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Exponential Stabilization for a Class of Nonlinear Composite Systems with Unmatched Uncertainties *

JIANG Bin and WANG Xianlai

(Facul of Electrical Engineering & Energy, Tianjin University, Tianjin, 300072, PRC)

ZHANG Siying

(Departmont of Automatic Control, Northeastern University, Shenyang, 110006, PRC)

Abstract: In this paper, exponential stabilization for a class of nonlinear large-scale systems with unmatched uncertainties is investigated. Tow types of robust control algorithms are proposed to stabilize the uncertain nonlinear large-scale systems exponentially provided that the nominal systems are exponentially stable and the unmatched uncertainties are equivalently matched. The decentralized control utilizes the local state of each subsystems as the feedback information. The two-level control uses the local state as well as the states of the connecting subsystems. Finally, an illustrative example is given to demonstrate the utilization of the approach developed in this paper.

Key words: nonlinear composite systems; unmatched uncertainty; exponential stabilization; decentralized control

1 Introduction

With the enlargement of dimension of a control system, analysis and control for the system become very complicated. It is standard to divide it into a number of interconnected subsystems. In the last decade, decentralized stabilization problem has been an important issue in the design of nonlinear large-scale systems and many researchers have worked on this [1,2]. But in those papers, they did not consider any uncertainties in dynamic models. In fact, there is generally some degree uncertainty in modelling a physical dynamic system. These uncertainties, which are in general time varying, may include unknown parameters and input disturbances via the system model and its environment. Furthermore, the uncertainties in some practical systems do not satisfy the matching condition which can be seen in [3]. The decentralized robust stabilizating and tracking controllers were proposed for nonlinear large-scale systems with matched uncertainties[4] and unmatched uncertainties[5] respectively. In this paper, the exponential stabilization for a class of nonlinear large-scale systems with numatched uncertainties is discussed. By means of technique employed in [6], decentralized and two-level control strategies are presented to stabilize exponentially the uncertain composite systems respectively. Both kinds of controllers are continuous on the state.

This paper is organized as follows: The system description and some assumptions are introducted in Section 2. In Section 3, a robust decentralized controller is proposed to stabilize the composite systems with unmatched uncertainties exponentially. Section 4 contains a two-

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level controller, i. e. the local control and the global control. A illustrative example is given in section 5.

2 System Description and Preliminaries

Consider a class of uncertain large-scale system S which is composed of N nonlinear interconnected subsystems S_i described by

S_i:
$$\dot{x}_i(t) = f_i(x_i,t) + \zeta_i(x_i,t) + \eta_i(x_i,t)u_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} H_{ij}(x_j,t),$$

$$x_i(t_o) = x_{io}, \quad i = 1,\cdots,N \tag{1}$$

where $t \in \mathbb{R}$ is the "time", $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$ are ith subsystem state, input respectively, f_i, ζ_i, η_i, g_i, H_{ij} are matrices, or vectors of appropriate dimensions. ζ_i, η_i represent the uncertain parameters, H_{ij} stand for the natural interconnection between the subsystem S_i and S_j which may also possess uncertainty.

In this paper, there are following assumptions:

Assumption 1 The functions f_i, ζ_i, η_i, g_i, H_{ij} are continuous, uniformly bounded with respect to time, and locally uniformly bounded with respect to $x_i(x_j)$.

Assumption 2

i) $f_i(0,t) = 0, t \in \mathbb{R}$; (2)

ii) There exist a C^1 function $V_i(\cdot): \mathbb{R}^{n_i} \times \mathbb{R} \to \mathbb{R}^+$ which satisfies

$$(\alpha_i \| x_i \|)^2 \leqslant V_i(x_i,t) \leqslant (\beta_i \| x_i \|)^2, \tag{3}$$

$$\partial V_i(x_i,t)/\partial t + \nabla^{\mathrm{T}} V_i(x_i,t) f_i(x_i,t) \leqslant -2\gamma_i V_i(x_i,t) \tag{4}$$

for all $x_i \in \mathbb{R}^{n_i}$, $t \in \mathbb{R}$, where α_i β_i γ_i are positive scalar constants, $i = 1\cdots N$.

Assumption 3 There exist constants $a_{ij} \in \mathbb{R}^+$, such that for all $(x_i, x_j, t) \in \mathbb{R}^{n_j} \times \mathbb{R}$:

$$\| \nabla^{\mathrm{T}} V_i(x_i,t) H_{ij}(x_j,t) \| \leqslant [V_i(x_i,t)]^{1/2} 2a_{ij} [V_j(x_j,t)]^{1/2}. \tag{5}$$

Remark 1 The Assumption 2 assured that the origen $x_i = 0$ of the nominal isloated subsystems $\dot{x}_i = f_i(x_i,t)$ are exponentially stable.

Assumption 4 The η_i are the matched uncertainties, while ζ_i are unmatched uncertainties, both of them are bounded in Euclidean norm by known functions, that is, there exist continuous functions $\bar{\eta}_i$, ρ_i, p_{1i} such that for all $x_i \in \mathbb{R}^{n_i}$, $t \in \mathbb{R}$:

$$\eta_i(x_i,t) = g_i(x_i,t)\bar{\eta}_i(x_i,t), \tag{6}$$

$$\| \zeta_i(x_i,t) \| \leqslant \rho_{1i}(x_i,t), \quad \| \bar{\eta}_i(x_i,t) \| \leqslant \rho_i(x_i,t). \tag{7}$$

Assumption 5[6] The unmatched uncertain functions $\zeta_i (i = 1,\cdots,N)$ are epuivaletly matched, that is, the bounding function ρ_i' defined by

$$\| \nabla^{\mathrm{T}} V_i(x_i,t) \zeta_i(x_i,t) \| \leqslant \rho_i'(x_i,t), \quad i = 1,\cdots,N \tag{8}$$

for all $(x_i,t) \in \mathbb{R}^{n_i} \times \mathbb{R}$, where $V_i(x_i,t)$ are defined in Assumption 2,3, have the property that the functions $\rho_i'(x_i,t)/[\| \nabla^{\mathrm{T}} V_i(x_i,t) g_i(x_i,t) \|]$ are continuous, uniformly bounded with respect to time, and locally bounded with respect to x_i.

Remark 2 It is easy to see from Assumption 5 that a necessary condition for uncertainties to be equivalently matched is that

$$\nabla^{\mathrm{T}} V_i(x_i,t) \zeta_i(x_i,t) = 0,$$

if $$(x_i,t) \in \{(x_i,t) \in \mathbb{R}^n \times \mathbb{R} : \nabla^T V_i(x_i,t) g_i(x_i,t) = 0\}.$$

3 Decentralized Robust Stabilization

The control laws proposed in this section for each subsystems S_i rely only on the local state x_i. We consider, the decetralized control as follows:

$$u_i(x_i,t) = u_{1i}(x_i,t) + u_{2i}(x_i,t), \tag{9}$$

$$u_{1i} = \rho_i(x_i,t)[\mu_{1i}(x_i,t)/(\|\mu_{1i}(x_i,t)\| + \varepsilon_i\|x_i\|^2)], \tag{10}$$

$$u_{2i} = -[\rho_i'(x_i,t)/(\|\nabla^T V_i(x_i,t) g_i(x_i,t)\|)] \cdot \mu_{2i}(x_i,t)/(\|\mu_{2i}(x_i,t)\| + \varepsilon_i\|x_i\|^2)], \tag{11}$$

$$\mu_{1i} = g_i^T \nabla V_i \rho_i, \quad \mu_{2i} = g_i^T \nabla V_i [\rho_i'/(\|\nabla^T V_i g_i\|)], \tag{12}$$

where $\varepsilon_i (i = 1, \cdots, N)$ are psoitive constants which satisfy the inequality of the form

$$\theta = \psi - \varepsilon/\alpha^2 > 0, \tag{13}$$

where $\varepsilon = \max\{\varepsilon_1, \cdots, \varepsilon_N\}$, $\alpha = \min\{\alpha_1, \cdots, \alpha_N\}$, ψ denotes the half of minimum eigenvalue of matrix Q, i, e. $\psi = [\lambda_{\min}(Q)]/2$, $Q = T + T^T$, the matrix $T = [T_{ij}]_{NXN}$ is defined by

$$T_{ij} = \begin{cases} \gamma_i, & i = j, \\ -a_{ij}, & i \neq j. \end{cases} \tag{14}$$

Remark 3 It is evident that $u_i (i = 1, \cdots, N)$ are continuous and locally uniformly bounded.

Theorem 1 Under the control

$$u(x,t) = [u_1^T, u_2^T, \cdots, u_N^T]^T \tag{15}$$

defined in (9)～(12), the nonlinear composite system S satisfying Assumption 1～5 has a classical solution and the solution is exponentially stable provided the successive principal minors of the test matrix T defined in (14) are all positive. More precisely, we have

$$\|x(t)\| \leqslant (\beta/\alpha)\|x(t_o)\|\exp\{-\theta(t - t_o)\}, \tag{16}$$

where $x(t) = \mathrm{col}[x_1(t), \cdots, x_N(t)]$, $\beta = \max\{\beta_1, \cdots, \beta_N\}$, θ is defined in (13).

Proof According to theorem in [7], under the control (15), the closed-loop system has the existence of classical solutions. Let $x(t)$ be the solution of the colsed-loop system and define the Lyapunov function as

$$V(x,t) = \sum_{i=1}^{N} V_i(x_i,t). \tag{17}$$

From the Assumption 2 and 4, the Lyapunov derivation for the closed-loop system is

$$\begin{aligned} \dot V = \sum \dot V_i &= \sum_i [\partial V_i/\partial t + \nabla^T V_i \dot x_i] \\ &= \sum_i \{\partial V_i/\partial t + \nabla^T V_i [f_i + \zeta_i + g_i(\bar\eta_i + u_i) + \sum_j H_{ij}]\} \qquad (18) \\ &\leqslant -\sum_i 2\gamma_i V_i + \sum_i \nabla^T V_i [\zeta_i + g_i(\bar\eta_i + u_i) + \sum_j H_{ij}] \qquad (19) \end{aligned}$$

According to Assumption 4, 5 and equalities (9～12), it follows that

$$\begin{aligned} &\nabla^T V_i [\zeta_i + g_i(\bar\eta_i + u_i)] \\ &\leqslant \|\nabla^T V_i g_i\| \cdot \|\bar\eta_i\| + \nabla^T V_i g_i u_{1i} + \|\nabla^T V_i \zeta_i\| + \nabla^T V_i g_i u_{2i} \\ &\leqslant \|\mu_{1i}\| + \nabla^T V_i g_i u_{1i} + \rho_i' + \nabla^T V_i g_i u_{2i} \end{aligned}$$

$$= \| \mu_{1i} \| - (\| \mu_{1i} \|^2)/(\| \mu_{1i} \| + \varepsilon_i \| x_i \|^2)$$

$$+ \| \mu_{2i} \| - (\| \mu_{2i} \|^2)/(\| \mu_{2i} \| + \varepsilon_i \| x_i \|^2) \leqslant 2\varepsilon_i \| x_i \|^2 \tag{20}$$

In view of (5), for all $(x_i, x_j, t) \in \mathbb{R}^{n_i} \times \mathbb{R}^{n_j} \times \mathbb{R}$

$$\dot{V} \leqslant -\sum_i 2\gamma_i V_i + \sum_i 2\varepsilon_i \| x_i \|^2 + \sum_i V_i(x_i)^{1/2}[\sum_j 2a_{ij}V_j(x_j)^{1/2}]. \tag{21}$$

Letting

$$y^{\mathrm{T}} = [V_1(x_1)^{1/2}, \cdots, V_N(x_N)^{1/2}], \quad \varepsilon = \max\{\varepsilon_1, \cdots, \varepsilon_N\}, \tag{22}$$

then one can get

$$\dot{V} \leqslant -y^{\mathrm{T}}Qy + 2\varepsilon \| x \|^2 \leqslant -2\psi V(x,t) + 2\varepsilon \| x \|^2, \tag{23}$$

where $Q = T + T^{\mathrm{T}}, 2\psi = \lambda_{\min}(Q)$, matrix T is defined in (14).

From(23) and (3), one can obtain that

$$\dot{V} \leqslant -2\psi V(x,t) + 2\varepsilon/\alpha^2 V(x,t) = -2\theta V(x,t), \quad \theta > 0.$$

Thus the closed-loop system S is exponentially stable.

4 Two-Level Exponential Stabilization —— the Local and Global Design

The control presented in last section for subsystem S_i does not depent on the state of its neighbouring subsystems S_j. If the local communication is feasible, then the golbal design for the control can be achieved.

At first, the Assumption 6 is described as follows:

Assumption 6 For each subsystem S_j, there exist continuous functions $k_{ij}(\cdot)$ containing uncertainties and known functions $r_{ij}(\cdot)$, such that

$$H_{ij}(x_j, t) = g_i(x_i, t)k_{ij}(x_i, x_j, t), \tag{24}$$

$$\| k_{ij}(x_i, x_j, t) \| \leqslant r_{ij}(x_i, x_j, t). \tag{25}$$

We propose the two-level controller as follows:

$$u_i(x,t) = u_{1i}(x,t) + u_{2i}(x_1,t), \tag{26}$$

$$u_{1i}(x) = -(\rho_i + \sum_j r_{ij})[\mu_{1i}(x,t)/\| \mu_{1i}(x) \| + \varepsilon_i \| x_i \|^2)], \tag{27}$$

$$u_{2i}(x_i) = -[\rho_i'/(\| \nabla^{\mathrm{T}} V_i g_i \|)][\mu_{2i}(\| \mu_{2i} \| + \varepsilon_i \| x_i \|^2)], \tag{28}$$

where

$$\mu_{1i} = g_i^{\mathrm{T}} \nabla V_i (\rho_i + \sum_j r_{ij}), \quad \mu_{2i} = g_i^{\mathrm{T}} \nabla V_i [(\rho_i'/(\| \nabla^{\mathrm{T}} V_i g_i \|)].$$

Theorem 2 Subject to Assumption 1,2,3,4,5 and 6, the nonlinear composite system S under the control (26)~(28) is exponentially stable.

Owing to the page constraint, the proof of Theorem 2 is omitted.

Remark 4 In Theorem 2, the interconnections have been compensated by the global control $u_{1i}(x)(i = 1, \cdots, N)$, thus the exponential stability of the large-scale systems can be assured without referring to the test matrix T.

5 Illustrative Example

To illustrate the efficiency of our control technique, let us consider a nonlinear uncertain large-scale system S composed of two interconnected nonlinear uncertain dynamical subsystem $S_i(i = 1,2)$ which are described by

$$\begin{cases} \dot{x}_{i1} = x_{i1} + 3x_{i1}x_{i2}^2 + 3x_{j1} + 6x_{i2}x_{j1}x_{j2} + 5x_{i1}x_{i2}\eta_i + 5x_{i1}u_i, & (29a) \\ \dot{x}_{i2} = -5x_{i2} - 2x_{i1}^2x_{i2} + 3x_{j2} - 4x_{i1}x_{j1}x_{j2} + 5x_{i1}^2\zeta_i, & (29b) \end{cases}$$

where $u_i (i = 1, \cdots, N)$ are the control, $\eta_i, \zeta_i \in [-1, 1]$ are uncertainties, $x_i = \mathrm{col}[x_{i1}, x_{i2}], i, j = 1, 2; i \neq j$.

From equation (29a) the isolated nominal system is unstable, but can be stabilizable.

The control laws $u_i (i = 1, 2.)$ are designed as

$$u_i = u_{i0} + \bar{u}_i, \quad i = 1, 2, \tag{30}$$

where $u_{i0} = -1$ are control to stabilize the isolated nominal systems, $\bar{u}_i$ are the part of robust control comtrol to cpmpensate for uncertainties.

It is easy to verify that all of assumptions in Theorem 1 hold. In fact, by calculation, one can obtain

$$V_i = 2x_{i1}^2 + 3x_{i2}^2, \quad \alpha_i = 2^{1/2}, \quad \beta_i = 3^{1/2}, \quad \gamma_i = 4, \quad a_{12} = a_{21} = 3.$$

$$T = \begin{bmatrix} 4 & -3 \\ -3 & 4 \end{bmatrix} \text{ is positive definite matrix.}$$

$$\rho_i = |x_{i2}|, \quad \rho_i' = 30|x_{i2}||x_{i1}|^2, \quad \psi = 1.$$

Thus, the constants $\varepsilon_i (i = 1, 2)$ are chosen such that

$$\theta = \psi - \varepsilon/\alpha^2 = 1 - \varepsilon/2 > 0,$$

$$\text{i.e. } \max\{\varepsilon_1, \varepsilon_2\} < 2.$$

According to Theorem 1, the uncertain large-scale system S can be exponentially stabilized. The decentralized controller is designed as follows

$$u_i = -1 - 20x_{i1}^2x_{i2}^2/[20x_{i1}^2|x_{i2}| + \varepsilon_i(x_{i1}^2 + x_{i2}^2)] - 45x_{i1}^2x_{i2}^2/[30x_{i1}^2|x_{i2}| + \varepsilon_i(x_{i1}^2 + x_{i2}^2)], \quad i = 1, 2.$$

6 Conclusion

In this paper, we have generalized the work on the design of exponential stablilzation for nonlinear system with matched uncertainties to that of unmatched uncertain nonlinear composite system. Two kinds of robust control algorithms are proposed respectively. The decentralized control is based on the local state of each isolated subsystems. The two-level control relies on both the local state of each isolated subsystems and that of the neighbouring subsystems. both of them are continuous and uniformly bounded. A numberical example is given to illustrate the design procedure.

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一类带有非匹配不确定性的非线性组合系统的按指数镇定

姜 斌 王先来

（天津大学电气自动化与能源工程学院·天津，300072）

张嗣瀛

（东北大学自控系，沈阳，110006）

摘要：本文讨论一类带有非匹配不确定性的非线性大系统的按指数镇定．在标称系统是按指数稳定且非匹配不确定性为等效匹配的假设条件下提出了两种鲁棒控制算法．分散控制仅利用每个子系统的状态作为反馈信息，而双层控制使用每个子系统及相关子系统的状态．最后用一个说明性的例子来说明本文所提出方法的可用性．

关键词：非线性组合系统；非匹配不确定性；按指数镇定；分散控制

本文作者简介

姜 斌 1966年生．1988毕业于江西师范大学数学系，1991年于东北工学院应用数学系获硕士学位，1995年于东北大学自动控制系获博士学位，现为天津大学电气自动化与能源工程学院讲师．从事大系统理论与应用，非线性系统的鲁棒控制及机器人控制等方面的研究．

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