第 14 卷第 5 期
1997 年 10 月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol. 14, No. 5
Oct., 1997

A Modified Clarke-Gawthrop Type Self-Tuning Controller with Guaranteed Robust Stability *

SHAO Cheng and ZHANG Chengjin

(Research Center of Automation, Northeastern University · Shenyang, 110006, PRC)

Abstract: In the paper, a new self-tuning controller of the Clacke-Gawthrop type is proposed and shown to be stable with respect to unmodeled dynamics and bounded disturbances. The optimal control law of the Clarke-Gawthrop type is modified by introducing an estimate of the modeling error as a feedback, and a modified least squares scheme with a relative deadzone is employed. The robustness results are derived by neither requiring too much a priori knowledge of the plant parameters, nor using any assumptions on the adaptive signals. Some simulation results are given to illustrate the effectiveness of the new self-tuning controller.

Key words: self-tuning; robust stability; unmodeled dynamics

1 Introduction

Of adaptive control algorithms for the linear systems, an important class is the self-tuning regulation with minimum variance strategy introduced by Astrom and Wittenmark[1]. Since then Clarke and Gawthrop[2~3] developed the self-tuning controller which penalizes not only output error as does the self-tuning regulator, but also excessive control fluctuation, and allows the system to be non-minimum phase and the point to change. The deterministic convergence of the Clarke-Gawthrop type self-tuning controller was estabilished in Tsiligiannis and Svoronos[4] for the disturbance-free case, and its robust stability for bounded disturbance case was investigated in Shao[5]. Although Gawthrop and Lim[6] gave the robust stability of the self-tuning controller in the presence of plant nonlinerities, unmodeled disturbances and plant-model order mismatch, some crucial assumptions that the time delay is minimal ($k = 1$) and the quantity related to the plant inputs and outputs is small, were required by the proposed method.

This paper is to exam the robust stability problem of the Clarke-Gawthrop type self-tuning controller in the presence of unmodeled dynamics and bounded disturbances. A new self-tuning controller of the Clarke-Gawthrop type is proposed. It is shown that the self-tuning controller provides robust stability with respect to the high-order unmodeled dynamics and bounded disturbances under rather relaxed condition. The robustness results together with the simulation results demonstrate that the Clark-Gawthrop type self-tuning controller is both suitable to the nonminimum phase systems and insensitive to small plant changes, which gives support to use the Clarke-Gawthrop type self-tuning controller in practice.

* Supported by Natural Science Foundation of China.
Manuscript received Apr. 4, 1995, revised Jan. 24, 1997.

2 The Plant Description

Consider the plant represented by

$$y(t) = G(q^{-1})u(t-d) + v(t); \quad G(q^{-1}) = \frac{B(1+\mu B')}{A(1+\mu A')}, \tag{2.1}$$

where y and u are the scalar output and input, respectively. v is the bounded output disturbances, $d \geqslant 1$ is the plant delay, A, A', B, B' are polynomials of unit delay operator q^{-1} of orders n_A, $n_{A'}$, n_B and $n_{B'}$, respentively. $\mu \geqslant 0$ is a small singular perturbation scalar, which leads to the high-order unmodeled dynamics observed below. From (2.1) we have

$$y(t) = \frac{B}{A}u(t-d) + \eta_p(t), \tag{2.2}$$

$$\eta_p(t) = \mu\frac{B' - A'}{1+\mu A'}\frac{B}{A}u(t-d) + v(t). \tag{2.3}$$

Using (2.2) gives

$$\eta_p(t) = \mu\frac{B' - A'}{1+\mu B'}y(t) + \frac{1+\mu A'}{1+\mu B'}v(t). \tag{2.4}$$

Then a singular perturbation from $\mu > 0$ to $\mu = 0$ leads to the reduced-order model

$$y(t) = \frac{B}{A}u(t-d) + v(t). \tag{2.5}$$

The designer is assumed to be given only the reduced-order (2.5), and this without knowledge of the coefficients of A and B. Thus the modeling error η_p includes the high-order unmodeled dynamics related to y. The following assumptions are made regarding A and B.

A1) A is a monic polyomial, A and B are relatively prime.

A2) The order n_A and n_B, and the delay d are known.

Remark 1 Assumption A1) indicates that the reduced-order model is controllable, this is an obvious prerequisite for adaptive control. Assumption A2) provides necessary parameter structure of the reduced-order model.

3 A Modified Self-Tuning Controller

The objective here is to propose a self-tuning controller on the basis of the reduced-order model, or the knowledge of A and B, but to apply it to the plant (2.1) such that the closed loop system tracks the desired output and the robust stability is ensured in the presence of the unmodeled dynamics and bounded disturbances.

Let P be arbitrary monic polynomial in q^{-1} of order n. Introduce the polynomial identity

$$P = AF + q^{-d}G, \tag{3.1}$$

where the orders n_F and n_G of F and G satisfy $n_F = d-1$; $n_G = n_A - 1$ or n_p which is the greater, and F is monic also. Then from (2.2) one obtains

$$Py(t+d) = Gy(t) + BFu(t) + AF\eta_p(t+d), \tag{3.2}$$

which can be written in a regressive form as follows

$$\phi(t+d) = \theta^{\mathrm{T}}X(t) + \eta(t+d), \tag{3.3}$$

where $\phi(t+d) = Py(t+d)$; θ is the parameter vector conposed of the coefficients of G and BF; $X(t) = (y(t),\cdots,y(t-n_G),u(t),\cdots,u(t-n_B-d+1))$; $\eta(t+d) = AF\eta_p(t+d)$. The following results establish the fact that η is overbounded relatively to the values of y. We first give a preliminary lemma.

Lemma 1 Let $C(q^{-1})$ be a polynomial in q^{-1} with finite order n_c. Then for arbitrary $\sigma \in (0,1)$ there exists $\mu_0 > 0$ such that $C_\mu(z^{-1}) = 1 + \mu C(z^{-1}) \neq 0$ for all $|z| > \sigma$ and $\mu \in [0, \mu_0]$, i. e. $C_\mu(q^{-1})$ is strictly Hurwitz uniformly in $\mu \in [0, \mu_0]$.

The proof is omitted.

The relative boundedness of η is then given by the following lemma.

Lemma 2 There exist nonnegative constants K_1 and K_2 which are independent of μ, and μ_1 such that for all $\mu \in [0, \mu_1]$

$$|\eta(t)| \leqslant \mu K_1 \max_{0 \leqslant \tau \leqslant t} |y(\tau)| + K_2. \tag{3.4}$$

The proof is omitted.

Now let us investigate what happens if the Clarke-Gawthrop type controller is employed. Suppse $\eta(t)$ is a white noise sequence, an optimal control law can be obtained by means of minimizing the following quadratic cost function with respect to $u(t)$.

$$J = E\{[P(y(t+d) - y^*(t+d))]^2 + [Q'(u(t) - u(t-1))]^2\}, \tag{3.5}$$

where P and Q' are constant weighting polynomials in q^{-1}, $y^*(t)$ is the bounded desired output. Minimizing J as in Gawthrop(1980) gives

$$X^{\mathrm{T}}(t)\theta + Q(u(t) - u(t-1)) = Py^*(t+d), \tag{3.6}$$

where $Q = q'_0 Q'/b_0$; q'_0 and b_0 are the leading coefficients of Q' and B, respectively. Since $\eta(t)$ includes unmodeled dynamics, the control law (3. 6) will not be suitable for our purpose. In particularly, for the self-tuning case, replacing θ in (3. 6) by its estimate $\hat{\theta}(t)$ and then applying (3. 6) to (3. 3) results in

$$P(y(t+d) - y^*(t+d)) = X^{\mathrm{T}}(t)(\theta - \hat{\theta}(t)) - Q(u(t) - u(t-1)) + \eta(t+d). \tag{3.7}$$

Thus because of unmodeled dynamics η the tracking error $e(t) = y(t) - y^*(t)$ will hardly be ensued to tend to zero even though the parameter estimate $\hat{\theta}(t)$ converge to their true values. To remove the effect of unmodeled dynamics we modify the Clarke-Gawthrop type control law (3. 6) by means of introducing an estimate of η

$$\hat{\eta}(t) = \phi(t) - \hat{\theta}^{\mathrm{T}}(t)X(t-d), \tag{3.8}$$

and then a modified self-tuning control law of the Clarke-Gawthrop type is given by

$$X^{\mathrm{T}}(t)\hat{\theta}(t) + Q(u(t) - u(t-1)) = Py^*(t+d) - \hat{\eta}(t). \tag{3.9}$$

It is shown later that such a modification ensues both the robust stability with respect to unmodeled dynamics and the zero average tracking error. For parameter estimation the following modified least squares scheme is employed

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \lambda(t)L(t)\varepsilon(t); \tag{3.10a}$$

$$\varepsilon(t) = \phi(t) - \hat{\theta}^{\mathrm{T}}(t-1)X(t-d); \tag{3.10b}$$

$$L(t) = K(t-2)X(t-d)/[\alpha + X^{\mathrm{T}}(t-d)K(t-2)X(t-d)]; \tag{3.10c}$$

$$K(t-1) = K(t-2) - \lambda(t)L(t)[\alpha + X^{\mathrm{T}}(t-d)K(t-2)X(t-d)]L^{\mathrm{T}}(t); \tag{3.10d}$$

$$\lambda(t) = \begin{cases} 0, & \text{if } |\varepsilon(t)| < 2\beta(\mu^* \max\limits_{0 \leqslant \tau \leqslant t} |y(s)| + 1), \\ \gamma, & \text{otherwise}, \gamma \in [\alpha_0, 3(1-\sigma_0)/4], \quad 0 < \sigma_0 < 3/7, \end{cases} \tag{3.10e}$$

where α and β are positive adjustable parameters given by the designer and with $\beta \geqslant \max\{K_1, K_2\}$ (see (3.4)), amd $\{K(t)\}$ is a matrix sequence with arbitrary initial $K(-1) > 0$.

Remark 2 It is obvious that if the initial value of $\hat{b}_0(t)$ (the estimates of b_0) is taken not equal to g_0 (the leading coefficient of G), then by means of choosing $\lambda(t)$ or/and $\alpha, u(t)$ is always solvable from (3.9). Hence, the singularity in solving $u(t)$ from (3.9) can be overcome.

Remark 3 The condition $\beta \geqslant \max\{K_1, K_2\}$ is not crucial. In practice, we can begin with a large initial value of β, and then reduce it when the closed loop system reaches to the steady state, which may give a better control accuracy.

To deduce robust stability, the following assumption is necessary.

A3) The off-line choices of P and Q are such that

$$f(q^{-1}) = P(q^{-1})B(q^{-1}) + \tilde{Q}(q^{-1})A(q^{-1}), (\tilde{Q} = (1 - q^{-1})Q)$$

is stable, i.e. $f(z) \neq 0, |z| \leqslant 1$.

Remark 4 This condition is basic in the Clarke-Gawthrop type self-tuning controller (e.g. see[4~6]). However, it is made here on the reduced-order model, and the robust adaptive control issues with respect to high-order unmodeled dynamics are considered. Since no further assumptions are made on A and B, the reduced-order model can be unstable and nonminimum phase.

Regarding the unmodeled dynamics only assumption is the following.

A4) A sufficiently small upper bound μ^* of μ is available. (How small "sufficiently small" is, will be made more precise later.)

4 Robust Stability Analysis

To establish robust stability of the resulting closed loop system the following lemmas are necessary.

Lemma 3 If μ^* is sufficienfly small such that $\mu^* < \mu_1$, then for all $\mu \in [0, \mu^*]$ the parameter estimation scheme (3.10) has the following properties

$$\text{i) } \lim_{t\to\infty} \frac{\lambda^{1/2}(t)\varepsilon(t)}{[\alpha + X^{\mathrm{T}}(t-d)K(t-2)X(t-d)]^{1/2}} = 0. \tag{4.1}$$

$$\text{ii) } |(\hat{\theta}(t-1) - \hat{\theta}(t-d))^{\mathrm{T}}X(t-d)| \leqslant h(t)\,\|X(t-d)\|, \quad h(t) \to 0 \text{ as } t \to \infty, \tag{4.2}$$

where $\|\cdot\|$ denotes the vector-Euclidean norm.

iii) $\hat{\theta}(t)$ is bounded.

Proof i) Let $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$. Define $V(t) = \tilde{\theta}^{\mathrm{T}}(t)K^{-1}(t-1)\tilde{\theta}(t)$, then it follows from (3.10a)~(3.10d) that

$$\begin{aligned} V(t) - V(t-1) &= -\frac{3\lambda(t)[\alpha + (1 - 4\lambda(t)/3)X^{\mathrm{T}}(t-d)K(t-2)X(t-d)]\varepsilon^2(t)}{4[\alpha + X^{\mathrm{T}}(t-d)K(t-2)X(t-d)][\alpha + (1-\lambda(t))X^{\mathrm{T}}(t-d)K(t-2)X(t-d)]} \\ &\quad - \frac{\lambda(t)(\varepsilon^2(t)/4 - \eta^2(t))}{\alpha + (1-\lambda(t))X^{\mathrm{T}}(t-d)K(t-2)X(t-d)}, \end{aligned} \tag{4.3}$$

therefore, it follows from (3.4), (3.10e) and (4.3) that $V(t)$ is a bounded and non-increasing sequence and thus converges. In view of (4.3) one obtains

$$\lim_{t\to\infty}\frac{\lambda(t)[\alpha+(1-4\lambda(t)/3)X^{\mathrm{T}}(t-d)K(t-2)X(t-d)]\varepsilon^2(t)}{[\alpha+X^{\mathrm{T}}(t-d)K(t-2)X(t-d)][\alpha+(1-\lambda(t))X^{\mathrm{T}}(t-d)K(t-2)X(t-d)]}=0. \tag{4.4}$$

From (3. 10e) it is easy to verify that

$$1\geqslant\frac{\alpha+(1-4\lambda(t)/3)X^{\mathrm{T}}(t-d)K(t-2)X(t-d)}{\alpha+(1-\lambda(t))X^{\mathrm{T}}(t-d)K(t-2)X(t-d)}\geqslant\frac{\sigma_0}{1-\sigma_0}>0. \tag{4.5}$$

Then using (4. 4) and (4. 5) results in (4. 1).

ii) From (3. 10a)~(3. 10d) one obtains

$$|\hat\theta(t-1)-\hat\theta(t-d))^{\mathrm{T}}X(t-d)|$$
$$\leqslant\sum_{s=1}^{d-1}\frac{\lambda(t-s)\lambda_{\max}(K(-1))|\varepsilon(t-s)|}{[\alpha+X^{\mathrm{T}}(t-d-s)K(t-2-s)X(t-d-s)]^{1/2}}, \tag{4.6}$$

where $\lambda_{\max}(K(-1))$ denotes the maximum eigenvalues of $K(-1)$. Taking

$$h(t)=\sum_{s=1}^{d-1}\frac{\lambda(t-s)\lambda_{\max}(K(-1))|\varepsilon(t-s)|}{[\alpha+X^{\mathrm{T}}(t-d-s)K(t-2-s)X(t-d-s)]^{1/2}}$$

and using (4. 6) and (4. 1) results in (4. 2).

iii) This is straightforward from the boundedness of $V(t)$ and (3. 10d). Q. E. D.

Lemma 4 The tracking error and the input dynamics satisfy

$$(PB+\tilde QA)e(t)=B\Delta_d\varepsilon(t)+\tilde QA\eta_p(t)+\delta_1(t), \tag{4.7}$$
$$(PB+\tilde QA)u(t-d)=A\Delta_d\varepsilon(t)-PA\eta_p(t)+\delta_2(t), \tag{4.8}$$
$$|\delta_i(t)|\leqslant C'+\omega(t)\max_{0\leqslant\tau\leqslant t}\|X(\tau-d)\|,\quad (i=1,2), \tag{4.9}$$

where $\Delta_d=1-q^{-d}$, $\omega(t)\to 0$ as $t\to\infty$, and the constant $C'>0$ is independent of μ.

Proof Using (2. 2), (3. 6) one obtains

$$Ae(t)=Bu(t-d)+A\eta_p(t)-Ay^*(t). \tag{4.10}$$
$$Pe(t)=\Delta_d\varepsilon(t)+(\hat\theta(t-1)-\hat\theta(t-d))^{\mathrm{T}}X(t-d)-\tilde Qu(t-d)$$
$$+(\hat\theta(t-d)-\hat\theta(t-d-1))^{\mathrm{T}}X(t-2d). \tag{4.11}$$

Multiplying (4. 10) by $\tilde Q$ and (4. 11) by B, and then adding together results in (4. 7) with

$$\delta_1(t)=B[(\hat\theta(t-1)-\hat\theta(t-d))^{\mathrm{T}}X(t-d)$$
$$+(\hat\theta(t-d)-\hat\theta(t-d-1))^{\mathrm{T}}X(t-2d)]-\tilde QAy^*(t). \tag{4.12}$$

Multiplying (4. 10) by P and (4. 11) by $-A$, and then adding together results in (4. 8) with

$$\delta_2(t)=A[(\hat\theta(t-1)-\hat\theta(t-d))^{\mathrm{T}}X(t-d)$$
$$+(\hat\theta(t-d)-\hat\theta(t-d-1))^{\mathrm{T}}X(t-2d)]+PAy^*(t). \tag{4.13}$$

From (3. 10a)~(3. 10e) one obtains

$$|(\hat\theta(t-d)-\hat\theta(t-d-1))^{\mathrm{T}}X(t-2d)|$$
$$\leqslant\frac{\lambda^{1/2}(t-d)|\varepsilon(t-d)|}{[\alpha+X^{\mathrm{T}}(t-2d)K(t-2-d)X(t-2d)]^{1/2}}K\max_{0\leqslant\tau\leqslant t}\|X(\tau-d)\|, \tag{4.14}$$

where $K=\sigma_0^{1/2}\lambda_{\max}(K(-1))$. Noting (4. 1) and the boundedness of y^* it is evident that $\delta_i(t)$, $i=1,2$, defined in (4. 12) and (4. 13) satisfy (4. 9). Q. E. D.

The robust stability results are given in the following theorem.

Theorem 1 Subject to assumptions A1)~A4), there exists sufficiently small $\mu^*>0$ such that the self-tuning controller applied to plant (2. 1) ensures that

i) The closed loop system is globally robust stable in the sense that y and u are bounded for arbitrary bounded initial conditions and $\mu \in [0,\mu^*]$.

ii) The tracking error satisfies

$$\lim_{N\to\infty}\frac{1}{N}\sum_{t=0}^{N} e(t) = 0. \tag{4.15}$$

iii) In addition, for constant disturbancd v (unnecessarily equal to zero) and fixed reference signal y^* it follows that

$$\lim_{t\to\infty}(y(t) - y^*) = 0. \tag{4.16}$$

Proof i) From lemmas 2 and 4, and assumption A3) it follows that there exist μ^* ($\leqslant \mu_1$) and non-negative constants $K_1', K_2', K_3', K_4', K_1'', K_2'', K_3'', K_4''$, such that for arbitrary initial time $t_0 \geqslant 0$, and for all $t \geqslant t_0$

$$|y(t)| \leqslant K_1' + K_2' \max_{0\leqslant\tau\leqslant t}|\varepsilon(\tau)| + K_3'\mu \max_{0\leqslant\tau\leqslant t}|y(\tau)| + K_4' \max_{0\leqslant\tau\leqslant t}\omega(\tau) \max_{0\leqslant\tau\leqslant t}\| X(\tau - d)\|, \tag{4.17}$$

$$|u(t - d)| \leqslant K_1'' + K_2'' \max_{0\leqslant\tau\leqslant t}|\varepsilon(\tau)| + K_3''\mu \max_{0\leqslant\tau\leqslant t}|y(\tau)| + K_4'' \max_{0\leqslant\tau\leqslant t}\omega(\tau) \max_{0\leqslant\tau\leqslant t}\| X(\tau - d)\|. \tag{4.18}$$

If μ^* is small such that $K_3'\mu^* < 1/2$, then from (4.17) one obtains that for all $\mu \in [0,\mu^*]$

$$\max_{0\leqslant\tau\leqslant t}|y(\tau)| \leqslant 2K_1' + K_2' \max_{0\leqslant\tau\leqslant t}|\varepsilon(\tau)| + 2K_4' \max_{0\leqslant\tau\leqslant t}\omega(\tau) \max_{0\leqslant\tau\leqslant t}\| X(\tau - d)\|. \tag{4.19}$$

Substituting (4.19) into (4.18) it follows that there exist non-negative constants K_5'', K_6'', K_7'' such that for sufficiently large t and $\mu \in [0,\mu^*]$

$$|u(t - d)| \leqslant K_5'' + K_6'' \max_{0\leqslant\tau\leqslant t}|\varepsilon(\tau)| + K_7'' \max_{0\leqslant\tau\leqslant t}\omega(\tau) \max_{0\leqslant\tau\leqslant t}\| X(\tau - d)\|. \tag{4.20}$$

For sufficiently large t such that $K_7''\,\omega(t) < 1/2$ then

$$\max_{0\leqslant\tau\leqslant t}\| X(\tau - d)\| \leqslant 2K_5'' + 2K_6'' \max_{0\leqslant\tau\leqslant t}|\varepsilon(\tau)|, \tag{4,21}$$

which from (4.19) follows that there exist constants K_1 and K_2 such that

$$\max_{0\leqslant\tau\leqslant t}|y(\tau)| \leqslant K_1 + K_2 \max_{0\leqslant\tau\leqslant t}|\varepsilon(\tau)|. \tag{4.22}$$

Hence the boudedness of $X(t)$ can be ensured by that of $\varepsilon(t)$. The proof is given by contradiction. Assume that $\varepsilon(t)$ is unbounded. Then for arbitrarily large n define the sequence $t_n = \min\{t \mid |\varepsilon(t)| \geqslant n, t \geqslant 0\}$. Along it we have $\max_{0\leqslant\tau\leqslant t}|\varepsilon(\tau)| = |\varepsilon(t_n)|$ and $\lim_{t_n\to\infty}\varepsilon(t_n) = \infty$ and from (4.22) one obtains

$$\frac{2\beta(\mu^* \max_{0\leqslant\tau\leqslant t}|y(\tau)| + 1)}{|\varepsilon(t_n)|} \leqslant \frac{2\beta(\mu^* K_1 + 1)}{|\varepsilon(t_n)|} + 2\beta\mu^* K_2 \leqslant 1 \tag{4.23}$$

provided t_n is sufficiently large and μ^* is sufficiently small. It follows from (3.10e) that for sufficiently large t_n, $\lambda(t_n) \neq 0$, which from (4.21) implies that

$$\lim_{t_n\to\infty}\frac{\lambda^{1/2}(t_n)|\varepsilon(t_n)|}{[\alpha + X^{\mathrm{T}}(t_n - d)K(t_n - 2)X(t_n - d)]^{1/2}} \geqslant \frac{\sigma_0^{1/2}}{2\lambda_{\max}^{1/2}(K(-1))K_6''} > 0,$$

which contradicts to (4.1). This means that the assumption that $\varepsilon(t)$ is unbounded is false, and thus the proof of Theorem 1 i) is completed.

ii) Using (4.11) one obtains

$$\sum_{t=d}^{N} Pe(t) = \sum_{t=N-d-1}^{N} \varepsilon(t) - \sum_{t=0}^{d-1}\varepsilon(t) + \sum_{t=d}^{N}(\hat{\theta}(t-1) - \hat{\theta}(t-d))^{\mathrm{T}}X(t-d) - Qu(N-d)$$
$$+ Qu(0) + \sum_{t=d}^{N}(\hat{\theta}(t-d) - \hat{\theta}(t-2d))^{\mathrm{T}}X(t-2d). \tag{4.24}$$

Since all signals in the adaptive control system are bounded, it follows from (4. 2), (4. 14), (4. 1) and (4. 24) that

$$P(1)\lim_{N\to\infty}\frac{1}{N}\sum_{t=0}^{N}e(t) = \lim_{N\to\infty}\frac{1}{N}\sum_{t=d}^{N}Pe(t) = 0. \tag{4.25}$$

Assumption A3) implies $P(1) \neq 0$, the result thus follows.

iii) Using (4. 1) and the boundedness of y and u , gives $\lim_{t\to\infty}\lambda^{1/2}(t)\varepsilon(t) = 0$. Which from (3. 10e) follows that there exists a sufficiently large time t_0 such that for all $t \geqslant t_0, \lambda(t) = 0$. Therefore, when $t \geqslant t_0$ the parameter estimates defined in (3,9) enter the relative deadzone, and furthermore the control law (3. 9) becomes linear and time-invariant. Since the closed loop system is BIBO stable, then for constant v and fixed y^* it will approach to steady state. Using (4. 15) the conclusion is derived readily. Q. E. D.

5 Simulation Examples

Example 1 Consider a 3-order unstable minimum phase plant with delay 2

$$y(t) = \frac{B(1+\mu B')}{A(1+\mu A')}u(t-2) + v(t), \tag{5.1}$$

where $A = 1 + q^{-1} + q^{-2}, B = 1, A' = q^{-1}, B' = 2q^{-1}, v(t)$ is a white noise sequence with variance of 0. 01. We choose P and Q as $P(q^{-1}) = 6 + 5q^{-1} + q^{-2}, Q(q^{-1}) = 1$ and take $\alpha = 1, \beta = 1, \gamma = 1/2, \mu = \mu^* = 0.01$ with initial conditions $\hat{\theta}(0) = [-3.5, 0.3, 5.4, -0.2]^{\mathrm{T}}, K(1) = I > 0$. As shown in figure 1, the self-tuning controller works well even if there exist high-order unmodeled dynamics and disturbances.

Example 2 the self-tuning controller works in the case that the reduced-order model is of nonminimum phase is investigated. The plant is as (5. 1) where $B = 0.5 + q^{-1}$. For designing the self-tuning controller, $P(q^{-1}) = 9 + 3q^{-1} + q^{-2}, Q(q^{-1}) = 2, \alpha = 1, \beta = 5, \gamma = 1/2, \mu = \mu^* = 0.01$ with initial conditions $\hat{\theta}(0) = [6, -4, -6, 6, 4.5]^{\mathrm{T}}, K(1) = I > 0$. As shown in figure 2, the self-tuning controller can be suitable to the unstable nonminimum phase plant with satisfactory.

6 Conclusions

In this paper we show that the useful Clarke-Gawthrop self-tuning controller can be modified to give robust stability properties with respect to the high-order unmodeled dynam-

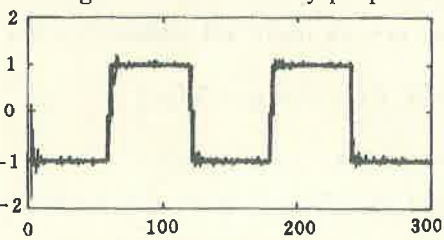

Fig. 1 The plant output and the reference output

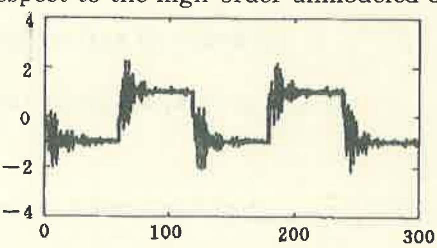

Fig. 2 The plant output and the reference output

ics and bounded external disturbances. In addition, such a modification removes all steady state errors appearing in the adaptive control system. Therefore, the robustness stability and simulation results give the support for the use of Clarke-Gawthrop self-tuning controller in practice.

References

1 Astrom, K. J. and Wittenmark, B.. On self-tuning regulators. Automatica, 1973, 9:185—195

2 Clarke, D. W. and Gawthrop, P. J.. Self-tuning controller. Proc. IEE, 1975, 122:929—935

3 Clarke, D. W. and Gawthrop, P. J.. Self-tuning control. Proc. IEE, 1979, 126:633—638

4 Tsiligiannis, C. A. and Svoronos, S. A.. Deterministic convergence of a Clarke-Gawthrop self-tuning controller. Automatica, 1986, 22(2):193—198

5 Shao, C.. Stable adaptive control system subject to bounded disturbances. Int. J. ACSP, 1991, 5( ):121—130

6 Gawthrop, P. J. and Lim, K. M.. Robustness of self-tuning controller. Proc. IEE, 1982, 129( ):21—27

鲁棒稳定的 Clarke-Gawthrop 改进型自校正控制器

邵　诚　张承进

（东北大学自动化研究中心·沈阳，110006）

摘要：本文提出了一种修改的 Clarke-Gawthrop 自校正控制器，并且证明了控制器对未建模动态和有界扰动的鲁棒稳定性．控制器结合了 Clarke-Gawthrop 二次指标函数，并通过引入模型误差反馈进行了修改，参数辨识采用了带有相对死区的修正最小二乘算法，鲁棒稳定性结果既没有要求系统参数太多的先验知识，也没有使用关于自适应信号的任何假设条件．

关键词：自校正；鲁棒稳定性；未建模动态

本文作者简介

邵　诚　1958 年生．博士，教授．东北大学自动化研究中心副主任，在国内外核心期刊及重要国际会议发表论文三十余篇．主要研究领域：鲁棒自适应控制、非线性系统的滤波和自适应控制等．

张承进　1962 年生．分别于 1983 年、1992 年在青岛海洋大学、山东矿业学院获学士、硕士学位．现为东北大学博士生．主要研究领域：自适应控制、时变系统控制的鲁棒等．