

## Suboptimal Decentralized Stabilization of Large-Scale Delay Stochastic Systems\*

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**Abstract:** In this paper, suboptimal decentralized stabilization of large-scale stochastic systems of Itô type with delays is investigated by the Lyapunov functionals based on matrix Riccati equations. A Criterion is established for the suboptimal decentralized stabilization. An algorithm is given to determine the suboptimal decentralized controllers. It is noted that this algorithm is based on solving Lyapunov matrix equations and Riccati matrix equations. A numerical example is given to show the applicability of the algorithm.

**Key words:** delay system; large-scale stochastic system; decentralized stabilization; optimization; algorithm

### 1 Introduction

Decentralized stabilization of deterministic large-scale systems has been widely investigated and many results have been obtained during the past decades. Decentralized local controllers are employed to stabilize various large-scale systems owing to the speciality of decentralization of information in large-scale systems. Investigation on decentralized stabilization of stochastic large-scale systems will not only develop the previous theory on large-scale systems but also provide more desirable tools for engineering practice. In the past years, some results have been obtained on the stability and control of large-scale stochastic systems (see [1~6]), while many problems require further investigation, e. g., stability of high order moments of stochastic systems with delays, decentralized stabilization of large-scale stochastic systems with delays or complicated interconnection. In this paper, suboptimal decentralized stabilization of large-scale stochastic systems with delay interconnected terms is investigated by Lyapunov functionals based on optimal feedback control of linear stochastic systems. An algorithm is given to determine decentralized controllers. A numerical example is given to illustrate the obtained results.

### 2 System Description and Problems

Consider the following large-scale system

$$(S): \quad dx_i(t) = [A_i x_i(t) + B_i u_i(t)]dt + \sum_{j=1}^{r_i} F_{ij} x_j(t) dW_{ij}(t) \\ + \sum_{j=1}^N A_{ij} x_j(t - \tau_{ij})dt, \quad i = 1, 2, \dots, N \quad (1)$$

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where  $x_i \in \mathbb{R}^{n_i}$ ,  $u_i \in \mathbb{R}^{m_i}$ ,  $A_i, F_{ij} \in \mathbb{R}^{n_i \times n_i}$ ,  $B_i \in \mathbb{R}^{n_i \times m_i}$ ,  $A_{ij} \in \mathbb{R}^{n_i \times n_j}$ , all the matrices are constant matrices,  $\tau_{ij} \geq 0$ ,  $W(t) = [W_{11}(t), \dots, W_{1r_1}(t), \dots, W_{N1}(t), \dots, W_{Nr_N}(t)]^T$  ( $t \geq 0$ ) is an  $(\sum_{i=1}^N r_i)$ -dimensional standard Wiener process with independent components defined on a complete probability space  $(\Omega, \mathcal{F}, P)$ ,  $A_{ii} = 0, i = 1, 2, \dots, N$ .

$$\begin{aligned} \text{Let } \sum_{i=1}^N n_i &= n, \quad \sum_{i=1}^N m_i = m, \quad x = (x_1^T, x_2^T, \dots, x_N^T)^T, \quad u = (u_1^T, u_2^T, \dots, u_N^T)^T, \\ A &= \text{block-diag}(A_1, A_2, \dots, A_N), \quad B = \text{block-diag}(B_1, B_2, \dots, B_N), \\ \bar{F}_{ij} &= E_{ij} \otimes F_{ij}, \quad j = 1, 2, \dots, r_i; \quad i = 1, 2, \dots, N, \\ \bar{A}_{ij} &= E_{ij} \otimes A_{ij}, \quad j = 1, 2, \dots, r_i; \quad i = 1, 2, \dots, N. \end{aligned}$$

Then the large-scale system (S) can be rewritten in the following equivalent form

$$dx(t) = [Ax(t) + Bu(t)]dt + \sum_{i=1}^N \sum_{j=1}^{r_i} \bar{F}_{ij} x(t) dW_{ij}(t) + \sum_{i=1}^N \sum_{j=1}^{r_i} \bar{A}_{ij} x(t - \tau_{ij}) dt. \quad (2)$$

It is assumed that  $(A_i, B_i)$  is assumed to be controllable. For the given positive definite matrices  $Q_i \in \mathbb{R}^{n_i \times n_i}$  and  $R_i \in \mathbb{R}^{m_i \times m_i}$ , it is known that for the following optimal stochastic control problem

$$dx_i = (A_i x_i + B_i u_i) dt + \sum_{j=1}^{r_i} F_{ij} x_i dW_{ij}, \quad i = 1, 2, \dots, N, \quad (3)$$

$$\text{s.t. } \min_u J = E \left[ \int_0^\infty e^{2\gamma t} (x_i^T Q_i x_i + u_i^T R_i u_i) dt \right] \quad (4)$$

its solution is given by

$$u_i = -R_i^{-1} B_i^T P_i x_i(t), \quad i = 1, 2, \dots, N \quad (5)$$

where  $P_i$  is the positive definite solution of the Riccati equation

$$(A_i + \gamma I_{n_i})^T P_i + P_i (A_i + \gamma I_{n_i}) - P_i B_i R_i^{-1} B_i^T P_i + \sum_{j=1}^{r_i} F_{ij}^T P_i F_{ij} + Q_i = 0, \quad i = 1, 2, \dots, N. \quad (6)$$

By using the feedback control law (5), the equilibrium of the closed-loop system of the  $i$ th isolated subsystem in (3) is mean-square exponentially asymptotically stable with decay  $r_i$ . For the definition of mean-square exponential asymptotic stability, see [1]. The local state feedback law (5) may or may not stabilize the large-scale system (S), according to the interconnection between the subsystems and the choice of performance parameters of the isolated subsystems, e.g.  $Q_i, R_i$ .

The problems we will solve in this paper are:

- 1) establish a criterion for the local controllers in the form of (5) which are used to stabilize the large-scale system (S);
- 2) give an optimal algorithm to choose appropriate parameters  $Q_1, Q_2, \dots, Q_N$  and to seek for possible decentralized stabilizing controllers which satisfy the criterion.

### 3 Notation, Lemmas and Definition

In this paper,  $\|x\|$  denotes the Euclidean norm of vector  $x$ ,  $\lambda_m(A)$  denotes the maximal eigenvalue of matrix  $A$ ,  $\lambda_n(A)$  denotes the minimal eigenvalue of matrix  $A$ ,  $I_n$  denotes the  $n$ -

identity matrix of order  $n$ ,  $G > 0$ ,  $G$  stands for that  $G$  is positive definite,  $A \otimes B$  is the Kronecker's tensor product of matrices  $A$  and  $B$ ,  $E_{ij} = (e_{ik}^{ij})_{n \times n}$ , where

$$e_{ik}^{ij} = \begin{cases} 1, & (i, k) = (i, j), \\ 0, & (i, k) \neq (i, j). \end{cases}$$

**Lemma 1** For any positive real number  $r$ , vectors  $X, Y \in \mathbb{R}^n$  and positive definite matrix  $P \in \mathbb{R}^{n \times n}$ ,  $(rX - Y)^T P (rX - Y) \geq 0$ , i. e. the following inequality holds

$$2X^T P Y \leq rX^T P X + \frac{1}{r}Y^T P Y.$$

**Lemma 2** (Hellman-Feynman Theorem<sup>[7,8]</sup>) If  $G = G(\beta_1, \beta_2, \dots, \beta_N) \in \mathbb{R}^{n \times n}$  is a symmetric matrix-valued function of  $\beta_1, \beta_2, \dots, \beta_N$ ,  $v$  is the eigenvector of  $G$  associated with the maximal eigenvalue of  $G$ , i. e.  $\lambda_M(G)$ , then

$$\frac{\partial \lambda_M(G)}{\partial \beta_j} = \frac{v^T (\partial G / \partial \beta_j) v}{v^T v}, \quad j = 1, 2, \dots, N. \quad (7)$$

**Proof** By  $Gv = \lambda_M(G)v$  we have

$$\frac{\partial G}{\partial \beta_j} v + G \frac{\partial v}{\partial \beta_j} = \frac{\partial \lambda_M(G)}{\partial \beta_j} v + \lambda_M(G) \frac{\partial v}{\partial \beta_j}. \quad (8)$$

By multiplying both sides of (8) with  $v^T$  and applying  $v^T G = (Gv)^T = \lambda_M(G)v^T$ , we immediately obtain the result.

**Definition** System (S) is said to be stabilized by local feedback controllers (5) if the equilibrium of the closed-loop system is mean-square asymptotically stable.

#### 4 Criterion

**Criterion** Suppose that  $(A_i, B_i)$  is controllable,  $Q_i$  and  $R_i$  are the given positive definite matrices,  $\gamma > 0$ ,  $P_i$  is the positive definite and symmetric solution of matrix Riccati equation (6) with  $\gamma_i = \gamma$ , then (S) is stabilized by the local feedback control law (5) provided that the matrix

$$G = \frac{N^2}{2\gamma} \sum_{i=1}^N \sum_{j=1}^N \bar{A}_{ij}^T P \bar{A}_{ij} - Q - PBR^{-1}B^T P \quad (9)$$

is negative definite, where

$$Q = \text{block-diag}(Q_1, Q_2, \dots, Q_N),$$

$$R = \text{block-diag}(R_1, R_2, \dots, R_N),$$

$$P = \text{block-diag}(P_1, P_2, \dots, P_N).$$

**Proof** Let the Lyapunov functional  $V$  be given by  $V = V_1 + V_2$ , where

$$V_1 = x^T P x \quad (10)$$

and

$$V_2 = \frac{N^2}{2\gamma} \sum_{i=1}^N \sum_{j=1}^N \int_{t-\tau_{ij}}^t x^T(s) \bar{A}_{ij}^T P \bar{A}_{ij} x(s) ds. \quad (11)$$

Let  $\mathcal{L}$  the adjoint partial differential operator of the closed-loop system of (S) by (5).

We have

$$\begin{aligned} \mathcal{L} V_1 = & x^T(t) [(A - BR^{-1}B^T P)^T P + P(A - BR^{-1}B^T P) + \sum_{i=1}^N \sum_{j=1}^N \bar{F}_{ij}^T P \bar{F}_{ij}] x(t) \\ & + 2x^T(t) P \sum_{i=1}^N \sum_{j=1}^N \bar{A}_{ij} x(t - \tau_{ij}) \end{aligned}$$

$$\begin{aligned}
&= x^T(t)[A^T P + PA - 2PBR^{-1}B^T P + \sum_{i=1}^N \sum_{j=1}^{r_i} \bar{F}_{ij}^T P \bar{F}_{ij}]x(t) \\
&\quad + \sum_{i=1}^N \sum_{j=1}^N 2x^T(t)P\bar{A}_{ij}x(t - \tau_{ij}) \\
&\leq x^T(t)[A^T P + PA - 2PBR^{-1}B^T P + \sum_{i=1}^N \sum_{j=1}^{r_i} \bar{F}_{ij}^T P \bar{F}_{ij}]x(t) \\
&\quad + \sum_{i=1}^N \sum_{j=1}^N [\frac{2\gamma}{N^2} x^T(t)Px(t) + \frac{N^2}{2\gamma} x^T(t - \tau_{ij})\bar{A}_{ij}^T P \bar{A}_{ij}x(t - \tau_{ij})] \\
&= x^T(t)[(A + \gamma I_n)^T P + P(A + \gamma I_n) - 2PBR^{-1}B^T P] \\
&\quad + \sum_{i=1}^N \sum_{j=1}^{r_i} \bar{F}_{ij}^T P \bar{F}_{ij}]x(t) + \frac{N^2}{2\gamma} \sum_{i=1}^N \sum_{j=1}^N x^T(t - \tau_{ij})\bar{A}_{ij}^T P \bar{A}_{ij}x(t - \tau_{ij}).
\end{aligned}$$

Thus

$$\mathcal{L}V = \mathcal{L}V_1 + \mathcal{L}V_2$$

$$\begin{aligned}
&\leq x^T(t)[(A + \gamma I_n)^T P + P(A + \gamma I_n) - 2PBR^{-1}B^T P + \sum_{i=1}^N \sum_{j=1}^{r_i} \bar{F}_{ij}^T P \bar{F}_{ij}] \\
&\quad + \frac{N^2}{2\gamma} \sum_{i=1}^N \sum_{j=1}^N \bar{A}_{ij}^T P \bar{A}_{ij}]x(t) \\
&= x^T(t)(-Q - PBR^{-1}B^T P + \frac{N^2}{2\gamma} \sum_{i=1}^N \sum_{j=1}^N \bar{A}_{ij}^T P \bar{A}_{ij})x(t) \\
&= x^T(t)Gx(t) \leq \lambda_M(G) \|x(t)\|^2.
\end{aligned}$$

Since  $G$  is negative definite, we get that  $\lambda_M < 0$ . This shows that  $\mathcal{L}V$  is negative definite. By the stability theorem of stochastic functional differential equations<sup>[9,10]</sup>, the equilibrium of (S) is mean-square asymptotically stable. The proof is complete.

## 5 Algorithm Analysis

If matrix  $G < 0$  for the given matrices  $Q, R$ , then the decentralized controllers (5) can be used to stabilize the large-scale system (S). When  $G$  is not negative definite, it does not mean that there are not decentralized controllers that stabilize the large-scale system. In this case, one may find new local controllers that stabilize the large-scale system (S) by changing some performance parameters of the subsystems. In the following algorithm, for the sake of convenience, we only change parameters in  $Q_1, Q_2, \dots, Q_N$  to optimize  $\lambda_M(G)$ .

$$\text{Set} \quad Q = \text{block-diag}(\beta_1 \tilde{Q}_1, \dots, \beta_N \tilde{Q}_N) \quad (12)$$

then (6) becomes

$$(A_i + \gamma I_{n_i})^T P_i + P_i(A_i + \gamma I_{n_i}) - P_i B_i R_i^{-1} B_i^T P_i + \sum_{j=1}^{r_i} F_{ij}^T P_i F_{ij} + \beta_i \tilde{Q} = 0, \quad i = 1, 2, \dots, N \quad (13)$$

and  $P_i$  is the function of  $\beta_i$ ,  $G$  is the function of  $\beta_1, \beta_2, \dots, \beta_N$ .

$$\text{Let} \quad \beta = (\beta_1, \beta_2, \dots, \beta_N)^T, \quad G = G(\beta). \quad (14)$$

In order to solve the optimal problem

$$\min_{\beta > 0} [\lambda_M(G(\beta))] \quad (15)$$

we apply the following gradient method

$$\beta(k+1) = \beta(k) - \rho \nabla \lambda_M[G(\beta(k))], \quad k = 0, 1, 2, \dots, \quad (16)$$

where  $\rho$  is a positive constant and where

$$\nabla \lambda_M[G(\beta(k))] = \left[ \frac{\partial \lambda_M(G)}{\partial \beta_1}, \dots, \frac{\partial \lambda_M(G)}{\partial \beta_N} \right]_{\beta=\beta(k)}^T. \quad (17)$$

By Lemma 2, if  $G(\beta(k))v_k = \lambda_M[G(\beta(k))]v_k$ , then

$$\frac{\partial \lambda_M[G(\beta(k))]}{\partial \beta_i} = \frac{v_k^T \frac{\partial G(\beta(k))}{\partial \beta_i} v_k}{v_k^T v_k}, \quad i = 1, 2, \dots, N. \quad (18)$$

By (9), we have

$$\frac{\partial G}{\partial \beta_i} = \frac{N^2}{2\gamma} \sum_{i=1}^N \sum_{j=1}^N \bar{A}_{ij}^T \frac{\partial P}{\partial \beta_i} \bar{A}_{ij} - \frac{\partial Q}{\partial \beta_i} - \frac{\partial P}{\partial \beta_i} B R^{-1} B^T P - P B R^{-1} B^T \frac{\partial P}{\partial \beta_i}, \quad i = 1, 2, \dots, N. \quad (19)$$

where

$$\frac{\partial P}{\partial \beta_i} = \text{block-diag}(0, \dots, 0, \frac{\partial P_i}{\partial \beta_i}, 0, \dots, 0), \quad (20)$$

$$\frac{\partial Q}{\partial \beta_i} = \text{block-diag}(0, \dots, 0, \frac{\partial Q_i}{\partial \beta_i}, 0, \dots, 0) = \text{block-diag}(0, \dots, 0, \tilde{Q}_i, 0, \dots, 0) \quad (21)$$

where  $\frac{\partial P}{\partial \beta_i}$  is determined by (13). By (13), we have

$$\begin{aligned} (A_i + \gamma I_{n_i})^T \frac{\partial P_i}{\partial \beta_i} + \frac{\partial P_i}{\partial \beta_i} (A_i + \gamma I_{n_i}) - \frac{\partial P_i}{\partial \beta_i} B_i R_i^{-1} B_i^T P \\ - P_i B_i R_i^{-1} B_i^T \frac{\partial P_i}{\partial \beta_i} + \sum_{j=1}^{r_i} F_{ij}^T \frac{\partial P_i}{\partial \beta_i} F_{ij} + \tilde{Q} = 0, \quad i = 1, 2, \dots, N \end{aligned}$$

i. e. 
$$\begin{aligned} (A_i + \gamma I_{n_i} - B_i R_i^{-1} B_i^T P_i)^T \frac{\partial P_i}{\partial \beta_i} + \frac{\partial P_i}{\partial \beta_i} (A_i + \gamma I_{n_i} - B_i R_i^{-1} B_i^T P_i) \\ + \sum_{j=1}^{r_i} F_{ij}^T \frac{\partial P_i}{\partial \beta_i} F_{ij} + \tilde{Q} = 0, \quad i = 1, 2, \dots, N. \end{aligned} \quad (22)$$

These equations are matrix Lyapunov equations. The existence of solutions of the Lyapunov matrix equations are guaranteed by that all eigenvalues of  $A_i + \gamma I_{n_i} - B_i R_i^{-1} B_i^T P_i$  ( $i = 1, 2, \dots, N$ ) possess negative real parts.

## 6 Algorithm for Selecting Decentralized Controllers

The algorithm is formulated as follows.

Suppose that  $(A_i, B_i)$  is controllable,  $i = 1, 2, \dots, N$ .  $\gamma, \rho > 0$  are given.

### Algorithm

Step 1 Given  $\tilde{Q}_i > 0, \beta_i(0) > 0, i = 1, 2, \dots, N$ ; set  $Q_i = \beta_i(0)\tilde{Q}_i$ , substitute these  $Q_i$  into (13);

Step 2 Solve Riccati equation (13) for each  $i \in \{1, 2, \dots, N\}$ ; Calculate  $G$  by (9) and (10);

Step 3 Calculate  $\lambda_M(G)$  by QL Algorithm, or Lanczos Algorithm<sup>[11]</sup> etc., if  $\lambda_M(G) < 0$ , go to Step 8; otherwise, go to Step 4;

Step 4 Solve (22) for each  $i \in \{1, 2, \dots, N\}$  and calculate  $\frac{\partial P_i}{\partial \beta_i}$  by (20);

Step 5 Calculate  $\frac{\partial G}{\partial \beta_i}$  by (19)~(21);

Step 6 Calculate  $\frac{\partial \lambda_M[G(\beta(0))]}{\partial \beta_i}$  by (18) and  $\nabla \lambda_M[G(\beta(0))]$  by (17);

Step 7 Calculate  $\beta(1)$  by (16), set  $\beta(0) = \beta(1)$ , i. e.  $\beta_i(0) = \beta_i(1)$ ,  $i = 1, 2, \dots, N$ , go to Step 2;

Step 8  $K_i = -R_i^{-1}B_i^T P_i$ ,  $i = 1, 2, \dots, N$ .

## 7 Example

Consider the following interconnected (large-scale) delay stochastic system

$$(\Sigma): \begin{cases} dx_1(t) = [A_1 x_1(t) + B_1 u_1(t)]dt + F_{11} x_1(t) dW_{11}(t) + A_{12} x_2(t - \tau_{12})dt, \\ dx_2(t) = [A_2 x_2(t) + B_2 u_2(t)]dt + F_{21} x_2(t) dW_{21}(t) + A_{21} x_1(t - \tau_{12})dt \end{cases} \quad (23)$$

where  $x_1, x_2 \in \mathbb{R}^2$ ,  $u_1 \in \mathbb{R}^1$ ,  $u_2 \in \mathbb{R}^2$ ,  $A_1, A_2, F_{11}, F_{21}, A_{12}, A_{21} \in \mathbb{R}^{2 \times 2}$ ,  $B_1 \in \mathbb{R}^{2 \times 1}$ ,  $B_2 \in \mathbb{R}^{2 \times 2}$ ,  $0 \leq \tau_{12}, \tau_{21} \leq \tau$ ,  $[W_{11}(t), W_{21}(t)]^T (t \geq 0)$  is a two-dimensional standard Wiener process, and

$$\begin{aligned} A_1 &= A_2 = \begin{bmatrix} -2.00000 & 1.00000 \\ 0 & -2.00000 \end{bmatrix}, & B_1 &= B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ F_{11} &= \begin{bmatrix} 0.50000 & 0 \\ 0 & 0.50000 \end{bmatrix}, & F_{21} &= \begin{bmatrix} 0.32000 & 0 \\ 0 & 0.32000 \end{bmatrix}, \\ A_{12} &= \begin{bmatrix} 0.700711 & 0 \\ 0 & 0.700711 \end{bmatrix}, & A_{21} &= \begin{bmatrix} 0.49497 & 0 \\ 0 & 0.49497 \end{bmatrix}. \end{aligned}$$

It is obvious that  $(A_i, B_i)$  is controllable ( $i = 1, 2$ ). Set

$$R_1 = [1], R_2 = [2], \gamma = 2, \beta(0) = (\beta_1(0), \beta_2(0))^T = (0.75000, 1.00000)^T,$$

$$\tilde{Q}_1 = I_2, \quad \tilde{Q}_2 = \begin{bmatrix} 1.65360 & -0.20000 \\ -0.20000 & 1.65360 \end{bmatrix}.$$

In this example, by solving (13) and (22) we obtain

$$\begin{aligned} P_1 &= \begin{bmatrix} 1.00000 & 1.00000 \\ 1.00000 & 2.00000 \end{bmatrix}, & P_2 &= \begin{bmatrix} 3.46410 & 2.00000 \\ 2.00000 & 3.46410 \end{bmatrix}, \\ \frac{\partial P_1}{\partial \beta_1} &= \begin{bmatrix} 1.22449 & 0.65306 \\ 0.65306 & 0.61497 \end{bmatrix}, & \frac{\partial P_2}{\partial \beta_2} &= \begin{bmatrix} 1.83520 & -0.91856 \\ -0.91856 & 0.78026 \end{bmatrix}. \end{aligned}$$

Thus

$$G = \begin{bmatrix} G_{11} & 0 \\ 0 & G_{11} \end{bmatrix}$$

where  $G_{11} = 2A_{21}^T P_2 A_{21} - Q_1 - P_1 B_1 R_1^{-1} B_1^T P_1 = \begin{bmatrix} -0.50292 & -1.38000 \\ -1.38000 & -3.50292 \end{bmatrix}$ ,

$$G_{22} = 2A_{12}^T P_1 A_{12} - Q_2 - P_2 B_2 R_2^{-1} B_2^T P_2 = \begin{bmatrix} -2.65360 & -2.26410 \\ -2.26410 & -5.65360 \end{bmatrix}.$$

It is easy to verify that  $\lambda_M[G(\beta(0))] < 0$ . By applying the above algorithm, we obtain the following decentralized controllers of  $(\Sigma)$  after the third iteration:

$$\begin{cases} u_1 = -(1.00020, 1.99850)x_1(t), \\ u_2 = -(0.99875, 1.73215)x_2(t). \end{cases} \quad (24)$$

## 8 Conclusion

Suboptimal decentralized stabilization of Itô-type large-scale stochastic systems with multiple delays is investigated. A criterion for the decentralized controllers to stabilize the large-scale system is obtained. Based on this criterion, an algorithm is designed to determine the local controllers by optimizing performance parameters of isolated subsystems. A numerical example is given to illustrate the applicability of the algorithm.

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## 时滞随机大系统的分散次优镇定

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**摘要:** 本文基于 Riccati 矩阵方程、采用 Lyapunov 泛函研究具有时滞的 Itô 型随机大系统的分散次优镇定, 建立了分散次优镇定的判据, 给出了确定分散次优控制器的一个算法, 这个算法以求解 Lyapunov 矩阵方程与 Riccati 矩阵方程为基础。文中给出了数值算例以说明本文算法的用法。

**关键词:** 时滞系统; 随机大系统; 分散镇定; 最优化; 算法

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