Design of Variance-Constrained Fault-Tolerant Controllers for Discrete-Time Stochastic Systems *

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Abstract: In this paper, the problem of designing variance-constrained fault-tolerant controllers for discrete-time stochastic systems is considered. The purpose of the addressed problem is to find the feedback controller so that the closed-loop system satisfies the prespecified steady-state variance constraints, and remains asymptotically stable against possible sensor failures, simultaneously. The sufficient conditons for the existence of variance-constrained fault-tolerant controllers are characterized. The analytical expression of desired controllers is also pressented.

Key words: linear discrete-time systems; stochastic systems; variance-constrained design; fault-tolerant control

1 Introduction

In recent years, significant effort in the fileds of stochastic control theory has been devoted to the variance-constrained design, and the literature on this problem is extensive and reflects considerable current activity. The practical motivation for this problem is that, in many engineering control systems, the performance requirements are usually described in terms of upper bounds on the individual steady-state variance values. Traditional control design techniques, such as LQG control and H_2/H_∞ control, are difficult to solve the variance-constrained design problem, since they can not ensure that the prespecified individual variance constraint will be satisfied. The covariance control theory[1,2], which was first developed in 1987, has provided a much more straightforword and effective approach to the variance-constrained problem. The main idea of this theory is to specify a state covariance matrix X to the different requirements on the system robustness and performance, and then design a controller such that the state covariance of the resulting closed-loop system is equal to this specified X. Therefore, this closed-loop system will possess the desired performance requirements.

On the other hand, the designed MIMO systems may not achieve the desired performance requirements and even become unstable when the feedback signals are switched off by a failure in the actuator or in the sensor, even if the open-loop system is stable. The property remaining stable in the presence of failures in the actuator or the sensor is called integrity, and hence integrity is a type of fault-tolerant with respect to stability in the MIMO feedback control system[3,4]. However, very few papers are concerned with the problem of variance-constrained control design possessing integrity for linear stochastic systems. The problem of fault-tolerant variance-constrained control design for continuous-time systems has been re-

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searched in [5] by the authors, and thus the present paper will be concentrated on the discrete-time case.

2 Problem Formulation and Assumptions

Consider the stationary vector process x generated by

$$x(k+1) = Ax(k) + Bu(k) + Dw(k), \quad u(k) = Gx(k). \tag{1}$$

where $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$, $w \in \mathbb{R}^{n_w}$; $w(k)$ is a zero mean white noise sequence with covariance I; and $w(k)$ and $x(0)$ are uncorrelated. A,B,D,G are the matrices with appropriate dimensions and $DD^{\mathrm{T}} > 0$. The pairs (A,B) and (A,D) are, respectively, assumed to be stabilizable and controllable.

To represent a sensor failure in any of the feedback loops, we introduce a switching matrix F, inserted between the controller gain and the state, as $F = \mathrm{diag}[f_1\, f_2 \cdots f_{n_x}]$, where f_i for $i = 1,2,\cdots,n_x$, is either one or zero. the value $f_i = 0$ corresponds to a sensor failure and $f_i = 1$ corresponds to a normal situation in the ith-loop. Hence, the closed-loop system which represents possible sensor failures becomes

$$x(k+1) = (A + BGF)x(k) + Dw(k). \tag{2}$$

If $A + BGF$ is Schur stable, then the steady-state covariance $X = \lim\limits_{k\to\infty} E[x(k)x^{\mathrm{T}}(k)]$ exists and satisfies the discrete Lyapunov equation $X = (A+BGF)X(A+BGF)^{\mathrm{T}} + DD^{\mathrm{T}}$.

Let the notion Ω be the set of diagonal matrices whose diagonal elements are arbitrary compositions of 1 or 0 (except zero-matrix). We now can conclude the fault-tolerant variance-constrained problem as follows. The problem under consideration is to determine state feedback controller G such that 1) the closed-loop system (2) remains stable for arbitrary $F \in \Omega$ and 2) the individual state meets the prespecified steady-state variance constraint, i. e., $[X]_{ii} \leqslant \sigma_i^2 (i = 1,2,\cdots,n_x)$, where X is the steady-state covariance. It should be pointed out that the given variance constraint σ_i^2 can be determined by practical requirements and can not more than the minimal variance value obtained from the traditional minimal variance control theory.

3 Main Results

In this section, the detailed proofs of main theorems are omitted due to the space limitation.

Theorem 1 Consider the closed-loop system (2) where $F \in \Omega$ is the arbitrary given switching matrix. If there exists a scalar $\varepsilon > 0$ such that the following algebraic matrix equation

$$P = APA^{\mathrm{T}} + \varepsilon AP^2A^{\mathrm{T}} + (\varepsilon^{-1} + \| P \|)BGG^{\mathrm{T}}B^{\mathrm{T}} + DD^{\mathrm{T}} \tag{3}$$

has a positive definite solution $P > 0$, then 1) the closed-loop matrix $A + BGF$ is stable for arbitrary $F \in \Omega$, and 2) the steady-state covariance X exists and satisfies $X \leqslant P$.

The proof of Theorem 1 can be easily completed by considering the following facts

$$FF^{\mathrm{T}} \leqslant I, \quad 0 \leqslant (\varepsilon^{1/2}APF - \varepsilon^{-1/2}BG)(\varepsilon^{1/2}APF - \varepsilon^{-1/2}BG)^{\mathrm{T}},$$

$$0 \leqslant BG(\| FPF^{\mathrm{T}} \| I - FPF^{\mathrm{T}})G^{\mathrm{T}}B^{\mathrm{T}} \leqslant \| P \| BGG^{\mathrm{T}}B^{\mathrm{T}} - (BGF)P(BGF)^{\mathrm{T}},$$

and using the discrete-time Lyapunov stability theory.

Remark 1 Theorem 1 shows that the integrity constraint is automatically enforced when a positive definite solution to (3) is known to exist. Furthermore, all such solutions provide upper bounds for the actural closed-loop steady-state covariance X, and therefore this behaviour can be utilized to achieve the prespecified variance constraint.

Remark 2 We can achieve the purpose of mixed fault-tolerant variance-constrained design by using the following approach: choose the proper positive definite matrix P which satisfies

$$[P]_{ii} \leqslant \sigma_i^2, \quad (i = 1,2,\cdots,n), \tag{4}$$

and then find the controller G and the parameter ε meeting Eq. (3). If such a controller exists and can obtained, then it follows from Theorem 1 that $A + BGF$ is stable and $[X]_{ii} \leqslant [P]_{ii} \leqslant \sigma_i^2, i = 1,2,\cdots,n_x$, and therefore the design task will be accomplished. In fact, the problem under study in this paper can be converted to an auxiliary "P-matrix assignment" problem. To be able to work in a more definitive term, the following definitions is introduced.

Definition 1 The given positive definite matrix P satisfying (4) is said to be assignable if there exist a controller and a scalar $\varepsilon > 0$ such that (3) is satisfied.

The following result characterizes the assignability condition of a given positive definite matrix.

Theorem 2 Let the positive definite matrix P satisfying (4) be given. Then this matrix P is assignable if and only if there exists a constant $\varepsilon > 0$ such that

$$P - APA^{\mathrm{T}} - \varepsilon AP^2A^{\mathrm{T}} - DD^{\mathrm{T}} \geqslant 0, \tag{5}$$

$$(I - BB^{+})(P - APA^{\mathrm{T}} - \varepsilon AP^2A^{\mathrm{T}} - DD^{\mathrm{T}})(I - BB^{+}) = 0 \tag{6}$$

where B^{+} denotes the generalized inverse of B.

The analytical expression of controllers assigning the assignable matrix P is given as follows.

Theorem 3 If the specified positive definite matrix P satisfying (4) is assignable, then all controllers which assign matrix P can be parameterized as

$$G = (\varepsilon^{-1} + \| P \|)^{-1/2}TV + (I - B^{+} B)Z, \tag{7}$$

where T is the square root of $P - APA^{\mathrm{T}} - \varepsilon AP^2A^{\mathrm{T}} - DD^{\mathrm{T}}$, V is arbitrary orthogonal and Z is arbitrary with proper dimensions.

Theorem 2 and Theorem 3 can be proved by using the generalized inverse theory and the fact that $MM^{\mathrm{T}} = NN^{\mathrm{T}}$ iff there exists an orthogonal matrix V meeting $M = NV$. The following result, which gives the solution to the problem of fault-tolerant variance-constrained controller design, is easily seen in the view of Theorem 2 and Theorem 3.

Theorem 4 Consider the closed-loop system (2). Given the constraints on the steady-state variance $\sigma_i^2 (i = 1,2,\cdots,n_x)$. If there exists a positive definite matrix P satisfying (4) (5)(6), then the desired fault-tolerant variance-constrained controllers can be obtained by (7).

Remark 3 In the design of practical control systems, we are usually required to construct an assignable matrix P satisfying (4)(5) from the assignability condition (6), and then obtain the desired controllers from (7) immediately. Note that (6) is actually a generalized algebraic Riccati equations which also appeared in [2] with similar form, we can solve it by using the same method proposed in [2].

4 Conclusions

An algebraic design approach has been developed to ensure stability and variance constraints in the presence of sensor failures. The controllers which guarantee stability and variance constraints have been characterized and a design method has also been presented based on the generalized inverse theory. Further work will center on the study of convergence of algorithm for the proposed controller design procedure.

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离散随机系统的容错约束方差控制设计

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摘要:本文考虑线性离散随机系统的容错约束方差控制设计问题,即设计反馈控制器,使闭环系统在可能的传感器失效下不仅保持渐近稳定,而且满足预先给定的稳态方差约束.文中导出了期望的容错约束方差控制器存在的充分条件,并进一步给出了其参数化代数表达式.

关键词:线性随机离散系统;约束方差控制;容错控制;完整性

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