Time-Varying Adaptive Control of Uncertain Dynamic Nonholonomic Systems *

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Abstract: This paper considers the stabilization problem of the dynamic nonholonomic systems with unknown constant inertia parameters. New periodic time-varying adaptive stabilizing laws are presented for a class of the systems. Unlike the feedback laws in other papers, they are not high-gain ones. The stabilization problem of the general uncertain dynamic nonholonomic system is also addressed, and the existence of the periodic time-varying adaptive stabilizing law is proved. Simulation results of an exapmle show that the approach is effective.

Key words: dynamic nonholonomic systems; uncertain nonlinear systems; adaptive control; global stabilization

不确定非完整动力学系统的时变自适应控制

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摘要: 研究了具有未知惯性参数非完整动力学系统的镇定问题.对于一类非完整系统,给出了一种新的时变自适应律镇定律.不同于其它文中的控制律,该镇定律不是高增益的.文中也讨论了一般不确定非完整动力学系统的镇定问题,证明了时变周期镇定律的存在性.一个简单的例子说明了如何应用文中结果设计镇定律,仿真结果表明了本文所提设计方法的有效性.

关键词: 非完整动力学系统; 不确定非线性系统; 自适应控制; 全局镇定

1 Introduction

In recent years, there has been a growing interest in design of feedback control laws for nonholonomic systems[1]. But the literature on uncertain dynamic nonholonomic system is sparse. In this paper, the point stabilization problem of dynamic nonholonomic control systems with unknown constant parameters is considered. New adaptive stabilizing feedback laws for a class of the uncertain dynamic nonholonomic system are presented. Differing from the results in [2] and [3], our controllers can be proved to be globally stabilizing ones, that is, they can make all positions and velocities of the closed loop system asymptotically converge to zero. In addition, the design parameters in our controllers are only needed to be positive, but not large enough, so they are not high-gain controllers. For general systems, the existence of the adaptive time-varying stabilizing control laws is proved, at the same time the structure of the controller is also presented.

2 Problem statement

Consider the general mechanical system with nonholonomic constraints, expressed in the following form

$$H(x)\ddot{x} + c(x,\dot{x})\dot{x} + G(x) = B(x)\tau + J^{\mathrm{T}}(x)\lambda, \tag{1}$$

$$J(x)\dot{x} = 0 \tag{2}$$

where $x = [x_1, \cdots, x_n]^{\mathrm{T}}$ is generalized coordinates, $H(x)$ is an $n \times n$ positive definite inertia matrix, $C(x, \dot{x})\dot{x}$ presents centripetal and Coriolis torques, $G(x)$ is the gravitational torques, $B(x)$ is an $n \times r$ full rand input transformation matrix, $J(x)$ is an $(n-m) \times n$ full rand matrix, $2 \leqslant m < n, r \geqslant m$. λ is Lagrange multiplier, τ is control input and the superscript T denotes the transpose. The constraints (2) is assumed to be completely nonholonomic. Also, (1) possesses following properties: 1) $\dot{H} - 2C$ is skew-symmetric for a proper definition of C; 2) $H(x)\dot{\xi} + C(x,\dot{x})\xi + G(x) = Y(x,\dot{x},\xi,\dot{\xi})a$, where a is p-vector of inertia parameters, $Y(x,\dot{x},\xi,\dot{\xi})$ is a known matrix of $x, \dot{x}, \xi$, and $\dot{\xi}$.

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Following [4], let the vector fields $g_1(x), \cdots, g_m(x)$ form a basis of the null space of $J(x)$, the system (1) and (2) can be described by the following reduced system

$$\dot{x} = g_1(x)u_1 + \cdots + g_m(x)u_m =: g(x)u, \quad (3)$$

$$H_1(x)\dot{u} + C_1(x,\dot{x})u + G_1(x) = B_1(x)\tau \quad (4)$$

where $H_1(x) = g^{\mathrm{T}}(x)H(x)g(x)$, $C_1(x,\dot{x}) = g^{\mathrm{T}}(x)H(x)\dot{g}(x) + g^{\mathrm{T}}(x)C(x,\dot{x})g(x)$, $G_1(x) = g^{\mathrm{T}}(x)G(x)$, $B_1(x) = g^{\mathrm{T}}(x)B(x)$. Also, (4) shows the following two properties: 1) $H_1(x)$ is positive definite and $\dot{H}_1 - 2C_1$ is skew-symmetric. 2) $H_1(x)\dot{\xi} + C_1(x,\dot{x})\xi + G_1(x) = Y_1(x,\dot{x},\xi,\dot{\xi})a$, where $Y_1(x,\dot{x},\xi,\dot{\xi})$ is a known matrix of x, $\dot{x}$, ξ and $\dot{\xi}$. For many practical systems, $H_1(x) \geqslant \lambda I (\lambda > 0)$ is assumed. For (4) to be fully actuated, $B_1(x)$ is assumed to be an onto map.

Now, the problem addressed is how to design a stabilizing control law for the system (3) and (4) with unknown constant inertia parameters.

3 Special case

In order to obtain a constructively adaptive feedback law for the system (3) and (4), the following assumption must be made.

Assumption 1 rank $\{ad_{g_1}^j g_\kappa(x) \mid j \geqslant 0, 1 \leqslant \kappa \leqslant m\} = n, \quad \forall x \neq 0$

With Assumption 1, the lemma in [5] can be modified as follows.

Lemma 1 Let $\beta(t,x)$ be a time-varying T-periodic function such that

1) $\beta(t+T,x) = \beta(t,x)$, $\beta(t,0) = 0$, $\forall (t,x)$,

2) $\beta(-t,x) = -\beta(t,x)$, $\forall (t,x)$,

3) The solution of the following ordinary differential equation (ODE) exists for all $t \in \mathbb{R}$

$$\dot{x} = \beta(t,x)g_1(x). \quad (5)$$

Let $\phi(t_1,t_2,x)$ denote the solution to (5) with the initial state x at time t_2, i.e.

$$\frac{\partial \phi}{\partial t}(t_1,t_2,x) = \beta(t_1,\phi(t_1,t_2,x))g_1(\phi(t_1,t_2,x));$$

$$\phi(t_2,t_2,x) = x$$

then the function $V(t,x) = \frac{1}{2}\|\phi(0,t,x)\|^2$ is well defined from $\mathbb{R} \times \mathbb{R}^n$ to $[0,+\infty)$ and has following properties:

i) $V(t+T,x) = V(t,x)$.

ii) $V(t,x) = 0 \Longleftrightarrow x = 0$,

$$\frac{\partial V}{\partial x}(t,x) = 0 \Longleftrightarrow x = 0.$$

iii) $$\frac{\partial V}{\partial t}(t,x) + \frac{\partial V}{\partial x}(t,x)\beta(t,x)g_1(x) = 0,$$

$$V(0,x) = \frac{1}{2}\|x\|^2.$$

iv) For any positive real number K, the set $\{x \mid V(t,x) \leqslant K$, for some $t \in \mathbb{R}\}$ is bonuded.

Proof the proof is just like that in [5], so it is omitted here. Q.E.D.

With above lemma, the following theorem can be proved.

Theorem 1 Consider the system (3) and (4), under Assumption 1, if $\beta(t,x)$ satisfies conditions 1 ~ 3 of Lemma 1 and

$$\left.\begin{aligned} &L_{g_i}(x)V(t,x) \triangleq \frac{\partial V(t,x)}{\partial x}g_i(x) = 0, 1 \leqslant i \leqslant m, \\ &\frac{\partial^j}{\partial t^j}\beta(t,x) = 0, \ \forall j \geqslant 1 \end{aligned}\right\}$$

$$\Rightarrow x = 0 \quad (6)$$

then the feedback law

$$\tau = B_1^{\#}[\hat{H}_1\dot{\eta} + \hat{C}_1\eta + \hat{G}_1 - K_p(u-\eta) - (L_gV)^{\mathrm{T}}] \quad (7)$$

and the adaptive law

$$\dot{\hat{\alpha}} = -\Gamma^{-1}Y_1^{\mathrm{T}}(x,\dot{x},\eta,\dot{\eta})(u-\eta) \quad (8)$$

stabilize x and u to the origin, and $\hat{\alpha}$ is bounded, where K_p and Γ are positive constant matrices, $\hat{H}_1$, $\hat{C}_1$ and $\hat{G}_1$ are the corresponding values of H_1, C_1 and G_1 with estimated parameter $\hat{\alpha}$, # is any left inverse, $L_gV = [L_{g_1}V, \cdots, L_{g_m}V]$, and

$$\eta = \begin{bmatrix} \beta(t,x) - L_{g_1}(x)V(t,x) \\ -L_{g_2}(x)V(t,x) \\ \vdots \\ -L_{g_m}(x)V(t,x) \end{bmatrix}. \quad (9)$$

Proof Since functions β, V, and τ are T-periodic with respect to time variable t, the closed loop system (3), (4), (7) and (8) can be considered as a time-invariant system on $S^1 \times \mathbb{R}^{n+m+p}$ (S^1 is a one parameter T-period circle[5]):

$$\begin{cases} \dot{\theta} = 1, \\ \dot{x} = g(x)\eta(\theta,x) + g(x)\tilde{u}(\theta,x), \\ H_1\dot{\tilde{u}} = Y_1(x,\dot{x},\eta(\theta,x),\dot{\eta}(\theta,x))\tilde{a} - \\ \qquad C_1(x)\tilde{u} - K_p\tilde{u} - (L_gV)^{\mathrm{T}}, \\ \dot{\tilde{a}} = -\Gamma^{-1}Y_1^{\mathrm{T}}(x,\dot{x},\eta(\theta,x),\dot{\eta}(\theta,x))\tilde{u} \end{cases} \tag{10}$$

where $\tilde{a} = \hat{a} - a, \tilde{u} = u - \eta$. Let positive function

$$V_1(\theta,x,\tilde{u},\tilde{a}) = V(\theta,x) + \frac{1}{2}(\tilde{a}^{\mathrm{T}}\Gamma\tilde{a} + \tilde{u}^{\mathrm{T}}H_1\tilde{u})$$

the derivative of V_1 with respect to time along (10) is

$$\dot{V}_1 = \frac{\partial V}{\partial\theta}\dot{\theta} + \frac{\partial V}{\partial x}\dot{x} + \tilde{a}^{\mathrm{T}}\Gamma\dot{\tilde{a}} + \tilde{u}^{\mathrm{T}}H_1\dot{\tilde{u}} + \frac{1}{2}\tilde{u}^{\mathrm{T}}\dot{H}_1\tilde{u} = -\sum_{i=1}^{m}(L_{g_i}V)^2 - \tilde{u}^{\mathrm{T}}K_p\tilde{u} \leqslant 0.$$

Therefore V_1 is non-increasing. For any initial time t_0, initial states $x(t_0), \tilde{u}(t_0)$ and $\tilde{a}(t_0)$, the solution of the closed loop system is bounded (noting $H_1(x) \geqslant \lambda I$).

By the LaSalle invariant principle, any bounded solution of the system (10) converges to the largest invariant set included in

$$\mathscr{A} = \{(\theta,x,\tilde{u},\tilde{a}) \mid L_{g_i}V = 0 \quad (i = 1,2,\cdots,m), \tilde{u} = 0\}.$$

Mimicking the argument in [5], the largest invariant set included in $\mathscr{A}$ is $x = 0$ and $\tilde{u} = 0$. From $u = \tilde{u} + \eta$ and $\eta \to 0$, u converges to zero. In summary, the states x and u of the closed system converge to zero asymptotically, and state $\hat{\alpha}$ is bounded. Q.E.D.

Remark 1 A possible choice $\beta(t,x)$ is

$$\beta(t,x) = \frac{\| X \|^2}{(1 + \| x \|^2)(1 + \| g_1(x) \|^2)}\sin t.$$

The above feedback law depends on the expression of V, which in turn involves the flow of the ODE (5). Under the following asumption, the expression of V can be given easily.

Assumption 2 $g_1 = [1,0,\cdots,0]^{\mathrm{T}}$(i.e. $g_1 = \frac{\partial}{\partial x_1}$).

Theorem 2 Consider the system (3) and (4), under Assumption 1 and 2, and $\gamma(t,x)$ is such that

1) $\gamma(t + T, x_2,\cdots,x_n) = \gamma(t,x_2,\cdots,x_n)$, $\gamma(t,0) = 0, \ \forall (t,x)$.

2) $L_{g_i}(x)W(x) = 0, (1 \leqslant i \leqslant m)$ and $\frac{\partial^j\gamma}{\partial t^j}(t,x) = 0, (j \geqslant 1)$ imply $x = 0$, where $W(x) = \frac{1}{2}(k_2x_2^2 + \cdots + k_nx_n^2), k_j > 0(2 \leqslant j \leqslant n)$.

Then the feedback law (7) and the adaptive law (8) stabilize the closed loop system states x and u to zero, and $\hat{a}$ is bounded, where V and β are as follows

$$V(t,x) = \frac{1}{2}[(k_1x_1 + \gamma(t,x_2,\cdots,x_n))^2 + k_2x_2^2 + \cdots + k_nx_n^2],$$

$$\beta(t,x) = -\frac{1}{k_1}\frac{\partial\gamma}{\partial t}(t,x_2,\cdots,x_n).$$

Remark 2 A possible $\gamma(t,x)$ in Theorem 2 is $\gamma(t,x) = (\alpha_2x_2^2 + \cdots + \alpha_nx_n^2)\cos\omega t$, where $\omega > 0$, $\alpha_j > 0(2 \leqslant j \leqslant n)$.

4 General case

This section deals with the stabilization of the system (3) and (4) without Assumption 1. Due to complexity of the general case, only existence of the adaptive law is established, but how to construct the controller is not explicitly given.

For a given time-varying function $u(t,x):\mathbb{R}\times\mathbb{R}^n \to \mathbb{R}^m$, Let $\phi_u:\mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ be defined by

$$\frac{\partial\phi_u}{\partial t}(t,x) = \sum_{k=1}^{m}u_k(t,\phi_u(t,x))g_k(\phi_u(t,x)),$$

$$\phi_u(0,x) = x.$$

The following lemma is obtained in [6].

Lemma 2 For the system (3), there exists $\bar{u}$ satisfying condition 1 in Lemma 1 such that

$$\| \sum_{i=1}^{m}\bar{u}_i(t,x)g_i(x) \| \leqslant 1, \ \forall (t,x) \in \mathbb{R}\times\mathbb{R}^n,$$

$$\phi_{\bar{u}}(x,T) = x, \ \forall x \in \mathbb{R}^n$$

and $\forall x \in \mathbb{R}^n/\{0\}$, with input $w = \{w_1,\cdots,w_m\}$, the linear system

$$\dot{y} = \frac{\partial}{\partial x}[\sum_{i=1}^{m}\bar{u}_i(t,\phi_{\bar{u}}(t,x))g_i(\phi_{\bar{u}}(t,x))]y + \sum_{i=1}^{m}w_ig_i(\phi_{\bar{u}}(t,x))$$

is controllable with impulsive controls at time $t = \frac{3T}{4}$.

Theorem 3 For the system (3) and (4), the feedback law (7) and the adaptive law (8) can stabilize states x and u to the origin, and $\hat{a}$ is bounded. In the feedback law (7), V and $\eta = [\eta_1,\cdots,\eta_m]^{\mathrm{T}}$ are as follows.

$$V(t,x) = \frac{1}{2}\| \phi_{\bar{u}}^{-1}(x,t) \|^2,$$

$$\eta_k = \bar{u}_k(t,x) - L_{g_k}V(t,x), (1 \leqslant k \leqslant m).$$

Proof Let

$$V_1 = V(t,x) + \frac{1}{2}[(\hat{a} - a)^{\mathrm{T}}\Gamma^{-1}(\hat{a} - a) + (u - \eta)^{\mathrm{T}}H_1(u - \eta)]$$

with aid of the proof of Theorem 1 and that in [6], the theorem can be easily proved. Q.E.D.

5 An example

Consider the wheeled mobile robot in [4], the position of the trolley in the plane is characterized by 3 variables x_Q, y_Q and θ. The nonholonomic constraint is $\dot{x}_Q \cos\theta + \dot{y}_Q \sin\theta = 0$. Let $x = [x_1, x_2, x_3]^{\mathrm{T}} = [\theta, x_Q, y_Q]^{\mathrm{T}}$ and $J(x) = [0, \cos x_1, \sin x_1]$, this nonholonomic constraint can be written as (2). Choose

$$g(x) = [g_1, g_2(x)] = \begin{bmatrix} 1 & 0 \\ 0 & -\sin x_1 \\ 0 & \cos x_1 \end{bmatrix},$$

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ -\dot{x}_2 \sin x_1 + \dot{x}_3 \cos x_1 \end{bmatrix}$$

then $\dot{x} = g_1 u_1 + g_2(x) u_2$. Let the inertia parameter vector $a = [m, I_Q]^{\mathrm{T}}$, then

$$Y_1(\dot{\xi}) = \begin{bmatrix} 0 & \dot{\xi}_1 \\ \dot{\xi}_2 & 0 \end{bmatrix}.$$

Choose $\gamma(t, x_2, x_3) = (x_2^2 + x_3^2)\cos t$, $V(t,x) = \frac{1}{2}[(k_1x_1 + \gamma)^2 + k_2x_2^2 + k_3x_3^2]$, by Theorem 2, the feedback law (7) and the adaptive law (8) stabilize $[x_1, x_2, x_3] = [\theta, x_q, x_Q]$ to the origin of the coordinate.

Suppose $a = [2\mathrm{kg}, 2\mathrm{kg}\cdot\mathrm{m}]^{\mathrm{T}}$ and $R = L = 1\mathrm{m}$ for simplicity. In the simulation, choose design parameters $k_1 = 0.4, k_2 = 15, k_3 = 0.7, K_p = \mathrm{diag}(5,5), \Gamma = \mathrm{diag}(0.1, 0.1)$, and initial states $x(0) = [0.5\mathrm{rad}, 1\mathrm{m}, -1\mathrm{m}]^{\mathrm{T}}, u(0) = [-0.68\mathrm{m/s}, 13.5\mathrm{m/s}]^{\mathrm{T}}, \hat{a}(0) = [3\mathrm{kg}, 3\mathrm{kg}\cdot\mathrm{m}]^{\mathrm{T}}$. Responses of x and u are shown in Figs. 1 and 2 respectively.

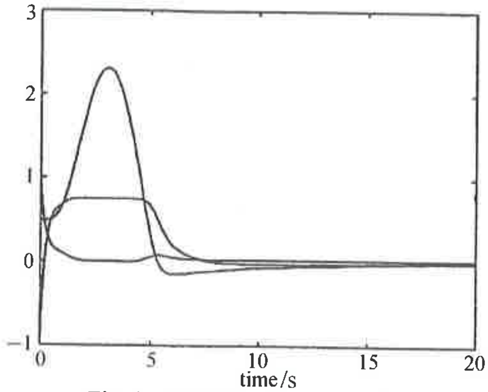

Fig. 1 Response of x_1, x_2 and x_3

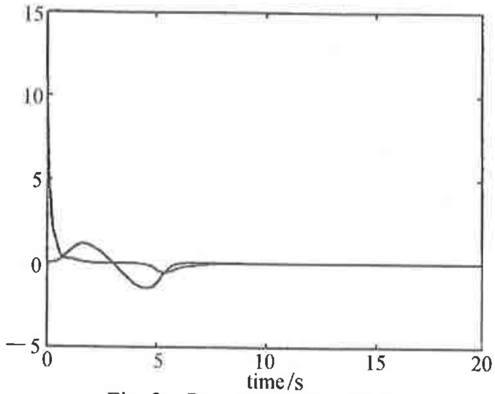

Fig. 2 Response of u_1 and u_2

6 Conclusions

In the paper, the stabilization problem of the dynamic nonholonomic systems with unknown constant inertia parameters is addressed. An explicit adaptive control design method is given for a class of systems. The feedback law and the adaptive law are simple in structure. For the general uncertain systems, the adaptive stabilization problem is also studied. To illustrate the design method, an example is considered. The simulation results show the approach is effective.

References

1 Kolmanovsky I and McClamroch N H. Developments in nonholonomic control problem. IEEE Control Systems, 1995, 15(6): 20 - 36

2 Colbauglo R, Barang E and Glass K. Adaptive control of nonholonomic mechanical system. Proc. IEEE Conf. on Decision and Control, Kobe, Japan, 1996, 1428 - 1434

3 Fierro R and Lewis F L. Practical point stabilization of a nonholonomic mobile robot using neural networks. Proc. IEEE Conf. on Decision and Control, Kobe, Japan, 1996, 1722 - 1727

4 Dong Wenjie and Huo Wei. Adaptive stabilization of dynamic nonholonomic chained systems with uncertainty. Proc. IEEE Conf. on Decision and Control, Hyatt Regency San Diego, USA, 1997, 2362 - 2367

5 Pomet J P. Explicit design of time-varying stabilizing control laws for a class of controllable systems without drift. Systems & Control Letters, 1992, 18(2): 147 - 158

6 Coron J M and Pomet P B. A remark on the design of time-varying stabilizing feedback laws for controllable systems without drift. Proc. NOLCOS, Bordeaux, France, 1992 397 - 401

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