第 14 卷第 5 期 控制理论与应用 Vol. 14, No. 5
1997 年 10 月 CONTROL THEORY AND APPLICATIONS Oct., 1997

Delay-Independent Stability of Linear Systems with Multiple Unknown but Constant Delays*

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Abstract: In this paper, we estabilsh some delay-independent stability criteria for linear time-delay systems including large-scale time-delay systems. The time delays under consideration are multiple and can be arbitrary unknown but constant. Therefore, the obtained results do not depend on the delays. For linear large-scale time-delay systems, an illustrative example shows that our results are better than the existing ones in the literature.

Key words: Multiple arbitrary unknown but constant delays; time-delay systems; large-scale systems; stability

1 Introduction

Over past thirty years, there has been a great amount of literature discussing stability of time-delay systems. The research approaches are either via frequency-domain or via time-domain. The main time-domain methods are Lyapunov methods [1]. The recent results have involved Lyapunov functional method [2~5], Lyapunov-Razumikhin method [6,7] and vector Lyapunov function/functional method [8~110].

In this paper, some delay-independent stability criteria for linear systems with multiple time delays including large-scale time-delay systems are established by Lyapunov functional method together with a vector inequality. The time delays under consideration can be arbitrary unknown but constant ones. An illustrative example shows that our results are less conservative than the existing ones in the literature. In the following, Section 2 gives the main results, Section 3 provides the illustrative example, and Section 4 concludes our discussion.

Notation x^{T} and M^{T} denote the transpose of a vector $x \in \mathbb{R}^n$ and a matrix $M \in \mathbb{R}^{n\times n}$, respectively. $\lambda_M(M)$ and $\lambda_m(M)$ denote the maximum and minimum eigenvalue of M, respectively. $\| x \|_2 = (\sum_{i=1}^n |x_i|^2)^{1/2}$ and $\| M \|_2 = [\lambda_M(M^{\mathrm{T}}M)]^{1/2}$.

2 Main Results

Let us consider the linear system with multiple delays described by

$$\begin{cases} \dot{x}(t) = Ax(t) + \sum_{i=1}^{p} B_i x(t-\tau_i)), \quad 0 \leqslant \tau_i \leqslant r < \infty, t \geqslant 0, \\ x(t) = \varphi(t), \quad t \in [-r, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, A, $B_i \in \mathbb{R}^{n\times n}, i = 1, 2, \cdots, p$, are constant matrices, $\tau_i, i = 1, 2, \cdots, p$, are

* This project was supported in part by NSFC, Guangdong Provincial Natural Science Foundation of China and Outstanding Young Teacher Foundation of SEDC.

Manuscript received Sep. 6, 1996, revised Feb. 20, 1997.

any unknown but constant delays, and $\varphi(t)$ denotes a continuous vector-valued initial function.

We assume that A is stable. Then, the Lyapunov equation

$$PA + A^{\mathrm{T}}P = -2Q, \tag{2}$$

has the unique $n \times n$ symmetric and positive definite solution P, where Q is an $n \times n$ symmetric and positive definite matrix.

Theorem 1 System (1) is asymptotically stable if

$$\| P[B_1 \cdots B_p] \|_2 < \frac{\lambda_m(Q)}{\sqrt{p}}, \tag{3}$$

where P and Q satisfy (2).

Proof Let

$$V(x(t)) = x^{\mathrm{T}}(t)Px(t) + \varepsilon \sum_{i=1}^{p} \int_{t-\tau_i}^{t} x^{\mathrm{T}}(s)x(s)\mathrm{d}s, \tag{4}$$

where P is the solution of (2) and $\varepsilon > 0$ is a positive constant. Along the trajectory of system (1), we obtain

$$\begin{aligned} \dot{V}(x(t)) &= 2x^{\mathrm{T}}(t)PAx(t) + 2x^{\mathrm{T}}(t)P\sum_{i=1}^{p} B_i x(t-\tau_i) \\ &\quad + \varepsilon \sum_{i=1}^{p} (x^{\mathrm{T}}(t)x(t) - x^{\mathrm{T}}(t-\tau_i)x(t-\tau_i)) \\ &= -2x^{\mathrm{T}}(t)Qx(t) + 2x^{\mathrm{T}}(t)P[B_1 \cdots B_p]\begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_p) \end{bmatrix} \\ &\quad + \varepsilon \sum_{i=1}^{p} (x^{\mathrm{T}}(t)x(t) - x^{\mathrm{T}}(t-\tau_i)x(t-\tau_i)). \end{aligned} \tag{5}$$

It is easy to show that

$$2u^{\mathrm{T}}Mv \leqslant \frac{1}{\varepsilon}u^{\mathrm{T}}MM^{\mathrm{T}}u + \varepsilon v^{\mathrm{T}}v, \quad u \in \mathbb{R}^n, v \in \mathbb{R}^m, \tag{6}$$

holds for any constant $\varepsilon > 0$ and any constant matrix $M \in \mathbb{R}^{n \times m}$. By using (6), we further obtain

$$\begin{aligned} \dot{V}(x(t)) &\leqslant -2x^{\mathrm{T}}(t)Qx(t) + \frac{1}{\varepsilon}x^{\mathrm{T}}(t)P[B_1 \cdots B_p][B_1 \cdots B_p]^{\mathrm{T}}Px(t) \\ &\quad + \varepsilon \sum_{i=1}^{p} x^{\mathrm{T}}(t-\tau_i)x(t-\tau_i) + \varepsilon \sum_{i=1}^{p} (x^{\mathrm{T}}(t)x(t) - x^{\mathrm{T}}(t-\tau_i)x(t-\tau_i)) \\ &= -2x^{\mathrm{T}}(t)Qx(t) + \frac{1}{\varepsilon}x^{\mathrm{T}}(t)P[B_1 \cdots B_p][B_1 \cdots B_p]^{\mathrm{T}}Px(t) + \varepsilon p x^{\mathrm{T}}(t)x(t). \end{aligned} \tag{7}$$

Let

$$\varepsilon = \left[\frac{\| P[B_1 \cdots B_p][B_1 \cdots B_p]^{\mathrm{T}}P \|_2}{p} \right]^{1/2}. \tag{8}$$

Substituting (8) into (7) yields

$$\dot{V}(x(t)) \leqslant -2[\lambda_m(Q) - (p \| P[B_1 \cdots B_p][B_1 \cdots B_p]^{\mathrm{T}}P \|_2)^{1/2}] \| x(t) \|_2^2. \tag{9}$$

Note that $\| P[B_1 \cdots B_p][B_1 \cdots B_p]^{\mathrm{T}}P \|_2^{1/2} = \| P[B_1 \cdots B_p] \|_2$. If (3) holds, then we have

$$\dot{V}(x(t)) \leqslant -2\rho \| x(t) \|_2^2, \tag{10}$$

where $\rho = \lambda_m(Q) - \sqrt{p}\,\| P[B_1 \cdots B_p] \|_2 > 0$. We complete the proof.

Corollary 1 Let P is the solution of (2) with a symmetric and positive definite matrix Q. System (1) is asymptotically stable if i)

$$\| [B_1 \cdots B_p] \|_2 < -\lambda_M[(A + A^{\mathrm{T}})/2] / \sqrt{p} \tag{11}$$

with $P = I_n$, or ii)

$$\| P[B_1 \cdots B_p] \|_2 < 1/\sqrt{p} \tag{12}$$

with $Q = I_n$, or iii)

$$\| [B_1 \cdots B_p] \|_2 \leqslant \eta/(k\sqrt{p}) \tag{13}$$

with $Q = I_n$ and $\left\| e^{(A+A^{\mathrm{T}})t} \right\|_2 \leqslant k e^{-2\eta t}, t \geqslant 0, k \geqslant 1, \eta > 0$, or iv)

$$\| [(1/d_1)B_1 \cdots (1/d_p)B_p] \|_2 < -\lambda_M[(A + A^{\mathrm{T}})/2] \tag{14}$$

with $P = I_n$, or

$$\| P[(1/d_1)B_1 \cdots (1/d_p)B_p] \|_2 < 1 \tag{15}$$

with $Q = I_n$, or

$$\| [(1/d_1)B_1 \cdots (1/d_p)B_p] \|_2 < \eta/k \tag{16}$$

with $Q = I_n$ and $\left\| e^{(A+A^{\mathrm{T}})t} \right\|_2 \leqslant k e^{-2\eta t}, t \geqslant 0, k \geqslant 1, \eta > 0$, where $d_i > 0$ are constants such that $\sum_{i=1}^{p} d_i^2 = 1$ in (14), (15) and (16).

Proof i) Let $P = I_n$. Note that $\lambda_m(Q) = -\lambda_M[(A + A^{\mathrm{T}})/2]$. Therefore, (3) implies (11). ii) Let $Q = I_n$. Then, it is easy to see that (3) implies (12). iii) Let $\left\| e^{(A+A^{\mathrm{T}})t} \right\|_2 \leqslant k e^{-2\eta t}, t \geqslant 0$, for some $k \geqslant 1$, and $\eta > 0$, and $Q = I_n$. It is well known that $P = 2\int_0^\infty e^{A^{\mathrm{T}}t} e^{At} \mathrm{d}t$ which gives

$$\| P \|_2 \leqslant 2\int_0^\infty \left\| e^{(A+A^{\mathrm{T}})t} \right\|_2 \mathrm{d}t \leqslant 2k \int_0^\infty e^{-2\eta t} \mathrm{d}t = k/\eta. \tag{17}$$

Note that $\| P[B_1 \cdots B_p] \|_2 \leqslant \| P \|_2 \| [B_1 \cdots B_p] \|_2$. We obtain (13) almost immediately from the proof of Theorem 1. iv) Let $V(x(t)) = x^{\mathrm{T}}(t)Px(t) + \varepsilon \sum_{i=1}^{p} \int_{t-\tau_i}^{t} d_i^2 x^{\mathrm{T}}(s)x(s)\mathrm{d}s$, where $d_i > 0$ are constants such that $\sum_{i=1}^{p} d_i^2 = 1$. Based on a similar procesure from (5) to (10) with $\varepsilon = (\| P[(1/d_1)B_1 \cdots (1/d_p)B_p][(1/d_1)B_1 \cdots (1/d_p)B_p]^{\mathrm{T}} P \|_2)^{1/2}$ and the above proofs: i), ii) and iii), it is not difficult to derive (14), (15) and (16), respectively.

Remark 1 It is easy to see that the results in Corollary 1 have been given [11] but we prove them by time-domain method.

Remark 2 Let $p = 1$ and $B_1 = E(t)$ with $\| E(t) \|_2 < \eta, \eta < 0$. Then, (3) implies Theorem 1 given in [12]. If let $B_i = \eta_i(t)E_i, i = 1, 2, \cdots, p$, and by introducing a similarity transformation matrix T, one can further improve (3) by choosing a proper matrix T and obtain Theorem 2 of [12].

In the following, we extend the above technique to linear large-scale time-delay system in the form

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + \sum_{j=1}^{N} A_{ij} x_j(t - \tau_{ij}), & t \geqslant 0, i = 1, 2, \cdots, N, \\ x_i(t) = \varphi_i(t), & t \in [-r, 0], \end{cases} \tag{18}$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $\sum_{i=1}^{N} n_i = n, \mathbb{R}^n = \mathbb{R}^{n1} \times \mathbb{R}^{n2} \times \cdots \times \mathbb{R}^{nN}, A_i \in \mathbb{R}^{n_i \times n_i}$ and $A_{ij} \in \mathbb{R}^{n_i \times n_j}$ are constant matrices, $0 \leqslant \tau_{ij} \leqslant r < \infty, i, j = 1, 2, \cdots, N$, denote arbitrary unknown but constant time delays, and $\varphi_i(t), i = 1, 2, \cdots, N$, denote the initial functions on $t \in [-r, 0]$.

Assume that A_i is stable for all $i = 1, 2, \cdots, N$. Then, the Lyapunov equation

$$P_iA_i + A_i^{\mathrm{T}}P_i = -2Q_i, \tag{19}$$

has the unique $n_i \times n_i$ symmetric and positive definite solution P_i, where Q_i is an $n_i \times n_i$ symmetric and positive definite matrix.

Theorem 2 System (18) is asymptotically stable if

$$\frac{\lambda_m^2(Q_i)}{\| P_i[A_{i1} \cdots A_{iN}] \|_2^2} > N, \quad i = 1, 2, \cdots, N, \tag{20}$$

where P_i and Q_i satisfy (19).

Proof Let

$$V(x(t)) = \sum_{i=1}^{N} [\gamma_i x_i^{\mathrm{T}}(t) P_i x_i(t)) + \sum_{j=1}^{N} \int_{t-\tau_{ij}}^{t} x_j^{\mathrm{T}}(s) x_j(s) \mathrm{d}s], \tag{21}$$

where P_i is the solution of (19) and $\gamma_i > 0, i = 1, 2, \cdots, N$, are positive constants. Along the trajectory of system (18), we obtain

$$\begin{aligned} \dot{V}(x(t)) = \sum_{i=1}^{N} \{ &-2\gamma_i x_i^{\mathrm{T}}(t) Q_i x_i(t) + \sum_{j=1}^{N} (x_j^{\mathrm{T}}(t) x_j(t) - x_j^{\mathrm{T}}(t-\tau_{ij}) x_j(t-\tau_{ij})) \\ &+ 2\gamma_i x_i^{\mathrm{T}}(t) P_i [A_{i1} \cdots A_{iN}] \begin{bmatrix} x_1(t-\tau_{i1}) \\ \vdots \\ x_N(t-\tau_{iN}) \end{bmatrix} \}. \end{aligned} \tag{22}$$

By inequality (6), we further obtain

$$\begin{aligned} \dot{V}(x(t)) \leqslant \sum_{i=1}^{N} [&-2\gamma_i x_i^{\mathrm{T}}(t) Q_i x_i(t) + \gamma_i^2 x_i^{\mathrm{T}}(t) P_i [A_{i1} \cdots A_{iN}][A_{i1} \cdots A_{iN}]^{\mathrm{T}} P_i x_i(t) \\ &+ \sum_{j=1}^{N} x_j^{\mathrm{T}}(t-\tau_{ij}) x_j(t-\tau_{ij}) \\ &+ \sum_{j=1}^{N} (x_j^{\mathrm{T}}(t) x_j(t) - x_j^{\mathrm{T}}(t-\tau_{ij}) x_j(t-\tau_{ij}))] \\ \leqslant -&\sum_{i=1}^{N} [2\gamma_i \lambda_m(Q_i) - \gamma_i^2 \| P_i [A_{i1} \cdots A_{iN}][A_{i1} \cdots A_{iN}]^{\mathrm{T}} P_i \|_2 - N] \| x_i(t) \|_2^2. \end{aligned} \tag{23}$$

Let

$$\gamma_i = \frac{\lambda_m(Q_i)}{\| P_i [A_{i1} \cdots A_{iN}] \|_2^2}, \quad i = 1, 2, \cdots, N. \tag{24}$$

We further obtain

$$\dot{V}(x(t)) \leqslant -\sum_{i=1}^{N} [\frac{\lambda_m^2(Q_i)}{\| P_i [A_{i1} \cdots A_{iN}] \|_2^2} - N] \| x_i(t) \|_2^2. \tag{25}$$

If (20) holds, then we have $\dot{V} < 0$, This gives the proof of the theorem.

Remark 3 In the derivation of Theorem 2, unlike some existing results[2,8,13,14], we do not use the properties of M-matrix.

Remark 4 For system (18), Ohta and Slijak[2] recently gave a stability condition that the $N \times N$ test matrix $O = (o_{ij})$ defined by

$$o_{ij} = \begin{cases} \lambda_m^{1/2}(Q_i P_i^{-1}) - \| P_i^{1/2} A_{ii} Q_i^{1/2} \|_2, & i = j \\ - \| P_i^{1/2} A_{ij} Q_j^{-1/2} \|_2, & i \neq j \end{cases} \tag{26}$$

is an M-matrix, where P_i and Q_i satisfy (19), and more recently, Xu and Xu[14] estabilished a stability result under the condition that the following $N \times N$ matrix $H = (\eta_{ij})$.

$$\eta_{ij} = \begin{cases} \lambda_m(Q_i) - \| P_i A_{ii}(z) \|_{2s}, & i = j \\ - \| P_i A_{ij}(x) \|_{2s}, & i \neq j \end{cases} \tag{27}$$

is an M-matrix, where P_i and Q_i satisfy (19) and $S = \{z \mid z \in C \text{ and } |z| = 1\}$. In the next section, we will give an example to show that our result is better than these results.

3 An Illustrative Example

Example 1 Consider system (18) with $N = 2$,

$$A_1 = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}, \quad A_{11} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.25 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -2 & -1 \\ 0 & -2 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 1.1 & 0 \\ 0.6 & 0.8 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 1 & 0.35 \\ 0 & 0.8 \end{bmatrix}.$$

respectively. Let $Q_1 = I_2$ and $Q_1 = I_2$, where I_2 denotes the 2×2 identity matrix. Solving two Lyapunov equations shown as (19), we obtain

$$P_1 = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 2.5 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.5000 & -0.1250 \\ -0.1250 & 0.5625 \end{bmatrix},$$

for $i = 1, 2$, respectively. By Theorem 2, we have

$$\frac{1}{\| P_1 [A_{11} A_{12}] \|_2^2} = 2.0014 > 2, \quad \frac{1}{\| P_2 [A_{21} A_{22}] \|_2^2} = 2.0173 > 2,$$

respectively. Therefore, the system is asymptotically stable. But (26) and (27) yield

$$O = \begin{bmatrix} 0.4369 & -0.4009 \\ -0.8454 & 0.5020 \end{bmatrix}, \quad H = \begin{bmatrix} 0.7172 & -0.6483 \\ -0.5378 & 0.4828 \end{bmatrix},$$

respectively. It is easy to check that both O and H are not M-matrix. For this example, our result is better than those obtained by Ohta and Siljak[2] and Xu and Xu[14].

4 Conclusion

Some stabiliy criteria for linear systems with multiple delays including linear large-scale time-delay systems have been provided in this paper. The time delays under consideration are arbitrary unknown but constant ones. Therefore, the obtained results are delay-independent. It is pointed out in the remarks that some of the obtained results are the same as some existing results in the recent literature but they are proved by Lyapunov functional method together with a vector inequality. For linear large-scale time-delay systems, an illustrative example shows that our result is less conservative than the existing ones in the literature.

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具有多个未知常时滞线性系统的时滞无关稳定性

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摘要:本文针对线性时滞系统包括时滞大系统建立了时滞无关的稳定性判据.所考虑的时滞可以是多个任意未知常时滞,故所得结果是时滞无关的.针对线性时滞大系统的一个说明例子比较证明了所建立的结果好于文献中存在的结果.

关键词:多个任意未知常时滞;时滞系统;大系统;稳定性

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