A Quasi-Infinite Horizon Predictive Control Scheme for Constrained Nonlinear Systems *

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Abstract: We present in this paper a quasi-infinite horizon model predictive control scheme for stable and unstable nonlinear systems subject to input and state constraints. Hard state constraints are relaxed in an optimal way to avoid infeasibility. With an additional terminal cost in the standard finite horizon objective functional and a terminal inequality constraint, the prediction is expanding quasi to infinity but the control profile to be determined by on-line optimization is only of finite horizon. Closed-loop stability is guaranteed by the feasibility of the optimization problem at the very beginning.

Key words: model predictive control; constrained nonlinear systems; stability; quasi-infinite horizon

约束非线性系统的一个准无限时域预测控制方案

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摘要: 提出一个具有准无限预测时域的模型预测控制方案.该方案可用于输入和状态约束非线性系统的控制.用优化柔性状态约束条件代替了硬性状态约束,以避免优化问题的不可解.开环优化问题含有附加的终端代价项和终端约束条件,这样预测时域延伸至准无限,而需在线优化获得的控制函数仅为有限时域.如果在最初时刻优化问题有解,则闭环系统具有保证稳定性.

关键词: 模型预测控制; 约束非线性系统; 稳定性; 准无限预测时域

1 Introduction

In practice, most plants suffer constraints on inputs and states, e.g., actuator saturation and some states are not allowed to exceed their limitations for a safe operation or environmental regulations. Many plants are additionally nonlinear, especially when the plant is operated at an optimal operating point that is desired for demanding economic considerations and higher product quality specifications. Thus, control approaches developed are required to be able to handle nonlinearity and constraints. Due to its ability to handle constraints in an explicit and optimal way, model predictive control (MPC) has become an attractive feedback strategy for linear or nonlinear plants subject to constraints. Since the late 70's, many MPC schemes have been suggested and found successful applications especially in process industry (see e.g. Ref. [1~4]).

The MPC problem is generally formulated as solving on-line a finite horizon open-loop optimal control problem subject to constraints. Such a finite horizon MPC scheme does not guarantee closed-loop stability[5]. Thus, Rawlings and Muske[6] derive an MPC scheme with infinite prediction horizon and finite control horizon, in which additional terminal equality constraints are used to force unstable modes to be zero at the end of the control horizon. Imposing terminal equality constraints on all states Mayne and Michalska[7] prove that MPC is able to stabilize a class of constrained nonlinear systems. In fact, the prediction in such an MPC scheme expands exactly to infinity. In the nonlinear case, however, the exact

* Supported by the State Education Commission and the National Natural Science Foundation of China (69804004).
Manuscript received May 16, 1997, revised May 20, 1998.

satisfaction of the terminal equality constraint is very difficult, if not impossible. Thus, Mayne and Michalska[8] replace the terminal equality constraint by a terminal inequality constraint such that the states at the end of a variable horizon are on the boundary of a terminal region. They suggest a dual-mode MPC scheme with a linear state feedback controller inside the terminal region and a predictive-controller outside the terminal region. The closed-loop control is then completed by switching between two controllers. In order to avoid this, the authors of this paper introduce a terminal cost into the finite horizon objective functional[9], which bounds the infinite horizon cost of the nonlinear plant model controlled by a local linear state feedback in a terminal region. A quasi-infinite horizon MPC scheme is suggested that guarantees closed-loop stability. Other methods to guarantee stability for constrained MPC can also be found in the literature. For more information see e.g. Ref. [2,10,11].

In all approaches above, closed-loop stability requires the feasibility of constraints. The satisfaction of input constraints can be assured easily during the computation of an optimal solution, while the state constraints eventually lead to infeasibility of the optimization problem. Two approaches have been developed: The first is suggested for constrained linear systems[6], where state constraints are removed over an early portion of the horizon, in order to enforce them after that; in the second method, violation of state constraints is allowed at each time but penalized (e.g. [12,13]). A comparison between both methods can be found elsewhere[14].

In this paper, we extend our work[9] to consider state constraints in the quasi-infinite horizon nonlinear MPC scheme. In order to avoid infeasibility, hard state constraints are allowed to be violated at each time, but the violation is penalized in the 2-norm. The open-loop optimal control problem involves then hard input constraints, soft state constraints and a terminal inequality constraint. The implicit prediction horizon in the proposed controller is quasi infinite, but the control profile to be determined on-line is only of finite horizon. It will be shown that asymptotic stability is guaranteed by the feasibility of the optimization problem at time $t = 0$, if the Jacobian linearization of the nonlinear system being controlled is stabilizable.

2 Control scheme

Consider multiple input plants described by the following nonlinear time-invariant model with state vector $x(t) \in \mathbb{R}^n$ and control vector $u(t) \in \mathbb{R}^m$, subject to hard input and state constraints

$$\dot{x}(t) = f(x(t), u(t)), x(0) = x_0, \tag{1a}$$

$$u(t) \in U, x(t) \in X, \ \forall t \geqslant 0, \tag{1b}$$

where it is assumed that $f: \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is twice continuously differentiable and w.l.o.g. $f(\mathbf{0},\mathbf{0}) = \mathbf{0}$, that $U \subset \mathbb{R}^m$ is compact and convex, $X \subseteq \mathbb{R}^n$ is connected, and that the point $(\mathbf{0},\mathbf{0})$ is contained in the interior of $X \times U$. Moreover, we consider the state feedback case.

We introduce first some notations used: For any vector $x \in \mathbb{R}^n$, $\| x \|$ denotes the 2-norm and the weighted norm $\| x \|_P$ is defined by $\| x \|_P^2 := x^{\mathrm{T}} P x$, where P is a positive definite matrix. For any Hermitian matrix A, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the largest and the smallest real parts of the eigenvalues, and $\| A \|$ stands for the induced 2-norm of A. In the framework of MPC, the open-loop optimal control problem at time t with initial condition $x(t)$ is formulated as

$$\min_{\bar{u}, \bar{s}} J(x(t), \bar{u}, \bar{s}) \tag{2a}$$

with

$$J(x(t), \bar{u}, \bar{s}) = \int_t^{t+T_p} \left(\| \bar{x}(\tau; x(t), t) \|_Q^2 + \| \bar{u}(\tau) \|_R^2 + \| \bar{s}(\tau) \|_W^2 \right) \mathrm{d}\tau + \| \bar{x}(t + T_p; x(t), t) \|_P^2 \tag{2b}$$

subject to

$$\dot{\bar{x}} = f(\bar{x}, \bar{u}), \ \bar{x}(t; x(t), t) = x(t), \tag{3a}$$

$$\bar{u}(\tau) \in U, \ \tau \in [t, t + T_p], \tag{3b}$$

$$\bar{x}(\tau; x(t), t) - \bar{s}(\tau) \in X, \tau \in [t, t + T_p], \tag{3c}$$

$$\bar{x}(t + T_p; x(t), t) \in \Omega, \tag{3d}$$

where $Q > 0$, $R > 0$ and $W \geqslant 0$ denote symmetric weighting matrices, T_p is a finite prediction horizon. In order to indicate that the predicted values need not and will not be the same as the actual values, (this is also true for the undisturbed case with no model-plant mismatch, if only a finite horizon is used), we denote the internal variables in the controller by a bar $(\bar{x}, \bar{u}, \bar{s})$. Thus, $\bar{x}(\cdot; x(t), t)$ is the trajectory of (3a) driven by

$\bar{u}(\cdot): [t, t + T_p] \to U$. Note the initial condition in (3a): The plant model in the controller is initialized by the actual system states $x(t)$.

In general, the above optimization problem can only be solved with numerical methods, especially when constrained nonlinear systems are considered. A time-continuous input parameterization leads to an infinitely dimensional optimization problem that is numerically insolvable. In order to get around that, minimizing (2b) subject to (3) will be done over step-shaped profiles of the open-loop control $\bar{u}$ and the state constraint violation $\bar{s}$, i.e., $\bar{u}(\tau) = \text{const}$ and $\bar{s}(\tau) = \text{const}$ for $\tau \in [t + i\delta, t + (i+1)\delta)$, $i = 0, 1, \cdots, N_p - 1$, where δ is the sampling time and $N_p = \frac{T_p}{\delta}$. An optimal solution (existence assumed) is denoted by $\bar{u}^*(\cdot; x(t)): [t, t + T_p] \to U$ and $\bar{s}^*(\cdot; x(t))$ on $[t, t + T_p]$. The corresponding optimal value is denoted by $J^*(x(t)) := J(x(t), \bar{u}^*, \bar{s}^*)$. According to the principle of MPC, the discrete closed-loop control is defined by

$$\bar{u}^*(\tau) := \bar{u}^*(\tau; x(t)), \quad \tau \in [t, t + \delta]. \tag{4}$$

The objective functional (2b) consists of a finite horizon standard cost to specify control performance, a terminal cost to penalize the states at the end of the finite horizon and a violation cost to penalize the violation of the state constraints. The soft state constraints (3c) imply that the hard ones in (1b) are relaxed by $\bar{s}(\cdot)$, whose weighted 2-norm will be minimized. The constraint (3d) is referred to as terminal inequality constraint and forces the states at the end of the finite horizon T_p to be in a terminal region defined by

$$\Omega := \{x \in \mathbb{R}^n \mid x^{\mathrm{T}} P x \leqslant \alpha, \alpha \in \mathbb{R}_0^+\}. \tag{5}$$

The positive definite symmetric terminal penalty matrix P is not a design parameter that can be chosen freely. It should be determined in such a way that the terminal cost $\| \bar{x}(t + T_p; x(t), t) \|_P^2$ bounds the infinite horizon cost for the unconstrained nonlinear plant model controlled by a local linear state feedback, if $\bar{x}(t + T_p; x(t), t) \in \Omega$, i.e.,

$$\int_{t+T_p}^{\infty} (\| \bar{x}(\tau; x(t), t) \|_Q^2 + \| \bar{u}(\tau) \|_R^2) \mathrm{d}\tau \leqslant \| \bar{x}(t + T_p; x(t), t) \|_P^2, \quad \bar{u} = K\bar{x}. \tag{6}$$

In the following, we outline a method to compute a terminal penalty matrix and a terminal region off-line. More details can be found elsewhere[9]. First, we linearize the plant model at the origin to get $A := \frac{\partial f}{\partial x}(\mathbf{0}, \mathbf{0})$ and $B := \frac{\partial f}{\partial u}(\mathbf{0}, \mathbf{0})$. It is assumed that

A0) the Jacobian linearized system (A, B) of (1a) is stabilizable.

This assumption ensures the existence of a linear state feedback $u = Kx$ such that $A_K := A + BK$ is asymptotically stable. Then, we choose a constant κ satisfying $0 \leqslant \kappa < -\lambda_{\max}(A_K)$, which implies the existence of the unique positive definite, symmetric solution of the Lyapunov equation

$$(A_K + \kappa I)^{\mathrm{T}} P + P(A_K + \kappa I) = -Q^*, \quad \text{with} \quad Q^* = Q + K^{\mathrm{T}} R K. \tag{7}$$

The solution P can be used as a terminal penalty matrix. Finally, we find a constant $\alpha \in (0, \infty)$ specifying a neighborhood Ω of the origin in the form of (5) such that

C0) $\Omega \subseteq X$; $Kx \in U$, for all $x \in \Omega$,

C1) $L_\phi := \sup \left\{ \frac{\| \phi(x) \|}{\| x \|} \,\middle|\, x \in \Omega, x \neq \mathbf{0} \right\}$ meets $L_\phi \leqslant \frac{\kappa \lambda_{\min}(P)}{\| P \|}$, where $\phi(x) := f(x, Kx) - A_K x$.

Remark 2.1 Since $(\mathbf{0}, \mathbf{0})$ is in the interior of $X \times U$ and $\phi(x)$ satisfies $\frac{\| \phi(x) \|}{\| x \|} \to 0$ as $\| x \| \to 0$, then, Ω with $\alpha > 0$ is not empty, if assumption A0) holds.

With the terminal penalty matrix P and the terminal region Ω determined above, we have

R0) the terminal region Ω defined by (5) is invariant for $\dot{x} = f(x, Kx)$,

R1) the infinite horizon cost for $\dot{x} = f(x, Kx)$ starting from Ω is bounded above as in (6).

Indeed, condition C0) implies that the plant model can be thought of as unconstrained in Ω. Thus, in order to find the upper bound in the form of (6), we differentiate $x^{\mathrm{T}} P x$ along any trajectory of $\dot{x} = f(x, Kx)$ starting from Ω and obtain

$$\frac{\mathrm{d}}{\mathrm{d}t} x(t)^{\mathrm{T}} P x(t) = x(t)^{\mathrm{T}} (A_K^{\mathrm{T}} P + P A_K) x(t) + 2 x(t)^{\mathrm{T}} P \phi(x(t)). \tag{8}$$

With a constant κ satisfying $\frac{L_\phi \cdot \| P \|}{\lambda_{\min}(P)} \leqslant \kappa < -\lambda_{\max}(A_K)$, we have

$$x(t)^{\mathrm{T}}P\phi(x(t)) \leqslant \kappa x(t)^{\mathrm{T}}Px(t). \quad (9)$$

Since P satisfies the Lyapunov equation (7), it follows from substituting (9) into (8) that

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t)^{\mathrm{T}}Px(t) \leqslant -x(t)^{\mathrm{T}}Q^{*}x(t). \quad (10)$$

Because of $Q^{*} > 0$, (10) implies that any trajectory of $\dot{x} = f(x, Kx)$ starting from Ω stays in Ω and converges to the origin. Recall the notation in the controller, then, integrating (10) from $t + T_p$ to ∞ with initial condition $\bar{x}(t + T_p; x(t), t) \in \Omega$ gives the result (6).

Remark 2.2 Substituting (6) into (2b) leads to

$$\min_{u,s}\int_{t}^{\infty}(\|\bar{x}(\tau;x(t),t)\|_{Q}^{2} + \|\bar{u}(\tau)\|_{R}^{2} + \|\bar{s}(\tau)\|_{W}^{2})\mathrm{d}\tau \leqslant \min_{u,s}J(x(t),\bar{u},\bar{s}).$$

This way, the prediction horizon of the proposed nonlinear MPC scheme expands quasi to infinity.

Remark 2.3 Condition C1) may be very conservative, due to the typically small value of $\frac{\lambda_{\min}(P)}{\|P\|}$. It is possible that for some systems this condition can only be met in an extremely small terminal region. From the above, we know that if inequality (9) is true, inequality (10) also holds. Hence, in order to get a less conservative terminal region, we may take a different approach: For a chosen κ, we make iterations of simple optimizations[8]

$$\max_{x}\{x^{\mathrm{T}}P\phi(x) - \kappa x^{\mathrm{T}}Px \mid x^{\mathrm{T}}Px \leqslant \alpha\}, \quad (11a)$$

$$\max_{x}\{d(Kx, U) \mid x^{\mathrm{T}}Px \leqslant \alpha\}, \quad (11b)$$

$$\max_{x}\{d(x, X) \mid x^{\mathrm{T}}Px \leqslant \alpha\}, \quad (11c)$$

by reducing α until the optimal values of (11) are non-positive. If a suitable α is found in this way, it specifies a terminal region in the form of (5), where R0) and R1) are true.

3 Asymptotic stability

In this section, we consider the stability property of the closed-loop system

$$\dot{x}(t) = f(x(t), u^{*}(t)) \quad (12)$$

with the model predictive control (4). The result is stated in the following.

Theorem 1 Suppose that

i) the nonlinear plant model (1a) has a unique solution for any piecewise continuous $u(\cdot): [0, \infty) \to U$ and any initial condition,

ii) Assumption A0) is satisfied,

iii) the open-loop optimal control problem described in Section 2 in feasible at time $t = 0$,

then, in the absence of disturbances, the closed-loop system (12) is nominally asymptotically stable. Let $D \subseteq \mathbb{R}^n$ denote the set of all initial states for which assumption iii) is satisfied, then, D gives a region of attraction for the closed-loop system.

Proof For $x(t) = \mathbf{0}$, the optimal solution of the optimization problem is $\bar{u}^{*}(\cdot; x(t)): [t, t + T_p] \to \mathbf{0}$. According to (4), we have $u^{*}(\cdot): [t, t + \delta] \to \mathbf{0}$. Since $f(\mathbf{0},\mathbf{0}) = \mathbf{0}$, then $x = \mathbf{0}$ is an equilibrium of the closed-loop system.

Given any initial state $x_0 \in D$. According to the principle of MPC, the optimization problem has to be solved repeatedly, updated with new measurements at $t = 0, \delta, 2\delta, \cdots$. In the following, we first show the feasibility of the optimization problem at each time.

Assumed that, at time t, the optimization problem with initial condition $\bar{x}(t; x(t), t) = x(t)$ is solved optimally. A finite horizon open-loop optimal control profile $\bar{u}^{*}(\cdot; x(t)): [t, t + T_p] \to U$ drives the plant model from $x(t)$ into the terminal region Ω along a finite horizon open-loop optimal state trajectory $\bar{x}^{*}(\cdot; x(t))$ on $[t, t + T_p]$, where the state constraints are violated in an optimal way by $\bar{s}^{*}(\cdot; x(t))$. Since $\bar{x}^{*}(t + T_p; x(t)) \in \Omega$ there is $s^{*}(t + T_p; x(t)) = 0$. Note that if the optimization problem is solved numerically, that is in general the case, $\bar{s}^{*}(\cdot; x(t))$ is step-shaped, i.e., only the discrete values of the state constraint violation are given. For the nominal system without disturbances, the closed-loop state trajectory on $[t, t + \delta]$ is then

$$x(\tau) = \bar{x}^{*}(\tau; x(t)), \quad \tau \in [t, t + \delta] \quad (13)$$

that violates the state constraints at time $t + \delta$ with $s(t + \delta) = \bar{s}^{*}(t + \delta; x(t))$. At time $t + \delta$, in order to solve the optimization problem with the new initial condition $\bar{x}(t + \delta; x(t + \delta), t + \delta) = x(t + \delta)$, a candidate control profile $\bar{u}(\cdot)$ on $[t + \delta, t + \delta + T_p]$ may be chosen as follows:

$$\bar{u}(\tau) = \begin{cases} \bar{u}^{*}(\tau; x(t)), & \text{for } \tau \in [t + \delta, t + T_p], \\ K\bar{x}(\tau; x(t+\delta), t+\delta), & \text{for } \tau \in [t+T_p, t+\delta+T_p]. \end{cases} \quad (14)$$

The generated state trajectory on $[t+\delta, t+T_p]$ is the restriction of $\bar{x}^*(\cdot; x(t))$ to $[t+\delta, t+T_p]$, i.e.

$$\bar{x}(\tau; x(t+\delta), t+\delta) = \bar{x}^*(\tau; x(t)),\quad \tau \in [t+\delta, t+T_p]. \tag{15}$$

Due to the invariance of the terminal region, $\bar{x}^*(t+T_p; x(t)) \in \Omega$ implies that the state trajectory on $[t+T_p, t+\delta+T_p]$ stays in Ω. Thus, the violation of the state constraints can be described by

$$\bar{s}(\tau) = \begin{cases} \bar{s}^*(\tau; x(t)), & \text{for } \tau \in [t+\delta, t+T_p], \\ \mathbf{0}, & \text{for } \tau \in [t+T_p, t+\delta+T_p]. \end{cases} \tag{16}$$

The input and state constraints are satisfied in Ω, thus, (14) and (16) constitute a feasible but perhaps not optimal solution to the optimization problem at time $t+\delta$. This result is also true, if we assume that a feasible (not necessarily optimal) solution to the optimization problem at time t is found. By induction, we conclude that, for the perfect nominal system without disturbances, assumption iii) implies the optimization problem is feasible at each time $t > 0$.

Now we show the non-increase of the optimal value function, that will be used to prove stability in the sense of Lyapunov. Due to $\bar{x}(t+T_p; x(t+\delta), t+\delta) = \bar{x}^*(t+T_p; x(t)) \in \Omega$ and the linear control on $[t+T_p, t+\delta+T_p]$, the state trajectory $\bar{x}(\cdot; x(t+\delta), t+\delta)$ on $[t+T_p, t+\delta+T_p]$ stays in Ω and obeys (10). Integrating (10) from $t+T_p$ to $t+\delta+T_p$ yields then the relationship:

$$\| \bar{x}(t+\delta+T_p; x(t+\delta), t+\delta) \|_P^2 - \| \bar{x}^*(t+T_p; x(t)) \|_P^2 \leqslant - \int_{t+T_p}^{t+\delta+T_p} \| \bar{x}(\tau; x(t+\delta), t+\delta) \|_{Q^*}^2 \mathrm{d}\tau. \tag{17}$$

Using (14) ~ (17), the open-loop objective value at time $t+\delta$ can be evaluated by

$$J(x(t+\delta)) \leqslant J^*(x(t)) - \int_t^{t+\delta} (\| \bar{x}^*(\tau; x(t)) \|_Q^2 + \| \bar{u}^*(\tau; x(t)) \|_R^2 + \| \bar{s}^*(\tau; x(t)) \|_W^2) \mathrm{d}\tau. \tag{18}$$

By the principle of optimality, the optimal solution at time $t+\delta$ will be not worse than the chosen one above. Then, (18) becomes

$$J^*(x(t+\delta)) - J^*(x(t)) \leqslant - \int_t^{t+\delta} (\| x(\tau) \|_Q^2 + \| u(\tau) \|_R^2) \mathrm{d}\tau, \tag{19}$$

where (4), (13) and $W \geqslant 0$ are used. By $Q > 0$ and $R > 0$, the inequality (19) implies that the value function $J^*(x(t))$ is not increasing (a monotonicity property). Thus, we define a function $V(x)$ for the closed-loop system (12) as $V(x) := J^*(x)$. From Lemma A.1[11], $V(x)$ has the properties that $V(\mathbf{0}) = 0$, $V(x) > 0$ for $x \neq \mathbf{0}$ and $V(x)$ is continuous at $x = \mathbf{0}$. Hence, we can take the standard argument used for example in Ref. [15] to show that the equilibrium $x = \mathbf{0}$ is stable. Moreover, from the monotonicity property of $V(x)$, we have $V(x(\infty)) \leqslant V(x(0)) - \int_0^\infty \| x(t) \|_Q^2 \mathrm{d}t$. Due to $V(x(\infty)) \geqslant 0$ and the boundedness of $V(x(0))$, the integral $\int_0^\infty \| x(t) \|_Q^2 \mathrm{d}t$ exists and is bounded. Because of the stability of the equilibrium $x = \mathbf{0}$, the compactness of U and the continuous differentiability of f, it is shown[9] that $\| x(t) \|_Q^2$ is uniformly continuous in t on $[0, \infty)$. It follows then from Barbalat's Lemma[15] that $\| x(t) \| \to 0$ as $t \to \infty$. Thus, the equilibrium point $x = \mathbf{0}$ of the system (12) is asymptotically stable. Note that in the above a continuous differentiability assumption on $V(x)$ is not used. Finally, using the same argument as in Ref. [9], we can prove by contradiction that D is invariant for the closed-loop system and hence belongs to the region of attraction. Q.E.D.

Remark 3.1 If the plant model is constrained stabilizable, the prediction horizon T_p can be chosen such that assumption iii) is satisfied.

Remark 3.2 It is clear that (19) is also valid for feasible solutions, as long as the optimization problem is initialized by the shifted feasible solution from the previous step. This means that not the optimality but the feasibility of the optimization problem is required for closed-loop stability.

4 Example

We now consider the undamped system[7] and assume the control u and the state x_1 have to satisfy constraints as follows:

$$\dot{x}_1 = -x_2 + u(0.5 + 0.5x_1), \tag{20a}$$

$$\dot{x}_2 = x_1 + u(0.5 - 2.0x_2), \tag{20b}$$

$$-1.0 \leqslant u \leqslant 1.0,\quad -1.0 \leqslant x_1 \leqslant 1.0. \tag{20c}$$

This is an academic example for demonstrating the proposed nonlinear MPC scheme. Some realistic case studies can be found elsewhere[11].

For the implementation of the proposed quasi-infinite horizon nonlinear MPC scheme, we need parameters Q, R, W, P and Ω. Following the method given in Section 2, we choose the weighting matrices for the control performance as $Q = \begin{pmatrix} 0.2 & 0.0 \\ 0.0 & 1.0 \end{pmatrix}$, $R = 0.5$, $W = 0.5$. Using the LQ technique, we get a feedback gain $K = [0.798 \quad 1.328]$ for the Jacobian linearized system of (20). From $\lambda(A_K) = -0.53 \pm 0.99\,\mathrm{i}$, we choose $\kappa = 0.5 (< -\lambda_{\max}(A_K))$. Then, solving the Lyapunov equation (7) and following Remark 2.3, we obtain a terminal penalty matrix and a terminal region as follows such that inequality (9) and condition C0) is satisfied:

$$P = \begin{pmatrix} 10.5654 & -2.2067 \\ -2.2067 & 28.1390 \end{pmatrix}, \tag{21}$$
$$\Omega = \{x \in \mathbb{R}^2 \mid x^{\mathrm{T}} P x \leqslant 0.65\}.$$

The discrete closed-loop control is defined by (4). Note that the linear state feedback gain K is not directly used to calculate the closed-loop control. The constrained optimization problem in Section 2 is solved with a sampling period of $\delta = 0.2$ time-units. For a prediction horizon of $T_p = 4.4$ time-units, the problem is feasible at time $t = 0$. Time profiles of the constrained nonlinear system controlled by the proposed predictive controller are shown in Fig. 1, for two initial states (solid lines and dash-dotted lines, respectively). The dashed lines represent upper and lower bounds on the state x_1. It can be seen that the input constraint holds hard and the constraint on the state x_1 is only violated at the beginning.

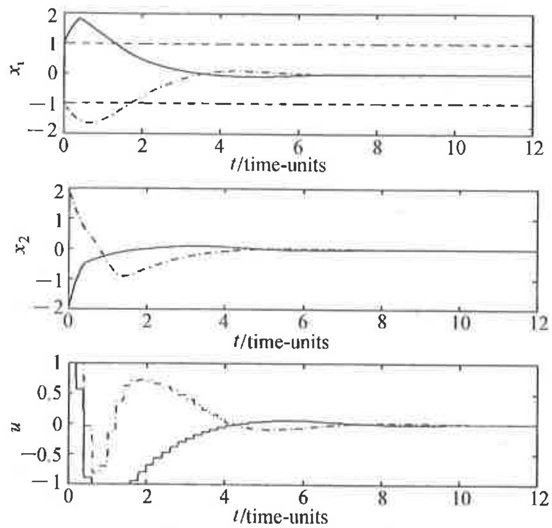

Fig. 1 Time profiles for closed-loop system from two initial states

It is shown that the proposed quasi-infinite horizon nonlinear MPC scheme has computational advantages[9], when compared to other nonlinear MPC approaches. Handling state constraints is however computationally extremely expensive. Table 1 shows the comparison of elapsed CPU time with and without the state constraint (SC), when using the same optimization routine and the same integration algorithm with the same numerical parameters. The total simulation time is 12 time-units. It should be pointed out that the integration step might play an important role in on-line computation time. In order to evaluate the objective functional and the nonlinear constraints, an integrator is needed to solve the nonlinear differential equations over the finite prediction horizon. Clearly, the smaller the integration step is, the more computation time the evaluation needs. For the results in Table 1, a very small integration step (0.001 time-units) was chosen, (for the sake of using a time-continuous model). Roughly, if the integration step doubles, only the half of the elapsed CPU time is needed.

Table 1 Comparison of elapsed CPU time

Initial state		Elapsed CPU time/s	
$x_1(0)$	$x_2(0)$	without SC	with SC
1.0	-2.0	609.32	6483.80
-1.0	2.0	482.46	6326.39
1.0	2.0	459.77	6423.23
-1.0	-2.0	425.68	6342.88

5 Conclusions

For nonlinear systems subject to input and state constraints, we have proposed an MPC scheme with a quasi-infinite prediction horizon but the control profile to be determined on-line is only of finite horizon. In order to avoid infeasibility, the hard state constraints are "softened", i.e., the violation of the hard state constraints is allowed but penalized by an inclusion in the performance objective, with the price being a significant increase in on-line computation time. If the Jacobian linearization of the nonlinear system is stabilizable, a terminal penalty matrix P can be chosen as the unique positive definite solution of an appropriate Lyapunov equation and an invariant terminal region can be computed off-line. The closed-loop system is shown to be asymptotically stable in the sense of Lyapunov, independent of the choice of performance parameters Q and R. The region of attraction is maximal in the sense that the hard state constraints are relaxed and that closed-loop stability requires just the feasibility (not necessarily the optimality) of the posed optimization problem.

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