), y(\tau)\}$ for the controller. By $\{t_*(\tau), y(\tau)\}$ the controller can construct the function $u^0(t|\tau, x^*(\tau)), t\in T_\tau$. We believe that at the moment τ the controller gives the control

$$u^*(\tau) = \begin{cases} u^0(\tau|\tau, x^*(\tau)), \text{ if } \tau - t_-(\tau) < \nu, \\ u^0(\tau - 0), \text{ if } \tau - t_-(\tau) > \nu. \end{cases}$$

Acting such a way controller workes out the control $u^*(t), t\in T$, which accepts only three values $-1\ u_0, 1$. What is more, that the distance between its switching points is no less than ν.

One of the important elements at solving the defining equations (14), (15) § 2. 3 is analysis of change of the structure $S(\tau)=\{K^+(\tau), K^-(\tau), K^0(\tau), p(\tau)\}$. Due to capacity limit this part is not given here.

3 Example

Let us illustrate the acting of controller using the control problem of oscillatory motion. It is required in the fixed time duration with minimal fuel consumption to quiet the harmonious oscillator under perturbances.

The mathematical model of this problem is as follows:

$$\int_0^{3\pi} |u(t)|\mathrm{d}t \to \min,$$

$$\ddot{x}+x=u+w, \quad x(0)=3\sqrt{3}, \quad \dot{x}(0)=0,$$

$$x(3\pi)=0, \quad \dot{x}(3\pi)=0, \quad |u(t)|\leqslant 1, \quad t\in[0,3\pi].$$

Introduce the phase variables $x_1=x, x_2=\dot{x}$ and the addtional variable $x_3=\int_0^t |u(s)|\mathrm{d}s$, the problem becomes into the canonical form (1)~(4) with the parameters:

$$n=3,\ m=2,\ c=(0,0,-1),\ b_1=(0,1,1),\ b_2=(0,1,-1),\ v_1=v_2=0,$$

$$u_0=0,\quad T=[0,t^*]=[0,3\pi],\quad x_0=3\sqrt{3},0,0),\quad g=(0,0),$$

$$A=\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\quad H=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}.$$

The optimal program control of the problem

$$u^0(t)=\begin{cases}1, & t\in(\pi/6,5\pi/6)\cup(13\pi/6,17\pi/6),\\ -1 & t\in(7\pi/6,11\pi/6),\\ 0, & \text{at the other moments.}\end{cases}$$

The set of its switching points is

$$t_*(0)=\{\pi/6,\ 5\pi/6,\ 7\pi/6,\ 11\pi/6,\ 13\pi/6,\ 17\pi/6\}.$$

At the $u^0(\cdot)$-support $t_{sp}\{\pi/6, 7\pi/6\}$, the potential vector is $y(0)=(-2,0)$.

As perturbances let us take two functions:

$$\text{I}: \quad w_1(t)=\begin{cases}\sin 3t, & t\in[0,\pi],\\ 0, & t\in(0,3\pi],\end{cases}$$

$$\text{II}: \quad w_2(t)=\begin{cases}\sin 3t, & t\in[0,2\pi/3],\\ 0, & t\in(2\pi/3,3\pi].\end{cases}$$

Remark In fact, the influence of perturbance is considered by system state in the real time regime. In the example, for imitating the measured state of the system forced by the control and perturbance, we take the two functions as perturbances.

Calculations were carried out with the parameters $h=\pi/100$, $\nu=5h$. Every Step 3 iteratons of the Newton method were used.

Data of the optimal program control $u^0(t), t\in T$ are inculded in line 0 of the table.

Data of the controls $u^*(t), t\in T$ worked out by optimal controller (5) under the influence of perturbances $w_1(t), w_2(t), t\in T$ are given in the lines Ⅰ, Ⅱ of the table.

Table 1 Switch points of controls for oscillatory motion system

	t_1	t_2	t_3	t_4	t_5	t_6
0	0.52(0+)*	2.62(+0)	3.67(0−)	5.76(−0)	6.81(0+)	8.90(+0)
Ⅰ	0.66(0+)	2.73(+0)	3.52(0−)	5.69(−0)	6.66(0+)	8.83(+0)
Ⅱ	0.66(0+)	2.87(+0)	3.48(0−)	6.00(−0)	6.62(0+)	9.15(+0)

* The symbol (0+) means that at the moment t_1 the control changes value from u_0 to +1.

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分段线性系统最优反馈控制的综合

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摘要: 本文以明斯克一批学者创建的最优控制问题的算法为基础提出了一个用于解决反馈最优控制问题的新方法. 对综合问题给出了一个动态提法,构造了一个用于解决实时最优控制问题的算法. 研究对象为具有分段线性输入装置系统在具有控制及终端状态约束下的端点最优控制问题.

关键词: 控制系统; 综合; 决定方程; 控制器

本文作者简介

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