Dominant Pole Placement Control of a Flexible Arm

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Abstract: Control of a flexible arm is an active research topic with many available results. A popular approach is to apply modal analysis to a flexible arm and suppress its first few vibration modes. This research presents a simple method to control a flexible arm by focusing on the dominant poles. The controller is able to deal with as many modes as the feedback sensor system allows. It can be incorporated with an adaptive law to deal with uncertainties hidden in the mode frequencies and damping ratios. Stability of the closed loop system is established in the Lyapunov sense.

Key words: adaptive control; flexible arm control; stability analysis

柔性臂的主极点设置控制

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摘要: 柔性臂控制是一个活跃的研究课题,已经有不少结果.最多用的方法是通过模态分析以控制柔性臂低价振型(一般为首一或二个振型).本文提出一种"主极点"(Dominant Poles)控制器,它可以控制任何数目的低价振型而不失稳定性,该控制器可以方便地转变为自适应控制器以应付数学模型中模态频率与阻尼系数与实际柔性臂之间的误差.本文用李雅普诺夫理论证明了本控制系统的全局稳定性(Global stability).

关键词: 自适应控制; 柔性臂控制; 稳定性分析

1 Introduction

Control of lightweight flexible-link robot arms has been an active research area since 1970[1]. While flexible-link robots are most suitable for space applications, their potential industrial applications are being seriously studied due to their high payload-to-arm ratio, faster executable motions and lower power consumption. As compared to the rigid-link arms, control of a flexible arm is more complex because there are two variables, i.e. the hub angle and the deflection to be controlled. Even worse, infinite number of oscillating modes are arising from the elastic behavior that makes the development of control laws for flexible-link arms far more difficult.

Numerous research works have been reported in the past two decades on the design of control laws for the flexible-link arms. These include linear control laws based on LQG[2,3] and stable factorization[4] techniques, and nonlinear control laws which utilize inverse dynamics[5,6], self-tuning[7], sliding mode[8,9] and singular perturbation[10] techniques. However, most of the control laws mentioned above assumed that the links had to be relatively stiff. With this assumption, the truncation of the infinite dimensional model was deemed to be justifiable, and the resulting reduced order model was then used for controller design purpose[11]. The destabilizing spillover problem resulting from this model truncation was intentionally avoided by using a comb filter[12]. The above approach suffers from the drawback that stability of the entire system can only be guaranteed provided that the discarded portion of the system is stable. In this paper, dominant pole placement control is proposed to suppress the first few vibration modes, and most important of all it can assure overall system stability. An adaptive law is also presented to deal with the uncertainties in the system physical parameters such as mode frequencies and damping ratios and its stability is also established in the Lyapunov sense.

The paper is organized as follows: Section 2 formulates the problem under investigation and proposes a dominant-pole placement control law. Section 3 presents the adaptive version of the control law and its stability analysis. Section 4 provides simulation results for a sin-

Manuscript received Jun. 3, 1998, revised Dec. 10, 1998.

gle link flexible arm, together with a brief conclusion.

2 Problem statement

The presentation of the proposed controller is based on the Euler-Bernoulli model of a single link flexible arm pinned on a hub that rotates an angle of θ in response to the external control toque τ. The deflection of the arm, measured with respect to a base line passing the hub axis, is represented by

$$y(x,t) = w(x,t) + x\theta(t). \tag{1}$$

The model is a fourth-order partial differential equation with four boundary conditions[2]:

$$EI\frac{\partial^4 y}{\partial x^4} + \rho\frac{\partial^2 y}{\partial t^2} = 0, \tag{2}$$

$$EI\frac{\partial^2 y}{\partial x^2}\bigg|_{x=0} + \tau - I_H\ddot{\theta} = 0, w(0) = 0, \tag{3}$$

$$EI\frac{\partial^2 y}{\partial x^2}\bigg|_{x=L} = 0 \text{ and } EI\frac{\partial y^3}{\partial xy^3}\bigg|_{x=L} = 0.$$

Its general solution $y(x,t)$ may be expressed in mode space as

$$y(x,t) = \sum_{i=0}^{\infty}\phi_i(x)q_i(t), \tag{4}$$

where $\{q_i\}_{i=0}^{\infty}$ are the generalized coordinates, $\{\phi_i\}_{i=0}^{\infty}$ are the mode functions[2]. Particularly, $q_0 = \theta$ represents the hub angle and $\phi_0(x) = x$ the rigid-body mode. According to Cannon and Schmitz[2], the generalized coordinates satisfy dynamic equations

$$\begin{aligned} \ddot{q}_0 &= \frac{1}{I_T}\tau, \\ \ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i &= \frac{1}{I_T}\frac{\mathrm{d}\phi_i(0)}{\mathrm{d}x}\tau. \end{aligned} \tag{5}$$

The control objective is to synthesize τ and force $y(L,t) = \sum_{i=0}^{\infty}\phi_i(L)q_i(t) \to y_d = \theta_d L$ as fast as possible where θ_d is the desired hub angle. Introducing

$$Q_{n+1} = [\theta - \theta_d, q_1, \cdots, q_n]^{\mathrm{T}},$$

$$D_{n+1} = \mathrm{diag}(0, 2\xi_1\omega_1, \cdots, 2\xi_n\omega_n),$$

$$\Omega_{n+1} = \mathrm{diag}(0, \omega_1^2, \cdots, \omega_n^2),$$

$$B_{n+1} = \frac{1}{I_T}[1, \frac{\mathrm{d}\phi_1}{\mathrm{d}x}, \cdots, \frac{\mathrm{d}\phi_n}{\mathrm{d}x}]_{x=0}^{\mathrm{T}},$$

$$Q_c = [q_{n+1}, q_{n+2}, \cdots]^{\mathrm{T}},$$

$$D_c = \mathrm{diag}(2\xi_{n+1}\omega_{n+1}, 2\xi_{n+2}\omega_{n+2}, \cdots),$$

$$\Omega_c = \mathrm{diag}(\omega_{n+1}^2, \omega_{n+2}^2, \cdots),$$

and

$$Bc = \frac{1}{I_T}[\frac{\mathrm{d}\phi_{n+1}}{\mathrm{d}x}, \frac{\mathrm{d}\phi_{n+2}}{\mathrm{d}x}, \cdots]_{x=0}^{\mathrm{T}},$$

one can express (5) as

$$\ddot{Q}_{n+1} + D_{n+1}\dot{Q}_{n+1} + \Omega_{n+1}Q_{n+1} = B_{n+1}\tau \tag{6}$$

and

$$\ddot{Q}c + D_c\dot{Q}_c + \Omega_c Q_c = B_c\tau. \tag{7}$$

A flexible arm has infinitely many modes of which only the first n ones are available. Coordinates of the first n modes constitute an approximation space vector Q_{n+1} whereas coordinates of the remaining modes form the complement space vector Q_c.

The design process focuses on the sub-space spanned by Q_{n+1}. It starts with a normalized vector $b = \frac{1}{\| B_{n+1} \|}B_{n+1}$. This is an $(n+1)$-dimensional constant vector. There exist n orthogonal unitlength vectors perpendicular to b. Let $a_1, a_2, \cdots, a_n$ denote these vectors and construct an $(n+1)\times n$ matrix $A = [a_1, a_2, \cdots, a_n]$. One can write

$$A^{\mathrm{T}}b = 0 \quad \text{and} \quad A^{\mathrm{T}}A = I_n, \tag{8}$$

where I_n is an $n \times n$ identity matrix. A projection of Q_{n+1} onto b and A leads to

$$Q_{n+1} = bb^{\mathrm{T}}Q_{n+1} + AA^{\mathrm{T}}Q_{n+1} = b\alpha + A\beta, \tag{9}$$

where

$$\alpha = b^{\mathrm{T}}Q_{n+1} \quad \text{and} \quad \beta = A^{\mathrm{T}}Q_{n+1}. \tag{10}$$

Substituting (9) and (10) respectively, one re-writes (6) as

$$\begin{aligned} \ddot{\alpha} + c_D\dot{\alpha} + c_\Omega\alpha &= c_B\tau - \tau_\beta, \\ \ddot{\beta} + C_D\dot{\beta} + C_\Omega\beta &= -\tau_\alpha, \end{aligned} \tag{11}$$

where $c_D = b^{\mathrm{T}}D_{n+1}b > 0, c_\Omega = b^{\mathrm{T}}\Omega_{n+1}b > 0, C_D = A^{\mathrm{T}}D_{n+1}A > 0, C_\Omega = A^{\mathrm{T}}\Omega_{n+1}A > 0, c_B = \| B_{n+1} \| > 0, \tau_\beta = b^{\mathrm{T}}(D_{n+1}A\dot{\beta} + \Omega_{n+1}A\beta)$ and $\tau_\alpha = A^{\mathrm{T}}(D_{n+1}b\dot{\alpha} + \Omega_{n+1}b\alpha)$. A simple control law is proposed here as

$$\begin{aligned} \tau &= -k_D\dot{\alpha} - k_\Omega\alpha + \frac{1}{c_B}\tau_\beta = \\ &-k_D\dot{\alpha} - k_\Omega\alpha + \frac{1}{c_B}b^{\mathrm{T}}(D_{n+1}\dot{z} + \Omega_{n+1}z), \end{aligned} \tag{12}$$

where

$$\begin{aligned} z &= A\beta = AA^{\mathrm{T}}Q_{n+1} = (I - bb^{\mathrm{T}})Q_{n+1} \\ \text{and} \quad \dot{z} &= (I - bb^{\mathrm{T}})\dot{Q}_{n+1} \end{aligned} \tag{13}$$

can be computed without an explicit search for A. Closed-loop stability is easy to analyze by the following approach. Substituting (12) into (7) and (11), the closed-loop system can be decoupled to

$$\ddot{\alpha} + (c_D + c_B k_D)\dot{\alpha} + (c_\Omega + c_B k_\Omega)\alpha = 0, \qquad (14)$$

$$\ddot{\beta} + C_D\dot{\beta} + C_\Omega\beta = -\tau_\alpha, \qquad (15)$$

$$\ddot{Q}_c + D_c\dot{Q}_c + \Omega_c Q_c = B_c\left(\frac{1}{C_B}\tau_\beta - k_D\dot{\alpha} - k_\Omega\alpha\right). \qquad (16)$$

Fig. 1 illustrates the cascade relation of (14) ~ (16). It starts with (14), a homogeneous equation. This sub-system has a zero input response due to the initial offset $\theta - \theta_d$ in the first element of Q_{n+1}. The output of (14) is asymptotically stable by a proper choice of k_D and k_Ω. The fact that $c_D = b^T D_{n+1} b > 0, c_\Omega = b^T\Omega_{n+1} b > 0$, $C_D = A^T D_{n+1} A > 0$ and $C_\Omega = A^T\Omega_{n+1}A > 0$ is very important. The convergence of α to zero ensures the convergence of β and Q_c to zero and the convergence of y to y_d. The control law (12) only places two poles of (14). Since the poles of (15) and (16) are due to the high frequency vibration modes, it is recommended that the poles of (14) be placed as the dominant poles of the entire system, hence the name of the controller.

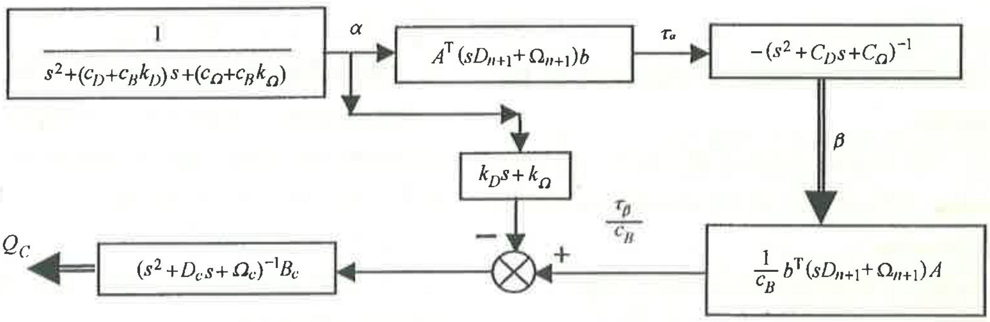

Fig. 1 A cascade of three subsystems

3 Adaptive control

Control law (12) requires exact knowledge of the flexible arm model in terms of $b, c_D, c_B, c_\Omega, D_{n+1}$ and Ω_{n+1}. While b and c_B can be computed using analytical values, c_D, c_Ω, D_{n+1} and Ω_{n+1} have to be measured via experiments. A good controller should be able to tolerate uncertainties hidden in c_D, c_Ω, D_{n+1} and Ω_{n+1} without degrading its performance. This is the focus of the current section. Without the exact knowledge of c_D, c_Ω, D_{n+1} and Ω_{n+1}, the control law has to substitute estimated values of k_D, k_Ω, D_{n+1} and Ω_{n+1} into (12) to get

$$\tau = -\hat{k}_D\dot{\alpha} - \hat{k}_\Omega\alpha + \frac{1}{c_B}\hat{\tau}_\beta = -\hat{k}_D\dot{\alpha} - \hat{k}_\Omega\alpha + \frac{1}{c_B}b^T(\hat{D}_{n+1}\dot{z} + \hat{\Omega}_{n+1}z), \qquad (17)$$

where $\hat{k}_D, \hat{k}_\Omega, \hat{D}_{n+1}$ and $\hat{\Omega}_{n+1}$ represent, respectively, the estimated values of k_D, k_Ω, D_{n+1} and Ω_{n+1}. Let the $\Delta k_D = \hat{k}_D - k_D, \Delta k_\Omega = \hat{k}_\Omega - k_\Omega, \Delta D = \hat{D}_{n+1} - D_{n+1}$ and $\Delta\Omega = \hat{\Omega}_{n+1} - \Omega_{n+1}$ denote parameter errors. One may express (17) as

$$\tau = -k_D\dot{\alpha} - k_\Omega\alpha + \frac{1}{c_B}\tau_\beta - \Delta k_D\dot{\alpha} - \Delta k_\Omega\alpha + \frac{1}{c_B}b^T(\Delta D\dot{z} + \Delta\Omega z). \qquad (18)$$

Since ΔD and $\Delta\Omega$ are diagonal matrices, quadratic forms $b^T\Delta D\dot{z}$ and $b^T\Delta\Omega z$ have equivalent expressions

$$b^T\Delta D\dot{z} = [b\dot{z}]^T\Delta d \text{ and } b^T\Delta\Omega z = [bz]^T\Delta\omega \qquad (19)$$

where $[bz]$ is a column vector whose ith component is the product of the ith components of b and z. The same construction applies to $[b\dot{z}]$. The diagonal elements of mitrices ΔD and $\Delta\Omega$ are represented by vectors Δd and $\Delta\omega$ respectively. As such, (18) becomes

$$\tau = -k_D\dot{\alpha} - k_\Omega\alpha + \frac{1}{c_B}\tau_\beta - \Delta k_D\dot{\alpha} - \Delta k_\Omega\alpha + \frac{1}{c_B}([b\dot{z}]^T\Delta d + [bz]^T\Delta\tilde{\omega}), \qquad (20)$$

which contains $2(n+2)$ unknown parameters. Substituting (20) into (7) and (11), the entire model of the flexible arm, subject to the uncertainties, can be obtained as

$$\ddot{\alpha} + c_D^*\dot{\alpha} + c_\Omega^*\alpha = -c_B(\Delta k_D\dot{\alpha} + \Delta k_\Omega\alpha) + [b\dot{z}]^T\Delta d + [bz]^T\Delta\tilde{\omega}, \qquad (21)$$

$$\ddot{\beta} + C_D\dot{\beta} + C_\Omega\beta = -\tau_\alpha, \qquad (22)$$

$$\ddot{Q}_c + D_c\dot{Q}_c + \Omega_c Q_c = B_c\left(\frac{1}{C_B}\hat{\tau}_\beta - \hat{k}_D\dot{\alpha} - \hat{k}_\Omega\alpha\right), \qquad (23)$$

where $c_D^* = c_D + c_B k_D$ and $c_\Omega^* = c_\Omega + c_B k_\Omega$ are desired coefficients of the characteristic equation that specify the dominant poles. The above equations are similar to the cascade system (14), (15) and (16). Only this time, there exists coupling between (21) and (22), due to uncertainties hidden in Δk_D, Δk_Ω, Δ_d and $\Delta\omega$. An adaptive law is needed to deal with these uncertainties. It is given by

$$\dot{\hat{k}}_D = \dot{\alpha} s, \quad \dot{\hat{k}}_\Omega = \alpha s, \qquad \dot{\hat{d}} = -s[b\dot{z}], \quad \dot{\bar{\omega}} = -s[bz], \tag{24}$$

where $\hat{d}$ and $\bar{\omega}$ are two vectors whose elements correspond to the diagonal elements of $\hat{D}_{n+1}$ and $\hat{\Omega}_{n+1}$ respectively; these adaptation processes involve an intermediate variable $s = \dot{\alpha} + k\alpha$ with a constant gain $0 < k < c_D^*$ that ensures

$$\eta = c_D^* - k > 0. \tag{25}$$

Stability of the closed-loop system is established by the following Lemmas.

Lemma 1 The cascade system (21), (22) and (23) is globally stable. All variables of the closed-loop system are uniformly bounded.

Proof Consider a Lyapunov function candidate

$$L = \frac{1}{2}[s^2 + (\eta k + c_\Omega^*)\alpha^2 + c_B(\Delta k_D)^2 + c_B(\Delta k_\Omega)^2 + \Delta d^{\mathrm{T}}\Delta d + \Delta\bar{\omega}^{\mathrm{T}}\Delta\bar{\omega}], \tag{26}$$

and evaluate its time derivative along the trajectory of (21). One obtains

$$\dot{L} = -\eta\dot{\alpha}^2 - c_\Omega^* k\alpha^2 + c_B(\Delta k_D\dot{\hat{k}}_D - \Delta k_D s\dot{\alpha} + \Delta k_\Omega\dot{\hat{k}}_\Omega - \Delta k_\Omega s\alpha) + \dot{\hat{d}}^{\mathrm{T}}\Delta d + s[b\dot{z}]^{\mathrm{T}}\Delta d + \dot{\bar{\omega}}^{\mathrm{T}}\Delta\bar{\omega} + s[bz]^{\mathrm{T}}\Delta\bar{\omega} \tag{27}$$

which is actually

$$\dot{L} = -\eta\dot{\alpha}^2 - c_\Omega^* k\alpha^2 \leqslant 0 \tag{28}$$

upon substitution of adaptive law (24). Since L is decreasing monotonically, it is uniformly bounded. Adaptive errors Δk_D, Δk_Ω, Δd and $\Delta\bar{\omega}$ as well as tracking errors $\dot{\alpha}$ and α are all uniformly bounded. It then follows that $\tau_\alpha = A^{\mathrm{T}}(D_{n+1}b\dot{\alpha} + \Omega_{n+1}b\alpha)$ is also uniformly bounded. Uniform boundedness of τ_β follows that of τ_α, since it is the output of (22) when excited by τ_α. The two signals combine to excite (23) and cause a uniformly bounded Q_c. Q.E.D.

Lemma 2 For the cascade system (21), (22) and (23), variables α, β and Q_c will converge to zero.

Proof The first step is to prove $\dot{\alpha} \to 0$ and $\alpha \to 0$ using the Barbalat's Lemma[13]. It requires a uniformly continuous $\dot{L}$, or a uniformly bounded $\ddot{L}$. From (28) and (21), one obtains

$$\ddot{L} = -2\eta\dot{\alpha}\ddot{\alpha} - 2c_\Omega^* k\alpha\dot{\alpha} = -2\dot{\alpha}\{c_\Omega^* k\alpha + \eta([b\dot{z}]^{\mathrm{T}}\Delta d + [bz]^{\mathrm{T}}\Delta\bar{\omega} - c_B\Delta k_D\dot{\alpha} - c_B\Delta k_\Omega\alpha - c_D^*\dot{\alpha} - c_\Omega^*\alpha)\},$$

whose right side is a nonlinear combination of variables that have been proven uniformly bounded by Lemma 1. Therefore $\ddot{L}$ is uniformly bounded and $\dot{L}$ uniformly continuous. The convergence of $\dot{L} \to 0$ is implied by (28) plus a uniformly continuous $\dot{L}$. As the result, $\dot{\alpha}$, α and τ_α will converge to zero. The output of subsystems (22) and (23), namely $\dot{\beta}$, β, τ_β and Q_c, will all converge to zero due to the cascade effect. Q.E.D.

4 Simulation and conclusion

The proposed control law is simulated in Matlab with a flexible arm model. The dimension of the flexible arm is $L \times W \times H = 1 \times 0.00635 \times 0.0381\ \mathrm{m}^3$ with Young's Modulas $E = 7 \times 10^{10}\ \mathrm{N/m}^2$ and density $\rho = 0.653225$ kg/m evaluated along its length. Mode functions and frequencies are obtained by standard analytical calculations. The damping ratio is assumed to be 0.06 for all modes.

The simulated flexible arm has five modes. The controller is synthesized by (12) and (13) with $n = 2,3,4$ and 5 respectively. Analytically, the closed-loop is equivalent to a cascade model (14) ~ (16). Practically, the control law (12) and (13) is independent of (14) ~ (16). The simulation synthesizes a torque to drive the five-mode model (5) instead of (14) ~ (16). With $n < 5$, the simulation tests the effects of the controller on higher order modes which are ignored by (12) and (13) yet included in the simulated arm. With $n = 5$, the simulation examines the best response of the closed-loop system when the controller makes use of all available model information.

Fig. 2 plots unit step responses of $y(L,t) = \sum_{i=0}^{5}\phi_i(L)q_i(t)$ subject to the proposed control law. It is observed that the rising time reduces as the number of modes used by the controller (12) and (13) increases. The controller has no negative effects on those modes that exist in the closed-loop system yet ignored by (12) and (13). In all cases, $y(L,t) = \sum_{i=0}^{5}\phi_i(L)q_i(t) \to y_d = 1$ within a short period of time.

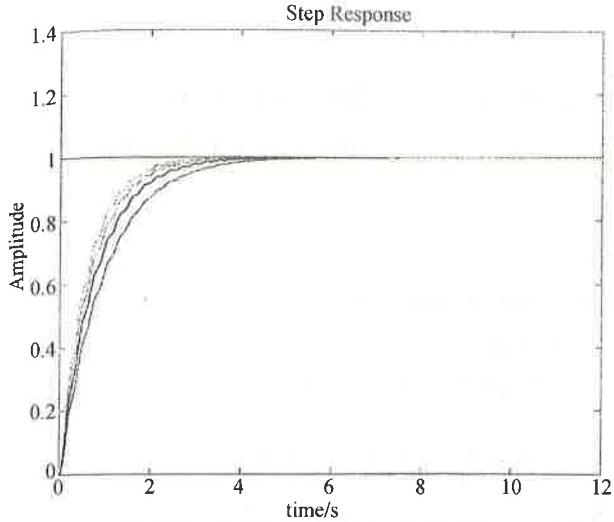

Fig. 2 Step responses of the closed-loop system

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