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Robust Sampled-Data Estimation for Continuous Systems with Noise Intensity Uncertainty *

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Abstract: In this paper, the problem of robust sampled-data estimation for continuous systems with noise intensity uncertainty is discussed by taking account of intersample behavior. The primary purpose of this study is to design discrete filters for this kind of uncertain systems such that the estimation error variance of each state is not greater than a specified value, and therefore the steady-state behavior of the filtering result will be satisfactory. This paper first studies the case that only the intensity of the model noise is uncertain. Then it is shown that the design method in this paper is also suitable for the systems with measurement noise intensity uncertainty. Finally, a numerical example is provided to demonstrate the usefulness and effectiveness of the design method in this paper.

Key words: sampled-data estimation; robust estimation; intersample behavior; noise intensity

噪声强度不确定的连续系统的鲁棒采样估计

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摘要: 基于内采样特性讨论噪声强度不确定的连续系统的鲁棒采样估计问题,主要目的是为这类不确定系统设计离散滤波器,使每个状态的估计误差方差不大于预先指定值,从而获得满意的稳态滤波特性.文中首先研究了仅有模型噪声强度不确定时的情形,随后说明当测量噪声强度不确定时可用同样方法进行研究.最后用数值例子说明了本文设计方法的有用性和有效性.

关键词: 采样估计; 鲁棒估计; 内采样特性; 噪声强度

1 Introduction

Owing to the advances in digital computers, discrete-time estimation and control of continuous systems have been developed and used in numerous applications. In most treatments of sampled-data estimation, the continuous-time plant is in some way discretized, and the estimator for the discretized plant is designed[1,2]. Generally, this treatment describes the behavior of the overall system only at the sampling instants, and the intersample behavior is lost in the process of discretization. Therefore, the study on sampled-data estimation involving the intersample behavior should be placed more emphasis.

Another motivation for this paper is the recent development of error covariance assignment (ECA) theory[3]. In practical applications, the statistical characteristics of measurement noise are easy to obtain, but it is not true for the system model noise. Unfortunately, the ECA theory is not suitable to the constrained variance estimation under noise intensity perturbations, since assignability conditions of error covariance depend directly on the intensity of model noise.

[4] and [5] study the problem of robust constrained variance estimation for continuous and discrete systems with model noise intensity uncertainty respectively. This paper studies the robust sampled-data estimation problem for continuous systems with model noise intensity uncertainty. It is also pointed out that the same approach can be utilized to study the case that the measurement noise intensity is also uncertain.

2 The discretization of the continuous system

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Consider the following continuous stochastic system and the measurement equation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + w(t), \\ y(t) &= Cx(t) + v(t) \end{aligned} \qquad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state, $y(t) \in \mathbb{R}^m$ is the measured output. $w(t)$ and $v(t)$ are uncorrelated zero mean Gaussian white noise processes with respective intensity $W \geqslant 0$ and $V > 0$. It is assumed that the model noise intensity varies in a certain range and its maximum value is $\overline{W}$, i.e., $0 \leqslant W \leqslant \overline{W}$.

Because direct sampling of signals containing white noises is not allowed[6], we utilize an averaging-type A/D device of the form[7] $y(k\tau) \triangleq \frac{1}{\tau}\int_{(k-1)\tau}^{k\tau} y(t)\mathrm{d}t$, where $\tau > 0$ is the sampling period. The present state dependent discrete-time equivalent model of the continuous system (1) is [2]

$$x((k+1)\tau) = A_\tau x(k\tau) + w_\tau(k\tau), \qquad (2)$$

$$y(k\tau) = C_\tau x(k\tau) + v_\tau(k\tau) \qquad (3)$$

where

$$A_\tau \triangleq \mathrm{e}^{A\tau}, \quad C_\tau \triangleq \frac{1}{\tau}C\int_0^\tau \mathrm{e}^{A(\xi-\tau)}\mathrm{d}\xi,$$

$$w_\tau(k\tau) \triangleq \int_0^\tau \mathrm{e}^{A(\tau-\xi)} w(k\tau+\xi)\mathrm{d}\xi,$$

$$v_\tau(k\tau) \triangleq \frac{1}{\tau}\int_0^\tau v((k-1)\tau+\xi)\mathrm{d}\xi - \frac{1}{\tau}C\int_0^\tau\int_\xi^\tau \mathrm{e}^{A(\xi-\eta)} w((k-1)\tau+\eta)\mathrm{d}\eta\mathrm{d}\xi$$

and $w_\tau(k\tau)$ and $v_\tau(k\tau)$ are zero mean white noise sequences and

$$\mathrm{E}\left\{\begin{bmatrix} w_\tau(k\tau) \\ v_\tau(k\tau) \end{bmatrix}\begin{bmatrix} w_\tau^{\mathrm{T}}(k\tau) & v_\tau^{\mathrm{T}}(k\tau) \end{bmatrix}\right\} = \begin{bmatrix} W_\tau & 0 \\ 0 & V_\tau \end{bmatrix} \triangleq W_d$$

where

$$W_\tau \triangleq \int_0^\tau \mathrm{e}^{A\xi} W \mathrm{e}^{A^{\mathrm{T}}\xi}\mathrm{d}\xi,$$

$$V_\tau \triangleq \frac{1}{\tau}V + \frac{1}{\tau^2}\int_0^\tau F(\xi) W F^{\mathrm{T}}(\xi)\mathrm{d}\xi,$$

$$F(\xi) \triangleq C\int_0^\xi \mathrm{e}^{A(\eta-\xi)}\mathrm{d}\eta.$$

By defining $\overline{W}_\tau \triangleq \int_0^\tau \mathrm{e}^{A\xi}\overline{W}\mathrm{e}^{A^{\mathrm{T}}\xi}\mathrm{d}\xi$ and $\overline{V}_\tau \triangleq \frac{1}{\tau}V + \frac{1}{\tau^2}\int_0^\tau F(\xi)\overline{W}F^{\mathrm{T}}(\xi)\mathrm{d}\xi$, it follows from $0 \leqslant W \leqslant \overline{W}$ that $W_\tau \leqslant \overline{W}_\tau$ and $V_\tau \leqslant \overline{V}_\tau$ and hence $W_d \leqslant \overline{W}_d \triangleq \begin{bmatrix} \overline{W}_\tau & 0 \\ 0 & \overline{V}_\tau \end{bmatrix}$.

3 Robust sampled-data estimation

Suppose that the discrete state estimator of the system (1) is of the form

$$\hat{x}[k+1] = G\hat{x}[k] + Ky(k\tau). \qquad (4)$$

By denoting $\varepsilon_d[k] \triangleq x(k\tau) - \hat{x}[k]$, it can be derived from (2), (3) and (4) that

$$\varepsilon_d[k+1] = G\varepsilon_d[k] + (A_\tau - G - KC_\tau)x(k\tau) + w_\tau(k\tau) - Kv_\tau(k\tau). \qquad (5)$$

The augmented system of (2) and (5) is given by

$$x_d[k+1] = A_1 x_d[k] + D_1 w_d[k] \qquad (6)$$

where $w_d[k]$ is a zero mean white noise process with covariance $W_d \geqslant 0$ and

$$x_d[k] \triangleq \begin{bmatrix} x(k\tau) \\ \varepsilon_d[k] \end{bmatrix}, \quad w_d[k] \triangleq \begin{bmatrix} w_\tau(k\tau) \\ v_\tau(k\tau) \end{bmatrix},$$

$$A_1 \triangleq \begin{bmatrix} A_\tau & 0 \\ A_\tau - G - KC_\tau & G \end{bmatrix}, \quad D_1 \triangleq \begin{bmatrix} I & 0 \\ I & -K \end{bmatrix}.$$

It is supposed that $w(t)$ and $v(t)$ have no correlation with the sample time state before the time t, i.e.

$$\mathrm{E}\left\{\begin{bmatrix} x(k_1\tau) \\ \varepsilon_d[k_1] \end{bmatrix}\begin{bmatrix} w^{\mathrm{T}}(k_2\tau+\xi) & v^{\mathrm{T}}(k_2\tau+\xi) \end{bmatrix}\right\} = 0,$$

$k_1 \leqslant k_2, \quad 0 \leqslant \xi < \tau.$

Definition 1 Consider the augmented discrete system (6). The sample time estimation covariance X_d is defined as

$$X_d \triangleq \lim_{k\to\infty} \mathrm{E}\{x_d[k]x_d^{\mathrm{T}}[k]\} = \lim_{k\to\infty}\mathrm{E}\left\{\begin{bmatrix} x(k\tau) \\ \varepsilon_d[k] \end{bmatrix}\begin{bmatrix} x^{\mathrm{T}}(k\tau) & \varepsilon_d^{\mathrm{T}}[k] \end{bmatrix}\right\} \triangleq \begin{bmatrix} X_{d1} & X_{d3} \\ X_{d3}^{\mathrm{T}} & X_{d2} \end{bmatrix}.$$

It is easy to see that the sample time estimation covariance in Definition 1 is in essence the state covariance of the discrete system (6) and only contains the signals at sampling instants. If X_d exists, X_d satisfies the following discrete Lyapunov equation

$$X_d = A_1 X_d A_1^{\mathrm{T}} + D_1 W_d D_1^{\mathrm{T}}. \qquad (7)$$

Define $\varepsilon_s(t) \triangleq x(t) - \hat{x}(t)$, where $\hat{x}(t) = \hat{x}[k]$, $k\tau \leqslant t < (k+1)\tau$. We now give the definition of the sampled-data estimation covariance.

Definition 2 The sampled-data estimation covariance X_s is given by

$$X_s \triangleq \lim_{k\to\infty}\frac{1}{\tau}\int_{k\tau}^{(k+1)\tau}\mathrm{E}\{x_s(t)x_s^{\mathrm{T}}(t)\}\mathrm{d}t,$$

$$x_s(t) \triangleq \begin{bmatrix} x(t) \\ \varepsilon_s(t) \end{bmatrix}.$$

The sampled-data estimation covariance takes complete account of the intersample behavior and is a more accurate criterion of the sampled-data estimation. To this end, the main purpose of this paper can be stated as follows: given the continuous system (1) with model noise intensity uncertainty and the upper bound $\bar{X}_s > 0$ of the sampled-data estimation covariance, seek all the discrete estimators (4) such that the sampled-data estimation covariance $X_s > 0$ satisfies $X_s \leqslant \bar{X}_s$. We call this the Constrained Variance Robust Sampled-Data Estimation (CVRSDE) problem.

4 The conversion of the CVRSDE problem

The following lemma shows the mathematical relation between the sampled-data estimation covariance X_s and the sample time estimation covariance X_d.

Lemma 1 Suppose that the sampled-data estimation covariance $X_s > 0$ exists. Then the covariance X_s can be obtained by the following equation

$$X_s = \frac{1}{\tau}\int_0^\tau [C_s(u)X_dC_s^{\mathrm{T}}(u) + W_s(u)]\mathrm{d}u \quad (8)$$

where $0 \leqslant u < \tau$ and X_d is the sample time estimation covariance and

$$C_s(u) \triangleq \begin{bmatrix} \mathrm{e}^{Au} & 0 \\ \mathrm{e}^{Au} - I & I \end{bmatrix},$$

$$W_s(u) \triangleq \begin{bmatrix} W_1(u) & W_1(u) \\ W_1(u) & W_1(u) \end{bmatrix},$$

$$W_1(u) \triangleq \int_0^u \mathrm{e}^{A\xi}W\mathrm{e}^{A^{\mathrm{T}}\xi}\mathrm{d}\xi.$$

We recognize from Lemma 1 that $C_s(u)$ and $W_s(u)$ do not depend on G and K. Thus, the CVRSDE problem can be solved by checking the following.

Theorem 1 Consider the system (1). For the desired upper bound $\bar{X}_s > 0$ of the sampled-data estimation covariance, $X_s \leqslant \bar{X}_s$ holds if the following two conditions are met.

i) There exists a positive definite matrix $\bar{X}_d > 0$ satisfying

$$\bar{X}_s = \frac{1}{\tau}\int_0^\tau [C_s(u)\bar{X}_dC_s^{\mathrm{T}}(u) + \bar{W}_s(u)]\mathrm{d}u \quad (9)$$

where $\bar{W}_s(u) \triangleq \begin{bmatrix} \bar{W}_1(u) & \bar{W}_1(u) \\ \bar{W}_1(u) & \bar{W}_1(u) \end{bmatrix},$

$$\bar{W}_1(u) \triangleq \int_0^u \mathrm{e}^{A\xi}\bar{W}\mathrm{e}^{A^{\mathrm{T}}\xi}\mathrm{d}\xi.$$

ii) There exists a set of matrices (G, K) satisfying

$$A_1X_dA_1^{\mathrm{T}} - X_d + D_1W_dD_1^{\mathrm{T}} = 0,\ X_d \leqslant \bar{X}_d. \quad (10)$$

Proof We need only show that

$$X_s \leqslant \bar{X}_s \text{ if } X_d \leqslant \bar{X}_d.$$

Subtracting (9) from (8) leads to

$$X_s - \bar{X}_s = \frac{1}{\tau}\int_0^\tau [C_s(u)(X_d - \bar{X}_d)C_s^{\mathrm{T}}(u) + W_s(u) - \bar{W}_s(u)]\mathrm{d}u. \quad (11)$$

Noting that $W \leqslant \bar{W}$, we have $W_1(u) - \bar{W}_1(u) = \int_0^u \mathrm{e}^{A\xi}(W - \bar{W})\mathrm{e}^{A^{\mathrm{T}}\xi}\mathrm{d}\xi \leqslant 0$. Therefore,

$$W_s(u) - \bar{W}_s(u) = \begin{bmatrix} W_1(u) - \bar{W}_1(u) & W_1(u) - \bar{W}_1(u) \\ W_1(u) - \bar{W}_1(u) & W_1(u) - \bar{W}_1(u) \end{bmatrix} \leqslant 0.$$

Since $X_d \leqslant \bar{X}_d$, it can be seen from (11) that

$$X_s - \bar{X}_s = \frac{1}{\tau}\int_0^\tau [C_s(u)(X_d - \bar{X}_d)C_s^{\mathrm{T}}(u) + W_s(u) - \bar{W}_s(u)]\mathrm{d}u \leqslant 0.$$

Consequently, we have $X_s - \bar{X}_s \leqslant 0$ or $X_s \leqslant \bar{X}_s$. This completes the proof.

It is not difficult to test that $\bar{X}_d$ is unique if there exists $\bar{X}_d$ satisfying (9). The proof follows immediately from Theorem 2 in [8].

From Theorem 1, for the given upper bound $\bar{X}_s > 0$, we can transform it to the constraint $\bar{X}_d$ on the sample time estimation covariance from (9) and then determine the set of matrices (G, K) from (10).

By defining $\hat{A} \triangleq \begin{bmatrix} A & 0 \\ A & 0 \end{bmatrix}$ and $\hat{Q} \triangleq \tau\bar{X}_s - \int_0^\tau \bar{W}_s(u)\mathrm{d}u$, (9) is equivalent to $\int_0^\tau \mathrm{e}^{\hat{A}u}\bar{X}_d\mathrm{e}^{\hat{A}^{\mathrm{T}}u}\mathrm{d}u = \hat{Q}$. We can compute $\bar{X}_d$ using the computational algorithm provided in [8]. In what follows, we will study the existence conditions and the analytical expression of the set of matrices (G, K) satisfying (10).

5 The design of desired estimators

Suppose that the positive definite matrix P satisfying

$$P \leqslant \bar{X}_d. \quad (12)$$

Using the matrix P, (7) can be expressed as

$$(X_d - P) - A_1(X_d - P)A_1^{\mathrm{T}} + P - A_1PA_1^{\mathrm{T}} - D_1W_dD_1^{\mathrm{T}} = 0.$$

It follows from Lyapunov stability theory that if A_1 is stable and

$$P - A_1 P A_1^{\mathrm{T}} - D_1 W_d D_1^{\mathrm{T}} > 0 \tag{13}$$

holds, then we have $X_d - P < 0$ and $X_d < P \leqslant \bar{X}_d$. Furthermore, if

$$P - A_1 P A_1^{\mathrm{T}} - D_1 \bar{W}_d D_1^{\mathrm{T}} > 0 \tag{14}$$

holds, (13) is automatically satisfied when $0 \leqslant W \leqslant \bar{W}$. Now, the design task will be accomplished by finding the set of matrices (G, K) satisfying (14) and the stability of A_1.

By defining

$$A_d \triangleq \begin{bmatrix} A_\tau & 0 \\ A_\tau & 0 \end{bmatrix},\ B \triangleq \begin{bmatrix} 0 \\ -I \end{bmatrix},\ H \triangleq [G \quad K],$$

$$M \triangleq \begin{bmatrix} I & -I \\ C_\tau & 0 \end{bmatrix},\ D \triangleq \begin{bmatrix} I & 0 \\ I & 0 \end{bmatrix},\ J \triangleq \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}$$

we have $A_1 = A_d + BHM$ and $D_1 = D + BHJ$. Thus, (14) can be rewritten as

$$P - (A_d + BHM) P (A_d + BHM)^{\mathrm{T}} - (D + BHJ) \bar{W}_d (D + BHJ)^{\mathrm{T}} > 0. \tag{15}$$

By defining $R \triangleq A_d P M^{\mathrm{T}} + D\bar{W}_d J^{\mathrm{T}}$ and $S \triangleq MPM^{\mathrm{T}} + J\bar{W}_d J^{\mathrm{T}}$, (15) is equivalent to

$$P - A_d P A_d^{\mathrm{T}} - D\bar{W}_d D^{\mathrm{T}} + RS^{-1}R^{\mathrm{T}} > [(BH + RS^{-1})T][(BH + RS^{-1})T]^{\mathrm{T}} \tag{16}$$

where T is the square root of $S > 0$, i.e., $S \triangleq TT^{\mathrm{T}}, T \in \mathbb{R}^{(n+m)\times(n+m)}$.

Assume that

$$P - A_d P A_d^{\mathrm{T}} - D\bar{W}_d D^{\mathrm{T}} + RS^{-1}R^{\mathrm{T}} > 0. \tag{17}$$

Then, we can always choose a positive definite matrix Q satisfying

$$\begin{cases} P - Q - A_d P A_d^{\mathrm{T}} - D\bar{W}_d D^{\mathrm{T}} + RS^{-1}R^{\mathrm{T}} \geqslant 0, \\ \operatorname{rank}(P - Q - A_d P A_d^{\mathrm{T}} - D\bar{W}_d D^{\mathrm{T}} + RS^{-1}R^{\mathrm{T}}) \leqslant n + m. \end{cases} \tag{18}$$

It can be seen that if the equation

$$P - Q - A_d P A_d^{\mathrm{T}} - D\bar{W}_d D^{\mathrm{T}} + RS^{-1}R^{\mathrm{T}} = [(BH + RS^{-1})T][(BH + RS^{-1})T]^{\mathrm{T}} \tag{19}$$

holds, we can obtain the inequality (13), (14), (15) and (16). Noting that (19) is equivalent to $P - A_1 P A_1^{\mathrm{T}} - (Q + D_1 \bar{W}_d D_1^{\mathrm{T}}) = 0$ and P is positive definite, we can conclude from Lyapunov stability theory that A_1 is stable. Therefore, if (17) holds, then H to (19) is the desired solution to the CVRSDE problem.

By defining

$$P - Q - A_d P A_d^{\mathrm{T}} - D\bar{W}_d D^{\mathrm{T}} + RS^{-1}R^{\mathrm{T}} = LL^{\mathrm{T}},$$

$$L \in \mathbb{R}^{2n\times(n+m)},$$

(19) can be rearranged as $LL^{\mathrm{T}} = [(BH + RS^{-1})T][(BH + RS^{-1})T]^{\mathrm{T}}$, which is equivalent to[9] $(BH + RS^{-1})T = LU$ or

$$BH = (LU - RT^{-\mathrm{T}})T^{-1} \tag{20}$$

where U is some orthogonal matrix. There exists H satisfying (20), if and only if[10] $(I - BB^+)(LU - RT^{-\mathrm{T}})T^{-1} = 0$, or equivalently $(I - BB^+)LU = (I - BB^+)RT^{-\mathrm{T}}$, which holds if and only if[9]

$$[(I - BB^+)L][(I - BB^+)L]^{\mathrm{T}} = [(I - BB^+)RT^{-\mathrm{T}}][(I - BB^+)RT^{-\mathrm{T}}]^{\mathrm{T}}. \tag{21}$$

By considering the definition of L and T, it follows from (21) that

$$(I - BB^+)(\bar{X}_d - Q - A_d \bar{X}_d A_d^{\mathrm{T}} - D\bar{W}_d D^{\mathrm{T}})(I - BB^+) = 0. \tag{22}$$

If there exists a solution to (20), H can be expressed as[10]

$$H = B^+(LU - RT^{-\mathrm{T}})T^{-1} + (I - B^+B)Z \tag{23}$$

where Z is arbitrary. Since $I - B^+B = 0$, (23) is equivalent to

$$H = [G \quad K] = B^+(LU - RT^{-\mathrm{T}})T^{-1} \tag{24}$$

and the orthogonal matrix U can be expressed as[9]

$$U \triangleq V_1 \begin{bmatrix} I & 0 \\ 0 & U_0 \end{bmatrix} V_2^{\mathrm{T}},\ U_0 U_0^{\mathrm{T}} = I,$$

$$(I - BB^+)L = U_1 \Sigma_1 V_1^{\mathrm{T}},$$

$$(I - BB^+)RT^{-\mathrm{T}} = U_2 \Sigma_2 V_2^{\mathrm{T}}.$$

The main result of this paper is stated in the following theorem.

Theorem 2 For the given upper bound $\bar{X}_s > 0$ of the sampled-data estimation covariance, if there exists $\bar{X}_d > 0$ satisfying (9) and $P > 0$ and $Q > 0$ satisfying (12), (17), (18) and (22), then G and K determined by (24) satisfy the constraint on the sampled-data estimation covariance X_s, i.e., $X_s \leqslant \bar{X}_s$, when the model noise intensity W varies between 0 and $\bar{W}$.

Remark We only consider the case that the model noise intensity is uncertain above. In fact, if the measurement noise intensity is also uncertain or only it is uncertain, by defining $\bar{W}_\tau \triangleq \int_0^\tau e^{A\xi}\bar{W}e^{A^{\mathrm{T}}\xi}d\xi$ and $\bar{V}_\tau \triangleq \frac{1}{\tau}\bar{V} + \frac{1}{\tau^2}\int_0^\tau F(\xi)\bar{W}F^{\mathrm{T}}(\xi)d\xi$, where $\bar{V}$ is the possible maximum value of the measurement noise intensity, we can also obtain that $W_\tau \leqslant \overline{W}_\tau$ and $V_\tau \leqslant \overline{V}_\tau$, and hence $W_d \leqslant \overline{W}_d$ holds. The design task of the CVRSDE problem follows immediately from the design approach provided above.

6 A numerical example

Consider the following continuous system

$$\dot{x}(t) = -1.70275x(t) + w(t),$$
$$y(t) = 0.55x(t) + v(t),$$
$$\overline{W} = 0.8,\ V = 0.18.$$

For the sampling period $\tau = 0.3$, the present state dependent discrete-time equivalent model is given by $x((k+1)\tau) = 0.6x(k\tau) + w_\tau(k\tau)$, $y(k\tau) = 0.71779x(k\tau) + v_\tau(k\tau)$ and

$$\overline{W}_d = \begin{bmatrix} \overline{W}_\tau & 0 \\ 0 & \overline{V}_\tau \end{bmatrix} = \begin{bmatrix} 0.15034 & 0 \\ 0 & 0.63155 \end{bmatrix}.$$

The design task is to seek the sampled-data estimators (4) such that the sampled-data estimation covariance X_s satisfies $X_s \leqslant \overline{X}_s = \begin{bmatrix} 0.23585 & 0.19922 \\ 0.19922 & 0.35501 \end{bmatrix}$ while the model noise intensity W varies between 0 and $\overline{W}$.

It follows from Theorem 1 that $\overline{X}_s = \frac{1}{\tau}\int_0^\tau [C_s(u)\overline{X}_d C_s^{\mathrm{T}}(u) + \overline{W}_s(u)]\mathrm{d}u$, where

$$C_s(u) = \begin{bmatrix} \mathrm{e}^{-1.70275u} & 0 \\ \mathrm{e}^{-1.70275u} - 1 & 1 \end{bmatrix},$$
$$\overline{W}_s(u) = \begin{bmatrix} \overline{W}_1(u) & \overline{W}_1(u) \\ \overline{W}_1(u) & \overline{W}_1(u) \end{bmatrix},$$
$$\overline{W}_1(u) = 0.23491(1 - \mathrm{e}^{-3.40550u}).$$

According to the algorithm provided in [8], we obtain

$$\overline{X}_d = \begin{bmatrix} 0.23641 & 0.18963 \\ 0.18963 & 0.33527 \end{bmatrix}.$$

By using the design procedure in previous sections, we have

$$A_d = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ -1 \end{bmatrix},$$
$$H = [G\ \ K],\ M = \begin{bmatrix} 1 & -1 \\ 1.15416 & 0 \end{bmatrix},$$
$$D = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},\ J = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Subject to the constraint (12) and the conditions (17), (18) and (22), we can choose

$$Q = \begin{bmatrix} 0.005 & 0 \\ 0 & 0.012 \end{bmatrix},\ P = \begin{bmatrix} 0.22710 & 0.19421 \\ 0.19421 & 0.32816 \end{bmatrix}.$$

Consequently, we have

$$L = \begin{bmatrix} 0.11989 & 0 \\ -0.19615 & 0.24486 \end{bmatrix},$$
$$T = \begin{bmatrix} 0.40756 & 0.02720 \\ 0 & 0.86785 \end{bmatrix},$$
$$V_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ V_2^{\mathrm{T}} = \begin{bmatrix} 0.34114 & 0.94001 \\ 0.94001 & -0.34114 \end{bmatrix}.$$

By setting $U_0 = 1$, we obtain the first desired estimator $\hat{x}[k+1] = 0.82930\hat{x}[k] + 0.22008y(k\tau)$. The second desired estimator $\hat{x}[k+1] = -0.30022\hat{x}[k] + 0.44799y(k\tau)$ can be obtained by setting $U_0 = -1$.

7 Conclusion

This paper has extended the results of [4] and [5] to the sampled-data case. The primary purpose is to design the discrete estimator for the continuous system with the noise intensity uncertainty such that the estimation error covariance of each system state is no more than an expected value. The present design methodology is based on Lyapunov stability theory.

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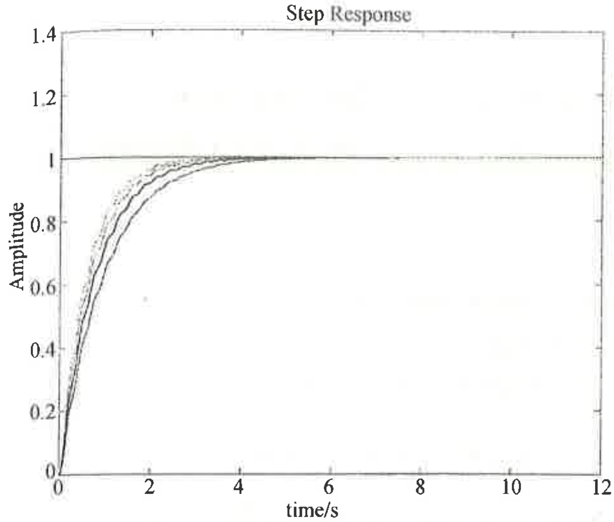

Fig. 2 Step responses of the closed-loop system

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