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Robust Stability of Multivariable Systems with Nonlinear and Linear Structured Uncertainties*

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Abstract: The robustness of absolute stability of Lurie system with multiple nonlinearities and structured linear uncertainties is considered. A unified criterion is obtained for robust absolute stability analysis of the nonlinear system in the presence of parametric uncertainties and unmodeled dynamics. The criterion includes multivariable Popov criterion and the mixed μ upper bound as special cases. An efficient algorithm based upon Rosenbrock's method and interior-point method is developed for computing the present criterion. An example is given to illustrate the application of the obtained result to the problem of analysis of worst-case H_∞ performance of nonlinear control systems with structured parametric and dynamic uncertainties.

Key words: robust stability; Lurie system; structured uncertainties; Popov criterion; mixed μ

1 Introduction

The absolute stability theory seems to be the first approach to stability analysis of uncertain systems. It was assumed in this theory that the linear part of the system is fixed, but the nonlinear part involves some sector-bounded uncertainty. Both Lyapunov function methods[1] and input-output analysis methods[2] have been developed to establish absolute stability criteria. One of the most important results in this field is V. M. Popov's criterion[3], which was later extended to systems with multiple nonlinearities by Jury and Lee[4], Popov[5].

It is of interest to extend absolute stability criteria for the case while the linear part of the system includes also some uncertainty. First contributions on robustness analysis of Lurie system with parameter uncertainty can be found in the pioneering work such as [6] and [7]. Recently, simulated by the great progress in robust stability analysis for linear systems, this problem has been studied by many authors along the line of Kharitonov's theorem[8]. Some extreme point results for robust absolute stability were obtained with the aid of the relationship between strictly positive realness property of rational functions and Hurwitz stability of polynomials with complex coefficients[9,10]. Unfortunately, it seems to be difficult to apply these results to multivariable systems with both parametric uncertainty and unmodeled dynamics.

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In this paper we treat multiple nonlinearities, parameter uncertainties and unmodeled dynamics in a unified framework. The main idea of this work is that if we extend constant matrices D, Q in multivariable Popov multiplier $D + j\omega Q$ to frequency-dependent ones, then Popov criterion gives an upper bound for robust stability of systems with parameter uncertainties and unmodeled dynamics. We prove that these two kinds of multipliers (multiplers with constant and frequency-dependent matrices) can be unified in one stability criterion. This unified criterion can be used not only for determining stability bounds in the presence of nonlinearities, uncertain parameters and unmodeled dynamics, but also for analyzing H_∞ performance of uncertain Lurie system.

Some other recent work related to the above problem should be metioned. In [11] the multivariable Popov criterion was interpreted in the sence of parameter-dependent Lyapunov functions and applied to robust stability analysis for constant real parameter uncertainty. Note that Popov criterion may be very conservative in this case since it looks upon parameter uncertainties as nonlinearities. Later, a generalized Popov multiplier framework was developed by using different methods[13~17] and connected to the mixed μ upper bound of Fan et al.[18]. The advantage of this approach is that a standard μ- synthesis problem can be solved in state space without $D, N-K$ iteration in the frequency domain. But in general the stability criteria in this framework can not be used for nonlinearities unless certain structure of multipliers is set for the case of monotonic and odd monotonic nonlinearities[12].

In this paper we denote the complex conjugate transpose of complex matrix M by M^H, and its largest singular value by $\bar{\sigma}(M)$. If M is Hermitian, we use $\bar{\lambda}(M)$ to denote its largest eigenvalue. $\mathbb{RH}_\infty^{n\times m}$ denotes the set of all $n \times m$ stable real rational transfer matrices. $\mathscr{F}_l(M, \Delta)$ and $\mathscr{F}_u(M, \Delta)$ denote lower and upper linear fractional transformations (LFTs) of matrices M and Δ respectively, i. e.

$$\mathscr{F}_l(M,\Delta) = M_{11} + M_{12}\Delta(I - M_{22}\Delta)^{-1}M_{21},$$
$$\mathscr{F}_u(M,\Delta) = M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12}.$$

2 Main Result

2.1 Unified Robust Absolute Stability Criterion

Denote the matrix of memoryless continuous nonlinear funtions by

$$F(v) = \mathrm{diag}[f_1(v_1), \cdots, f_i(v_i)], \tag{1}$$

where $f_i(v_i)$'s, without loss of generality, are assumed to be subject to the following sector condition:

$$0 \leqslant v_i f_i(v_i) \leqslant 2v_i^2, \quad f_i(0) = 0, \tag{2}$$

and the matrix of parametric and dynamic uncertainties by $\Delta(s) \in B\Delta(s) \subset \chi(s)$, where

$$\chi(s) = \{\Delta(s) = \mathrm{diag}[\delta_1 I_{k_1}, \cdots, \delta_{m_r} I_{k_{m_r}}, \Delta_{m_r+1}(s), \cdots, \Delta_{m_r+m_c}(s)]:$$
$$\delta_i \in \mathbb{R}, \Delta_{m_r+q}(s) \in \mathbb{RH}_\infty^{k_{m_r+q}\times k_{m_r+q}}\}, \tag{3}$$

$$B\Delta(s) = \{\Delta(s) \in \chi(s): \bar{\sigma}(\Delta(j\omega)) \leqslant 1, \forall\ \omega \in \mathbb{R}\}. \tag{4}$$

Then a multivariable Lurie system with parametric uncertainties and unmodeled dynamics can be formulated as in Fig. 1.

Let $N := m_v + \sum_{i=1}^{m_r} k_i + \sum_{q=1}^{m_c} k_{m_r+q}$, $M(s) \in \mathbb{R}\mathrm{H}_\infty^{N\times N}$. Let $M(s)$ be partitioned as

$$M(s) = \begin{bmatrix} M_{11}(s) & M_{12}(s) \\ M_{21}(s) & M_{22}(s) \end{bmatrix}$$

where

$$M_{11}(s) \in \mathbb{R}\mathrm{H}_\infty^{m_v\times m_v},$$
$$M_{12}(s) \in \mathbb{R}\mathrm{H}_\infty^{m_v\times (N-m_v)},$$
$$M_{21}(s) \in \mathbb{R}\mathrm{H}_\infty^{(N-m_v)\times m_v}$$

and $M_{22}(s) \in \mathbb{R}\mathrm{H}_\infty^{(N-m_v)\times(N-m_v)}$.

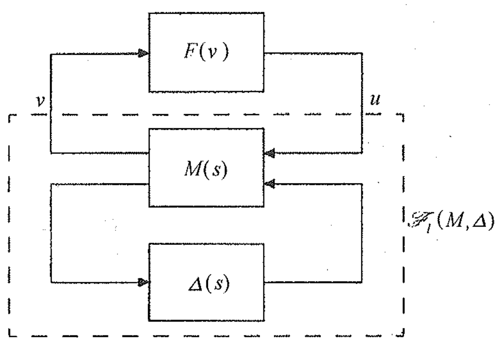

Fig.1 System with multiple nonline arities and structured linear uncertainties

Denote

$$\widetilde{M} = \begin{bmatrix} M_{11} - M_{12}(I+M_{22})^{-1}M_{21} & M_{12} - M_{12}(I+M_{22})^{-1}M_{22} \\ (I+M_{22})^{-1}M_{21} & (I+M_{22})^{-1}M_{22} \end{bmatrix}, \tag{5}$$

and define some sets of matrices

$$\boldsymbol{Q}_1 := \{\mathrm{diag}[q_1,\cdots,q_{m_v},I_{N-m_v}]: q_i \in \mathbb{R}, q_i > 0, i = 1,2,\cdots,m_v\}, \tag{6}$$

$$\boldsymbol{Q}_2 := \{\mathrm{diag}[I_{m_v},Q_1,\cdots,Q_{m_r},0_{m_c}]: Q_i = Q^H \in \mathbb{C}^{k_i\times k_i}, i = 1,\cdots,m_r\}, \tag{7}$$

$$\boldsymbol{D}_1 := \{\mathrm{diag}[d_1,\cdots,d_{m_v},I_{N-m_v}]: d_i \in \mathbb{R}, d_i > 0, i = 1,2,\cdots,m_v\}, \tag{8}$$

$$\boldsymbol{D}_2 := \{\mathrm{diag}[I_{m_v},D_1,\cdots,D_{m_r},d_{m_r+1}I_{k_{m_r+1}},\cdots,d_{m_r+m_c}I_{k_{m_r+m_c}}]:$$
$$D_i \in \mathbb{C}^{k_i\times k_i}, D_i = D_i^H > 0, i = 1,\cdots,m_r, d_{m_r+q} \in \mathbb{R}, d_{m_r+q} > 0, q = 1,\cdots,m_c\}. \tag{9}$$

Then robust absolute stability of the system shown in Fig. 1 can be verified by the following criterion, which is the main contribution of this paper.

Theorem 2.1 Suppose that $M(s) \in \mathbb{R}\mathrm{H}_\infty$. Then the multivariable Lurie system with structured parametric and dynamic uncertainties, shown in Fig. 1, is robustly absolutely stable for all nonlinearities $f_i(v_i)$'s satisfying assumption (2) and all linear uncertainties $\Delta(s) \in \boldsymbol{B}\Delta(s)$, if there exist $Q_1 \in \boldsymbol{Q}_1$ and $D_1 \in \boldsymbol{D}_1$ such that for all $\omega \in \mathbb{R}$ and some ω-dependent matrices $Q_2 \in \boldsymbol{Q}_2$ and $D_2 \in \boldsymbol{D}_2$ the following inequality holds:

$$I - (I + j\omega Q_1Q_2)D_1D_2\widetilde{M}(j\omega)D_2^{-1}D_1^{-1} - D_1^{-1}D_2^{-1}\widetilde{M}^H(j\omega)D_2D_1(I - j\omega Q_2Q_1) > 0. \tag{10}$$

Proof of the theorem is given in Appendix.

Remark Using loop transformation technique[2] more general Lurie system with nonlinearities described by

$$k_{i1}v_i^2 \leqslant v_i f_i(v_i) \leqslant k_{i2}v_i^2, \quad f_i(0) = 0, \quad 0 \leqslant k_{i1} < k_{i2} < \infty \tag{11}$$

can always be transformed to the system with nonlinearities described by (2).

Remark In Theorem 2.1 some common D_1 and Q_1 are required to guarantee condition (10) for all $\omega \in \mathbb{R}$, while D_2 and Q_2 can be chosen for different frequencies ω.

2.2 Special Cases

Now we specialize the main result to systems involving only nonlinearities or linear uncertainties and show the connection between our criterion and some famous results.

Case 1 $\Delta(s) = 0$.

In this case the system contains only nonlinearities. It is clear that $m_r = m_c = 0$ and $\widetilde{M} = M = M_{11}$. By definitions (7) and (9) we have $Q_2 = D_2 = I$. Thus Theorem 2.1 reduces to the following

Corollary 2.1 Suppose that $M(s) \in \mathbb{R}H_\infty$ and nonlinearities $f_i(v_i)$'s satisfy assumption (2). Then the system interconnected by $M(s)$ with $F(v)$, defined by (1), is absolutely stable if there exist $Q_1 \in \boldsymbol{Q}_1$ and $D_1 \in \boldsymbol{D}_1$ such that inequality

$$I - (I + j\omega Q_1)D_1 M(j\omega)D_1^{-1} - D_1^{-1}M^H(j\omega)D_1(I - j\omega Q_1) > 0 \tag{12}$$

holds for all $\omega \in \mathbb{R}$.

Since $D_1 > 0$, inequality (12) can be rewritten as

$$D_1^2 - (D_1^2 + j\omega D_1 Q_1 D_1)M(j\omega) - M^H(j\omega)(D_1^2 - j\omega D_1 Q_1 D_1) > 0. \tag{13}$$

From the fact that the map $\boldsymbol{D}_1 \times \boldsymbol{Q}_1 \to \boldsymbol{D}_1 \times \boldsymbol{Q}_1$ defined by $(D_1, Q_1) \mapsto (D_1^2, D_1 Q_1 D_1)$ is a bijection, it follows that inequality (13) holds if and only if there exist $D_1 \in \boldsymbol{D}_1$ and $D_2 \in \boldsymbol{D}_2$ such that

$$D_1 - (D_1 + j\omega Q_1)M(j\omega) - M^H(j\omega)(D_1 - j\omega Q_1) > 0. \tag{14}$$

This is in fact the famous multivariable Popov criterion given in [4].

Case 2 $F(v) = 0$.

In this case the system contains parametric uncertainties and unmodeled dynamics. So $m_v = 0$, $\widetilde{M} = (I + M)^{-1}M$ and $M = M_{22}$. By definitions (6) and (8) we have $D_1 = Q_1 = I$. We can denote ωQ_2 simply as Q_2 because $\omega Q_2 \in \boldsymbol{Q}_2$. In the sequel, Theorem 2.1 reduces to the following

Corollary 2.2 The system interconnected by $M(s)$ with parametric and dynamic uncertainties $\Delta(s)$ is robustly stable, if for each $\omega \in \mathbb{R}$ there exist $D_2 \in \boldsymbol{D}_2, Q_2 \in \boldsymbol{Q}_2$ such that inequality

$$(I - (I + jQ_2)D_2(I + M(j\omega))^{-1}M(j\omega)D_2^{-1} - D_2^{-1}M^H(j\omega)(I + M^H(j\omega))^{-1}D_2(I - jQ_2) > 0 \tag{15}$$

holds.

It is well known that the problem of robust stability of systems with parametric and dynamic uncertainties can be also handled by means of the mixed μ theory[19]. Using the upper bound for mixed μ proposed by Fan et al. [18] a sufficient condition for robust stability of the system is:

$$\sup_{\omega \in \mathbb{R}} \inf_{D,G} \bar{\lambda}(DMD^{-1}D^{-1}M^H D + j(GDMD^{-1} - D^{-1}M^H DG)) < 1 \tag{16}$$

where D, G have the same definition as D_2, Q_2. Next proposition shows that Corollary 2.2 gives the same robustness bound as the upper bound of mixed μ proposed by Fan et al..

Proposition 2.1 The following statements are equivalent to each other:

1) $I - (I + jQ_2)D_2(I + M)^{-1}MD_2^{-1} - D_2^{-1}M^H(I + M^H)^{-1}D_2(I - jQ_2) > 0$,

$\exists\, Q_2 \in \boldsymbol{Q}_2$, $\exists\, D_2 \in \boldsymbol{D}_2$,

2) $\inf_{D_2, Q_2} \bar{\lambda}(D_2 M D_2^{-1} D_2^{-1} M^H D_2 + j(Q_2 D_2 M D_2^{-1} - D_2^{-1} M^H D_2 Q_2)) < 1$.

Proof Denote $M_D = D_2 M D_2^{-1}$ for convenience. Then we have the following chain of equivalence:

$$I - (I + jQ_2)D_2(I + M)^{-1}MD_2^{-1} - D_2^{-1}M^H(I + M^H)^{-1}D_2(I - jQ_2) > 0$$
$$\Leftrightarrow I - (I + jQ_2)M_D(I + M_D)^{-1} - (I + M_D^H)^{-1}M_D^H(I - jQ_2) > 0$$
$$\Leftrightarrow (I + M_D^H)(I + M_D) - M_D^H(I - jQ_2)(I + M_D) - (I + M_D^H)(I - jQ_2)M_D > 0$$
$$\Leftrightarrow I - M_D^H M_D + jM_D^H Q_2 - jQ_2 M_D > 0, \quad \exists\, Q_2 \in \boldsymbol{Q}_2, \quad \exists\, D_2 \in \boldsymbol{D}_2.$$

An equivalent form of the last inequality is $\inf\limits_{D_2,Q_2} \bar{\lambda}(M_D^H M_D + j(Q_2M_D - M_D^H Q_2)) < 1$. Thus the proposition is proved. Q. E. D.

The proposition shows that the result of mixed μ upper bound given in [18] is a special case of our main theorem.

2.3 Computation of the Robust Absolute Stability Criterion

It is well known that Popov criterion is usually verified by using a graphic method. However, for multivariable systems this method requires certain row (column) diagonal dominant conditions which are not always satisfied in practice. And this method does not give any systematic approach to choosing matrices D_1 and Q_1 for all frequencies. Futher more, if the role of multiplier matrices D_2, Q_2 must be taken into consideration for each frequency as we have shown in Theorem 2. 1, it is very difficult to apply the graphic method. Below we will show that the criterion given by Theorem 2. 1 can be verified via an optimization procedure based on an interior-point method.

First we note that from definitions (6)~(9) it follows that $D_1D_2 = D_2D_1$, $Q_1Q_2 = Q_2Q_1$, $D_1Q_2 = Q_2D_1$ and $Q_1D_2 = D_2Q_1$. Using the commutativity shown above the condition (10) of Theorem 2. 1 can be restated as follows:

$$D_1^2D_2^2 - (D_1^2D_2^2 + j\omega D_1Q_1D_1D_2Q_2D_2)\widetilde{M}(j\omega) - \widetilde{M}^H(j\omega) \cdot (D_2^2D_1^2 - j\omega D_2Q_2D_2D_1Q_1D_1) > 0, \quad \forall\, \omega \in \mathbb{R}, \tag{17}$$

for some $D_1 \in \boldsymbol{D}_1$, $Q_1 \in \boldsymbol{Q}_1$ and ω- dependent matrices $D_2 \in \boldsymbol{D}_2$ and $Q_2 \in \boldsymbol{Q}_2$. Since the maps $\boldsymbol{D}_i \times \boldsymbol{Q}_i \rightarrow \boldsymbol{D}_i \times \boldsymbol{Q}_i$ defined by $(D_i, Q_i) \mapsto (D_i^2, D_iQ_iD_i)$, $i = 1,2$, are bijections, inequality (17) holds if and only if there exist some $D_1 \in \boldsymbol{D}_1$, $Q_1 \in \boldsymbol{Q}_1$ and ω- dependent matrices $D_2 \in \boldsymbol{D}_2$ and $Q_2 \in \boldsymbol{Q}_2$ such that

$$D_1D_2 - (D_1D_2 + j\omega Q_1Q_2)\widetilde{M}(j\omega) - \widetilde{M}^H(j\omega)(D_2D_1 - j\omega Q_2Q_1) > 0, \; \forall\, \omega \in \mathbb{R}. \tag{18}$$

Define

$$T(D_1, Q_1, \omega) = \inf_{D_2,Q_2} \bar{\lambda}((D_1D_2 + j\omega Q_1Q_2)\widetilde{M}(j\omega) + \widetilde{M}^H(j\omega)(D_2D_1 - j\omega Q_2Q_1) - D_1D_2), \tag{19}$$

then inequality (18) holds if and only if there exist $D_1 \in \boldsymbol{D}_1$ and $Q_1 \in \boldsymbol{Q}_1$ such that

$$\sup_{\omega\in\mathbb{R}} T(D_1, Q_1, \omega) < 0. \tag{20}$$

By definitions of D_1 and Q_1 they can be written as $D_1 = \mathrm{diag}[D_{11}, I]$, $Q_1 = \mathrm{diag}[Q_{11}, I]$. And D_{11}, Q_{11} can be determined by the absolute stability criterion for the nominal Lurie system (i. e., Lurie system without linear parametric and dynamic uncertainties). Using Corollary 2. 1 we have

$$(D_{11}, Q_{11}) = \arg \min_{D_{11},Q_{11}} \sup_{\omega\in\mathbb{R}} \bar{\lambda}((D_{11} + j\omega Q_{11})M(j\omega) + M^H(j\omega)(D_{11} - j\omega Q_{11}) - D_{11}) \tag{21}$$

where $M = M_{11}$ denotes the transfer matrix of the linear part of the nominal Lurie system. It is easy to see that this optimization problem is convex with respect to Q_{11} and D_{11}. However, it may be very difficult to calculate derivatives $\frac{\partial T}{\partial D_1}, \frac{\partial T}{\partial Q_1}$ which give the necessary conditions for a solution to (21). To solve this problem we can use, for example, Rosenbrock's method which is directly based on function values[20].

Finally, to verify inequality (20) we need to calculate

$$\min_{Q_2 \in \mathcal{Q}_2, D_2 \in \mathcal{D}_2} \bar{\lambda}((D_1 D_2 + j\omega Q_1 Q_2)\widetilde{M}(j\omega) + \widetilde{M}^H(j\omega)(D_2 D_1 - j\omega Q_2 Q_1) - D_1 D_2) \tag{22}$$

for given D_1, Q_1 and each $\omega \in \mathbb{R}$. Note that the optimization problem defined by (22) is also convex with respect to Q_2 and D_2, and can be solved by some standard algorithm, e. g., interior-point algorithm[21].

3 Example

In this section we will give an example to illustrate the application of the main result to analysis of the worst-case H_∞ performance of multivariable Lurie system with structured real parameter uncertainty.

Assume that the system is described by a state-space model:

$$\begin{cases} \dot{x} = (A + B_2 \Delta^r C_2)x + B_1 u + B_3 w, \\ y = C_1 x, \\ z = C_3 x, \\ u = f(y). \end{cases} \tag{23}$$

Here x is the state vector of the system. z, y, w, u denote, respectively, the controlled output, measured output, disturbance, and control variables. $w \in \mathscr{L}_2$. $A, B_i, C_j, i, j = 1,2,3$ are given matrices with appropriate dimensions. $\Delta^r = \mathrm{diag}[\delta_1^r, \cdots, \delta_{m_r}^r], \delta_i^r \in \mathbb{R}, |\delta_i^r| \leqslant 1$. $f(y) = [f_1(y_1), \cdots, f_{m_v}(y_{m_v})]^{\mathrm{T}}$ is the vector of continuous nonlinear functions satisfying sector-bounded condition (2). Our goal is to verify robust H_∞ performance of the system in the presence of nonlinearities and parameter uncertainty, i. e., verify whether the inequality

$$\max_{\Delta^r, f(y)} \int_0^{\mathrm{T}} \| z(\Delta^r, f(y), t) \| \,\mathrm{d}t < \gamma \int_0^T \| w(t) \| \,\mathrm{d}t$$

holds for all $T > 0, w \in \mathscr{L}_2[0, T]$, where γ is a constant value which describes the desired performance level. If we denote $C_2 x$ by y_p, $\Delta C_2 x$ by u_p, then system (23) can be rewritten as follows:

$$\begin{cases} \dot{x} = Ax + B_1 u + B_2 u_p + B_3 w, \\ y = C_1 x, \\ y_p = C_2 x, \\ z = C_3 x, \\ u = f(y), \\ u_p = \Delta^r y_p. \end{cases} \tag{24}$$

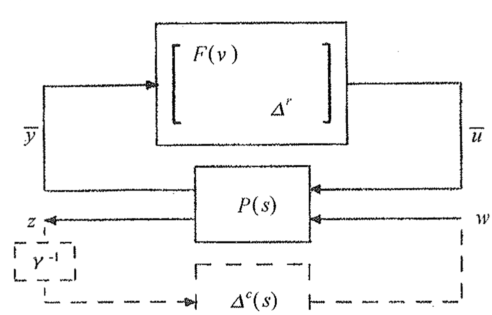

Fig. 2 Worst-case H_∞ performance of uncertain multivariable Lurie system

The input-output model of the system is shown in Fig. 2. In the system $\bar{y} = [y \quad y_p]^{\mathrm{T}}, \bar{u} = [u \quad u_p]^{\mathrm{T}}, F(y) = \mathrm{daig}[f_1(y_1), \cdots,$

$f_{m_v}(y_{m_v})]$, and

$$P(s) = C(sI - A)^{-1}B, \tag{25}$$

where $B = [B_1 \quad B_2 \quad B_3]$, $C = [C_1 \quad C_2 \quad C_3]^{\mathrm{T}}$. According to the Robust Performance Theorem[19] we know that the worst-case H_∞ performance problem of the system can be treated as a robust stability problem of the system with virtual dynamic uncertainty Δ^c (Fig. 2). So we can transform the system to the standard form shown in Fig. 1 with $\Delta = \mathrm{diag}[\delta_1^r, \cdots, \delta_{m_r}^r, \Delta^c] \in \boldsymbol{B\Delta}$ and $M(s) = \mathrm{diag}[I, \gamma^{-1}]P(s)$. Take

$$A = \begin{bmatrix} -5 & 4 & 0 \\ -1 & -8 & 0.4 \\ 0 & 2 & -9 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0.5 & 1 \\ 0.03 & 0.1 \\ 0 & 0 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0 & 0 \\ 0.3 & 1 \\ 1 & 0.5 \end{bmatrix}, \; B_3 = \begin{bmatrix} 1 \\ 0 \\ 0.3 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}, \; C_2 = \begin{bmatrix} 0.2 & 1 & 0 \\ 0 & 3 & 0 \end{bmatrix}, \; C_3 = [1 \quad 0.2 \quad 0].$$

Using the computation method developed in Section 2 we can verify that $\sup_{\omega \in \mathbb{R}} T(D_1, Q_1, \omega) = -0.009 < 0$ for $D_1 = \mathrm{diag}[40.0, 1.0]$, $Q_1 = \mathrm{diag}[2.1, 0.1]$, $\gamma = 0.285$. So we have

$$\max_{\Delta^r, f(y)} \int_0^T \| z(\Delta^r, f(y), t) \| \,\mathrm{d}t < 0.285 \int_0^T \| w(t) \| \,\mathrm{d}t, \quad \forall\, T > 0, \quad \forall\, w \in \mathscr{L}_2[0, T].$$

4 Conclusion

The main result of this paper is the robust absolute stability criterion given by Theorem 2.1 for multivariable Lurie system with structured parametric and dynamic uncertainties. In this criterion the classical Popov multiplier was extended to the form of $D_1D_2 + j\omega Q_1Q_2$ where D_1, Q_1 are independent of frequency ω but D_2, Q_2 should be chosen for each frequency. It was shown that the criterion can be verified via an optimization procedure based on Rosenbrock's and interior-point method. An example was given to illustrate how to apply the obtained result to the problem of worst-case H_∞ performance of uncertain Lurie system with multiple nonlinearities.

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Appendix Proof of the Main Theorem

Introduce the following definitions:

$$\chi_c := \{\mathrm{diag}[\Delta_1, \cdots, \Delta_{m_r+m_c}] : \Delta_i \in \mathbb{C}, i = 1, \cdots, m_r, \Delta_{m_r+q} \in \mathbb{C}^{k_{m_r+q} \times k_{m_r+q}}, q = 1, \cdots, m_c\},$$

$$\boldsymbol{B\Delta}^c := \{\Delta \in \chi_c : \bar{\sigma}(\Delta) \leqslant 1\},$$

$$\boldsymbol{D} := \{\mathrm{diag}[d_1, \cdots, d_{m_v}, d_{m_v+1} I k_{m_v+1}, \cdots, d_{m_v+m_r+m_c} I_{k_{m_v+m_r+m_c}}] :$$

$$0 < d_i \in \mathbb{R}, i = 1, \cdots, m_v + m_r + m_c\}.$$

Comparing these matrix sets with ones defined in Section 2 we have $\chi(s) \subset \chi_c$, $\boldsymbol{B\Delta}(s) \subset \boldsymbol{B\Delta}^c$, $\boldsymbol{D}_1 \subset \boldsymbol{D}$, $\boldsymbol{D}_2$

The following lemma can be derived from straightforward operation based on the definition of LFT

Lemma A. 1 If $I - M$, $I - M_{11}\Delta$ and $I - M_{22}\Delta$ are nonsingular square matrices, then the following lities hold:

i) $\mathscr{F}_l((I - M)^{-1}M, \Delta - I) = (I - \mathscr{F}_l(M, \Delta))^{-1}\mathscr{F}_l(M, \Delta)$;

ii) $\mathscr{F}_u((I - M)^{-1}M, \Delta - I) = (I - \mathscr{F}_u(M, \Delta))^{-1}\mathscr{F}_u(M, \Delta)$.

The following result was first presented by Redheffer[23].

Lemma A. 2 Let $M \in \mathbb{C}^{N \times N}$. If there exists $D = \mathrm{diag}[D_{11}, D_{22}] \in \boldsymbol{D}$ such that $\bar{\sigma}(DMD^{-1}) < 1$,

i) $\max\limits_{\Delta \in B\Delta^c} \bar{\sigma}(D_{11}\mathscr{F}_l(M,\Delta)D_{11}^{-1}) < 1$;

ii) $\max\limits_{\Delta \in B\Delta^c} \bar{\sigma}(D_{22}\mathscr{F}_u(M,\Delta)D_{22}^{-1}) < 1$.

Next lemma can be regarded as a reformulation of the result of Anderson[22].

Lemma A. 3 Suppose $G(j\omega) \in \mathbb{C}^{m\times m}$, then for any $\alpha > 0$ and any $\omega \in \mathbb{R}$, the following two statements are equivalent to each other:

i) $\bar{\sigma}((G(j\omega) - \alpha I)(G(j\omega) + \alpha I)^{-1}) < 1$,

ii) $G(j\omega) + G^H(j\omega) > 0$.

According to Popov criterion[4] (Corollary 2. 1) the system shown in Fig. 1 is absolutely stable if

i) $\mathscr{F}_l(M,\Delta) \in \mathbb{R}\mathrm{H}_\infty$ for all $\Delta(s) \in \boldsymbol{B\Delta}(s)$, i. e.,

$$\det(I - M_{22}(j\omega)\Delta(j\omega)) \neq 0, \quad \forall\, \Delta \in \boldsymbol{B\Delta}(j\omega), \quad \forall\, \omega \in \mathbb{R}; \tag{A4}$$

ii) there exist some $Q_1 = \mathrm{diag}[Q_{11}, I_{N-m_v}] \in \boldsymbol{Q}_1$ and $D_1 = \mathrm{diag}[D_{11}, I_{N-m_v}] \in \boldsymbol{D}_1$ such that for all $\Delta(s) \in \boldsymbol{B\Delta}(s)$ and all $\omega \in \mathbb{R}$ the following inequality holds

$$I - (I + j\omega Q_{11})D_{11}\mathscr{F}_l(M(j\omega),\Delta(j\omega))D_{11}^{-1} - D_{11}^{-1}\mathscr{F}_l^H(M(j\omega),\Delta(j\omega))D_{11}(I - j\omega Q_{11}) > 0. \tag{A5}$$

To simplify the statement of proof of Theorem 2. 1 we first prove the following proposition.

Proposition A. 1 Write $D_1 \in \boldsymbol{D}_2$ and $Q_2 \in \boldsymbol{Q}_2$ as diag $[I_{m_v}, D_{22}]$, and $\mathrm{diag}[I_{m_v}, Q_{22}]$ respectively. Let

$$\bar{\Delta}^c = (D_{22}\Delta D_{22}^{-1} + I)(I + jQ_{22})^{-1} - I, \tag{A6}$$

$$\bar{M} = (I + j\omega Q_1 Q_2)D_2\tilde{M}D_2^{-1}, \tag{A7}$$

then

i) $\bar{\Delta}^c \in \boldsymbol{B\Delta}^c$ for all $\Delta \in \boldsymbol{B\Delta}(j\omega), D_2 \in \boldsymbol{D}_2, Q_2 \in \boldsymbol{Q}_2$;

ii) $\mathscr{F}_l((I - \bar{M})^{-1}\bar{M}, \bar{\Delta}^c) = (I - (I + j\omega Q_{11})\mathscr{F}_l(M,\Delta))^{-1}(I + j\omega Q_{11})\mathscr{F}_l(M,\Delta)$.

Proof i) It is evident that $\bar{\Delta}^c$ is a block-diagonal matrix. Denote its ith element in diagonal by $\bar{\Delta}_i$. While $i > m_r$ we have $\bar{\sigma}(\bar{\Delta}_i) = \bar{\sigma}((d_i\Delta_i d_i^{-1} + I)(I + j0) - I) = \bar{\sigma}(\Delta_i) \leqslant 1$.

While $i \leqslant m_r$ we have $\bar{\Delta}_i = (D_i\delta_i^r D_i^{-1} + I)(I + jQ_i)^{-1} - I = (\delta_i^r + 1)(I + Q_iQ_i^H)^{-1}(I - jQ_i) - I$. So we get

$$\begin{aligned}\bar{\sigma}(\bar{\Delta}_i) &= \bar{\lambda}(((\delta_i^r + 1)(I + Q_iQ_i^H)^{-1}(I - jQ_i) - I)((\delta_i^r + 1)(I + jQ_i)(I + Q_iQ_i^H)^{-1} - I)) \\ &= \bar{\lambda}((\delta_i^r + 1)^2(I + Q_iQ_i^H)^{-1} - 2(\delta_i^r + 1)(I + Q_iQ_i^H)^{-1} + I) < 1.\end{aligned}$$

Thus conclusion i) of Proposition A. 1 is proved.

ii) According to Lemma A. 1 we have

$$\mathscr{F}_l((I - \bar{M})^{-1}\bar{M}, \bar{\Delta}_c) = (I - \mathscr{F}_l(\bar{M}, \bar{\Delta}_c'))^{-1}\mathscr{F}_l(\bar{M}, \bar{\Delta}_c'),$$

where

$$\bar{\Delta}_c' = (D_{22}\Delta D_{22}^{-1} + I)(I + j\omega Q_{22})^{-1}.$$

Substituting $Q_1 = \mathrm{diag}[Q_{11}, I], Q_2 = \mathrm{diag}[I, Q_{22}]$ and $D_2 = \mathrm{diag}[I, D_{22}]$ into (A7) we get

$$\bar{M} = \begin{bmatrix} (I + j\omega Q_{11})\tilde{M}_{11} & (I + j\omega Q_{11})\tilde{M}_{12} \\ (I + j\omega Q_{22})D_{22}\tilde{M}_{21}D_{22}^{-1} & (I + j\omega Q_{22})D_{22}\tilde{M}_{22}D_{22}^{-1} \end{bmatrix}.$$

Using the definition of linear fractional transformation, we get

$$\begin{aligned}&\mathscr{F}_l(\bar{M}, \bar{\Delta}_c') \\ &= (I + j\omega Q_{11})\tilde{M}_{11} + (I + j\omega Q_{11})\tilde{M}_{12}(D_{22}\Delta D_{22}^{-1} + I)(I + j\omega Q_{11})^{-1} \\ &\quad \cdot (I - (I + j\omega Q_{22})D_{22}\tilde{M}_{22}D_{22}^{-1}(D_{22}\Delta D_{22}^{-1} + I)(I + j\omega Q_{22})^{-1})^{-1}(I + j\omega Q_{22})D_{22}\tilde{M}_{21}D_{22}^{-1} \\ &= (I + j\omega Q_{11})\tilde{M}_{11} + (I + j\omega Q_{11}\tilde{M}_{12}(\Delta + I)(I - \tilde{M}_{22}\Delta)^{-1}\tilde{M}_{21}\end{aligned}$$

$$=(I+j\omega Q_{11})M_{11}+(I+j\omega Q_{11})M_{12}(\Delta+I)(I-M_{22}\Delta)^{-1}M_{21}-(I+j\omega Q_{11})M_{12}$$
$$\cdot\,[(I+M_{22})^{-1}-(I+M_{22})^{-1}M_{22}(\Delta+I)(I-(I+M_{22}))^{-1}M_{22}(\Delta+I))^{-1}(I+M_{22})^{-1}]M_{21}.$$

Using the following matrix identity

$$(A-BCD)^{-1}=A^{-1}+A^{-1}B(C^{-1}-DA^{-1}B)^{-1}DA^{-1},$$

we obtain

$$\begin{aligned}&\mathscr{F}_l(\overline{M},\overline{\Delta}_c')\\&=(I+j\omega Q_{11})[M_{11}+M_{12}(\Delta+I)(I-M_{22}\Delta)^{-1}M_{21}-M_{12}(I-M_{22}\Delta)^{-1}M_{21}]\\&=(I+j\omega Q_{11})(M_{11}+M_{12}\Delta(I-M_{22}\Delta)^{-1}M_{21})\\&=(I+j\omega Q_{11})\mathscr{F}_l(M,\Delta).\end{aligned}$$

Thus conclusion ii) of Proposition A. 1 is proved. Q. E. D.

Now we are ready to complete the proof of Theorem 2. 1.

Proof of Theorem 2. 1 Suppose there exist $D_1\in\boldsymbol{D}_1, Q_1\in\boldsymbol{Q}_1$ and ω -dependent matrices $D_2\in\boldsymbol{D}_2, Q_2\in\boldsymbol{Q}_2$ such that

$$I-(I+j\omega Q_1Q_2)D_1D_2\widetilde{M}(j\omega)D_2^{-1}D_1^{-1}-D_1^{-1}D_2^{-1}\widetilde{M}^H(j\omega)D_2D_1(I-j\omega Q_2Q_1)>,\quad\forall\,\omega\in\mathbb{R}.$$

By Lemma A. 3 this implies that

$$\bar{\sigma}((I-(I+j\omega Q_1Q_2)D_1D_2\widetilde{M}(j\omega)D_2^{-1}D_1^{-1})^{-1}(I+j\omega Q_1Q_2)D_1D_2\widetilde{M}(j\omega)D_2^{-1}D_1^{-1})<1,\forall\,\omega\in\mathbb{R}.$$

Using the commutativity between Q_i and D_i , and extracting D_1 and D_1^{-1}, we have

$$\bar{\sigma}(D_1(I-(I+j\omega Q_1Q_2)D_2\widetilde{M}(j\omega)D_2^{-1})^{-1}(I+j\omega Q_1Q_2)D_2\widetilde{M}(j\omega)D_2D_1^{-1})<1,\quad\forall\,\omega\in\mathbb{R}.$$

Writing D_1 as diag $[D_{11},I]$ and using Lemma A. 2 we get

$$\max_{\Delta^c\in B\Delta^c}\bar{\sigma}(D_{11}\mathscr{F}_l((I-\overline{M})^{-1}\overline{M},\Delta^c)D_{11}^{-1})<1,\quad\forall\,\omega\in\mathbb{R},\tag{A8}$$

and

$$\max_{\Delta^c\in B\Delta^c}\bar{\sigma}(\mathscr{F}_u((I-\overline{M})^{-1}\overline{M},\Delta^c))<1,\quad\forall\,\omega\in\mathbb{R},\tag{A9}$$

where $\overline{M}$ is given by (A7). Let $\Delta^c=-I\in\boldsymbol{B\Delta}^c$, it follows from conclusion ii) of Lemma A. 1 that

$$\begin{aligned}\mathscr{F}_u((I-\overline{M})^{-1}\overline{M},0-I)&=(I-\mathscr{F}_u(\overline{M},0))^{-1}\mathscr{F}_u(\overline{M},0)\\&=(I-\overline{M}_{22})^{-1}\overline{M}_{22}\\&=(I-(I+j\omega Q_{22})D_{22}\widetilde{M}_{22}D_{22}^{-1})^{-1}(I+j\omega Q_{22})D_{22}\widetilde{M}_{22}D_{22}^{-1}.\end{aligned}$$

Denote $\hat{M}=(I-(I+j\omega Q_{22})D_{22}\widetilde{M}_{22}D_{22}^{-1})^{-1}(I+j\omega Q_{22})D_{22}\widetilde{M}_{22}D_{22}^{-1}$, then according to (A9) we have $\bar{\sigma}(\hat{M})<1$. This implies that

$$\det(I-\hat{M}\Delta^c)\neq 0,\quad\forall\,\Delta^c\in\boldsymbol{B\Delta}^c.\tag{A10}$$

Take $\overline{\Delta}^c=(D_{22}\Delta D_{22}^{-1}+I)(I+jQ_{22})^{-1}-I$. By conclusion i) of Proposition A. 1 we know that $\overline{\Delta}_c\in\boldsymbol{B\Delta}^c$. Substituting $\overline{\Delta}^c$ into (A10) we have

$$\begin{aligned}&\det(I-(I-(I+j\omega Q_{22})D_{22}\widetilde{M}_{22}D_{22}^{-1})^{-1}(I+j\omega Q_{22})D_{22}\widetilde{M}_{22}D_{22}^{-1})(D_{22}\Delta D_{22}^{-1}+I)(I+j\omega Q_{22})^{-1}-I)\\&=\det(I-D_{22}\widetilde{M}_{22}D_{22}^{-1})^{-1}\det(I-D_{22}\widetilde{M}_{22}D_{22}^{-1})(D_{22}\Delta D_{22}^{-1}+I)\\&=\det(I-D_{22}(I+M_{22})^{-1}M_{22}^{-1}D_{22}^{-1})^{-1}\det(I-D_{22}(I+M_{22})^{-1}M_{22}^{-1}D_{22}^{-1})(D_{22}\Delta D_{22}^{-1}+I)\\&=\det(I-D_{22}(I+M_{22})^{-1}M_{22}^{-1}D_{22}^{-1})^{-1}\det(I+M_{22})^{-1}\det(I-M_{22}\Delta)\neq 0.\end{aligned}$$

Hence det $(I-M_{22}\Delta)\neq 0,\forall\,\Delta\in\boldsymbol{B\Delta}(j\omega)$. So we have proved (A4).

Now let us back up to inequality (A8) . Take $\overline{\Delta}_c=(D_{22}\Delta D_{22}^{-1}+I)(I+jQ_{22})^{-1}-I$. By conclusion ii) of Proposition A. 1 we have $\overline{\Delta}^c\in\boldsymbol{B\Delta}^c$ for all $\Delta\in\boldsymbol{B\Delta}(j\omega)$, $D_2\in\boldsymbol{D}_2$ and $Q_2\in\boldsymbol{Q}_2$. Substituting $\overline{\Delta}^c$ into (A9), it follows from conclusion ii) of Proposition A. 1 that

$$\begin{aligned}&\max_{\Delta\in B\Delta}\bar{\sigma}(D_{11}(I-(I+j\omega Q_{11})\mathscr{F}_l(M(j\omega),\Delta(j\omega)))^{-1}\\&\cdot\,(I+j\omega Q_{11})\mathscr{F}_l(M(j\omega),\Delta(j\omega))D_{11}^{-1})<1,\quad\forall\,\omega\in\mathbb{R}.\end{aligned}$$

Using Lemma A. 3, we have

$$I - (I + j\omega Q_{11}) D_{11} \mathscr{F}_l(M(j\omega), \Delta(j\omega)) D_{11}^{-1} - D_{11}^{-1} \mathscr{F}_l^H(M(j\omega), \Delta(j\omega))$$
$$\cdot D_{11}(I - j\omega Q_{11}) > 0, \quad \forall \omega \in \mathbb{R}, \quad \forall \Delta \in \boldsymbol{B\Delta}.$$

Thus we have proved (A5). And this closes the proof of Theorem 2.1.

具有线性和非线性结构式不确定性的多变量系统的鲁棒稳定性

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摘要：本文研究具有多重扇形非线性环节和结构式线性不确定性系统的鲁棒稳定性.给出了检验含参数不确定性和未建模动态的多变量 Lurie 系统鲁棒绝对稳定的统一判据,并证明了多变量 Popov 判据和混合 μ 上界均是该判据的特殊形式.基于 Rosenbrock 优化算法和内点法,本文还给出了计算这一判据的一种凸优化算法.最后用一计算示例表明本文结果还可用于确定含参数不确定性和未建模动态的 Lurie 系统的 H_∞ 性能.

关键词：鲁棒稳定性；Lurie 系统；结构式不确定性；Popov 判据；混合 μ

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中国智能机器人'98 学术研讨会 会议纪要

中国智能机器人'98 学术研讨会暨中国人工智能学会第三届智能机器人学术研讨会于 1998 年 5 月 30 日—6 月 2 日在杭州召开.这是一次关于智能机器人领域的全国性学术大会.来自全国各地从事智能机器人、智能控制研究和应用的论文作者代表、专家和领导,以及国家自然科学基金委员会和国家 863 计划空间机器人专家组代表近百人出席了这次盛会.

在大会开幕式上,浙江大学副校长顾伟康教授、国家自然科学基金委员会信息科学部自动化学科主任徐孝涵教授、国家 863 空间机器人专家组组长孙增圻教授、浙江大学计算机系主任陈纯教授、中南工业大学信息工程学院院长沈德耀教授分别在会上致词,对大会的召开表示祝贺.全国政协副主席、中国工程院院长、原国务委员兼国家科委主任宋健院士、中国科学院院长路甬祥院士、中国人工智能学会理事长涂序彦教授、国家科技部高技术司和自动化办公室及浙江省科委分别发来贺信或通过电话对大会的召开表示热烈祝贺.中国有色金属学会计算机学术委员会、湖南省自动化学会、湖南省计算机学会、中南工业大学科协和信息工程学院等单位送来贺信,热烈祝贺大会顺利召开.

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