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H_∞ Controller Design Based on Chain Scattering Description *

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Abstract: This paper concentrates on computational method, Q-parameter utilization problem, stable controller design and reduced order design in H_∞ controller based on chain scattering description of transfer function and (J, J')-loseless factorization.

Key words: chain scattering description; (J, J')-loseless factorization; H_∞ control

基于链式散射描述的 H_∞ 控制器设计

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摘要: 基于链式散射描述对 H_∞ 的标准问题,揭示了 Q 参数和控制器之间的关系. 利用 Q 参数得到稳定的控制器,并利用 Q 参数进行控制器降价. 还给出了有关例子.

关键词: 链式散射描述; (J, J')-无损因子分解; H_∞ 控制

1 Introduction

It was pointed in [1] that we could solve H_∞ control problem based on chain scattering description of transfer function. The same idea was also presented in [2,3]. Youla parameterization and model match problem are not necessary to solve H_∞ control problem. It gives a simple and unified framework for H_∞ control problem. Futhermore, as shown in this paper, the special structure of controller parameterization makes it possible to use free parameter $Q(s)$ to satisfy more specifications, such as to design stable controller and design reduced order controller.

Usually, closed loop transfer function $\Phi(s)$ is represented as Linear Fractional Transformations (LFT). If $\Phi(s)$ is represented as:

$$\Phi(s) = \mathrm{CSD}(S(s), K(s)) =: (S_{11}(s)K(s) + S_{12}(s))(S_{21}(s)K(s) + S_{22}(s))^{-1}, \tag{1}$$

we say equation (1) is Chain Scattering Description (CSD) of $\Phi(s)$, where

$$S(s) = \begin{bmatrix} S_{11}(s) & S_{12}(s) \\ S_{21}(s) & S_{22}(s) \end{bmatrix} \in \mathbb{R}^{(m+r)\times(p+q)},$$

$$K(s) \in \mathbb{R}^{p\times q}, \quad r = q.$$

We can obtain directly CSD from LFT for 1-block and 2-block problem[1]. It can be shown that if and only if we can get (J, J')-loseless factorization of $S(s)$ then we can get solutions to standard H_∞ control problem: Find $K(s)$ stabilizing closed loop system and $\|\Phi(s)\|_\infty < 1$[1]. For general 4-block problem, we need to introduce some auxiliary signals to get CSD, see [3,4] for detail. In this paper we focus our attention on computational issues. For simplicity we consider only 1-block and 2-block problem. But it should be noted that our method can easily be extended to 4-block problem.

Computation of (J, J')-loseless factorization in the state space was discussed in [2~6] based on Algebraic Riccati Equation. In all these methods it was assumed that there exists an nonsingular matrix $D_\pi \in \mathbb{R}^{(p+q)\times(p+q)}$ such that

$$D_\pi^{\mathrm{T}} J_{p,q} D_\pi = D^{\mathrm{T}} J_{m,r} D, \tag{2}$$

where $J_{m,r} = \mathrm{diag}\{I_m, -I_r\}$, $J_{p,q} = \mathrm{diag}\{I_p, -I_q\}$, $m \geqslant p \geqslant 0$, $r \geqslant q \geqslant 0$, $D \in \mathbb{R}^{(m+r)\times(p+q)}$. D_π is used to give solution of (J, J')-loseless factorization. In [2~4] a special D_π is solved for a particular D. In [7] a simple numerical algorithm is proposed to compute the solution of equation (2). In Section 2 we give a numerical algorithm to compute (J, J')-loseless factorization in $\mathbb{RL}_\infty$.

Next in Section 3 new state space parameterization of

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controller is obtained. We discuss the relation between the order of controller and parameter $Q(s)$.

In Section 4 we discuss how to get stable controller, whose order does not increase at the same time, by constant matrix parameter $D^q \in \mathbb{R}^{p\times q}, \bar{\sigma}(D_q) < 1$.

In Section 5 we propose a method to get reduced order controller by parameter $Q(s)$.

Finally we have implemented our algorithms in Matlab environment. Design examples of ordinary controller, stable controller and reduced order controller are given to demonstrate our method.

Notations: $\mathbb{R}\mathrm{H}_\infty$: set of stable rational proper matrices, $\mathbb{R}\mathrm{L}_\infty$: set of rational proper matrices without pole on the $j\omega$- axis, $\mathbb{R}^{p\times q}$: constant real $p \times q$ matrix, deg $(G(s))$: McMillan degree of $G(s)$, $\lambda(A)$: eigenvalue of A, $\rho(A)$: maximum eigenvalue of A, $\bar{\sigma}(D)$: maximum singular value of D, I_m: identical matrix in $\mathbb{R}^{m\times m}$, $X = \mathrm{Ric}(H)$: solution of algebraic Riccati Equation: $XA + A^{\mathrm{T}}X - XPX + Q = 0$ such that $A - PX$ is stable, where $H =: \begin{bmatrix} A & -P \\ -Q & -A^{\mathrm{T}} \end{bmatrix}$.

2 Computing (J, J')- loseless factorization in $\mathbb{R}\mathrm{L}_\infty$

From [7] and Theorem 2 in [6], we could give an algorithm to compute (J, J')- loseless factorization in $\mathbb{R}\mathrm{L}_\infty$.

Algorithm Given state space realization of partitioned matrix $S(s)$ (equation (1)) belongs to $\mathbb{R}\mathrm{L}_\infty$:

$$S(s) = C(sI - A)^{-1}B + D =: \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right], \quad (3)$$

where $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times (p+q)}$, $C \in \mathbb{R}^{(m+r)\times n}, D \in \mathbb{R}^{(m+r)\times(p+q)}$.

Step 1 Using the algorithm in [7] to solve equation (2) to get D_π. If D_π does not exist, $S(s)$ has no such factorization, return.

Step 2 Solving two algebraic Riccati equation: $X = \mathrm{Ric}(Hx)$, $Y = \mathrm{Ric}(Hy)$, where

$$Hx =: \begin{bmatrix} A & 0 \\ -C^{\mathrm{T}}J_{mr}C & -A^{\mathrm{T}} \end{bmatrix} - \begin{bmatrix} B \\ -C^{\mathrm{T}}J_{mr}D \end{bmatrix} \cdot (D^{\mathrm{T}}J_{mr}D)^{-1}[D^{\mathrm{T}}J_{mr}D \quad B^{\mathrm{T}}],$$

$$Hy =: \begin{bmatrix} A^{\mathrm{T}} & C^{\mathrm{T}}J_{mr}C \\ 0 & -A \end{bmatrix}.$$

If $X \geqslant 0, Y \geqslant 0$ and $\rho(XY) < 1$ hold, continue; else, $S(s)$ has no such factorization, return.

Step 3 We get state space realization of (J, J')-loseless factors: $\Theta(s)$ is (J, J')-loseless, $\Pi(s)$ and $\Pi(s)^{-1}$ is stable.

$$S(s) = \Theta(s)\Pi(s)^{-1} \quad (4)$$

where

$$\Theta(s) = \left[\begin{array}{cc|c} A + BF & 0 & B_1 \\ 0 & A & B_2 \\ \hline C + DF & C & D \end{array}\right] D_\pi^{-1},$$

$$\Pi(s) = \left[\begin{array}{c|c} A + YC^{\mathrm{T}}J_{mr}C + (B + YC^{\mathrm{T}}J_{mr}D)F(I - YX)^{-1} & -(B + YC^{\mathrm{T}}J_{mr}D) \\ \hline -F(I - YX)^{-1} & I \end{array}\right] D_\pi^{-1},$$

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} (I - YX)^{-1}(B + YC^{\mathrm{T}}J_{mr}D) \\ (I - YX)^{-1}Y(XB + C^{\mathrm{T}}J_{mr}D) \end{bmatrix}, \quad F = -(D^{\mathrm{T}}J_{mr}D)^{-1}(B^{\mathrm{T}}X + D^{\mathrm{T}}J_{mr}C).$$

Step 4 Find the minimal state space realization of $\Theta(s)$ and $\Pi(s)$.

Remark Our formulae for $S(s)$, $\Pi(s)$, which are natural and suitable for computation, are different from [6].

3 Computation of H_∞ controller

3.1 Algorithm to compute H_∞ controller

We shall outline the steps to compute H_∞ controller based on CSD of closed loop transfer function and (J, J') -loseless factorization.

Step 1 Computing the CSD of closed loop transfer function. we can get CSD directly from robust control problem (such as mixed sensitivity problem) or convert from ordinary state space realization of generalized plant[1,2]. Supposing we get state space realization (3).

Step 2 Computing (J, J') -loseless factorization of generalized plant $S(s)$ using the algorithm in section 2.

Step 3 Computing H_∞ controller $K(s)$. Supposing we get (J, J') loseless factorization (4), then all the H_∞ controllers are given by

$$K(s) = \mathrm{CSD}(\Pi(s), Q(s)) \tag{5}$$

where $Q(s) \in \mathbb{RH}^{p\times q}$ and $\| Q(s) \|_\infty \leqslant 1$. To efficiently compute $K(s)$, we shall give its state space realization.

3.2 State space parameterization of controller

Theorem 1 (State space parameterization of $K(s)$) assuming $\| Q(s) \|_\infty \leqslant 1$, let

$$\Pi(s) = \begin{bmatrix} \Pi_{11}(s) & \Pi_{12}(s) \\ \Pi_{21}(s) & \Pi_{22}(s) \end{bmatrix} =: \left[\begin{array}{cc|c} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right],$$

$$Q(s) =: \left[\begin{array}{c|c} A_q & B_q \\ \hline C_q & D_q \end{array}\right],$$

$$K(s) = \mathrm{CSD}(\Pi(s), Q(s)) =: \left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array}\right],$$

then

$$A_k = \begin{bmatrix} A & B_1 C_q \\ 0 & A_q \end{bmatrix} - \begin{bmatrix} B_1 D_q + B_2 \\ B_q \end{bmatrix} \cdot (D_{21} D_q + D_{22})^{-1} [C_2 \quad D_{21} C_q],$$

$$B_k = -\begin{bmatrix} B_1 D_q + B_2 \\ B_q \end{bmatrix} (D_{21} D_q + D_{22})^{-1},$$

$$C_k = [-C_1 \quad -D_{11} C_q] + (D_{11} D_q + D_{12})(D_{21} D_q + D_{22})^{-1} [C_2 \quad D_{21} C_q],$$

$$D_k = (D_{11} D_q + D_{12})(D_{21} D_q + D_{22})^{-1}.$$

Specially, if $A_q = B_q = C_q = 0$, we have

$$A_k = A - (B_1 D_q + B_2)(D_{21} D_q + D_{22})^{-1} C_2,$$

$$B_k = -(B_1 D_q + B_2)(D_{21} D_q + D_{22})^{-1},$$

$$C_k = -C_1 + (D_{11} D_q + D_{12})(D_{21} D_q + D_{22})^{-1} C_2,$$

$$D_k = (D_{11} D_q + D_{12})(D_{21} D_q + D_{22})^{-1}.$$

To simplify notation, A is substituted for $A + YC^{\mathrm{T}} J_{mr} C + (B + YC^{\mathrm{T}} J_{mr} D) F (I - YX)^{-1}$ in Section 3.

Proof We verify the theorem by direct computation. First, we get the following identical relation (Here A, B, C_1, C_2, D_1, D_2 are arbitrary matrices of compatible dimensions):

$$\left[\begin{array}{c|c} A & B \\ \hline C_1 & D_1 \end{array}\right] \left[\begin{array}{c|c} A & B \\ \hline C_2 & D_2 \end{array}\right]^{-1} = \left[\begin{array}{c|c} A - BD_2^{-1} C_2 & -BD_2^{-1} \\ \hline -C_1 + D_1 D_2^{-1} C_2 & D_1 D_2^{-1} \end{array}\right]. \tag{6}$$

Through direct computation we have:

$$\Pi_{11}(s) Q(s) + \Pi_{12}(s) = \left[\begin{array}{c|c} A & B_1 \\ \hline C_1 & D_{11} \end{array}\right] \left[\begin{array}{c|c} A_q & B_q \\ \hline C_q & D_q \end{array}\right] + \left[\begin{array}{c|c} A & B_2 \\ \hline C_1 & D_{12} \end{array}\right] = \left[\begin{array}{cc|c} A & B_1 C_q & B_1 D_q + B_2 \\ 0 & A_q & B_q \\ \hline C_1 & D_{11} C_q & D_{11} D_q + D_{12} \end{array}\right],$$

$$\Pi_{21}(s) Q(s) + \Pi_{22}(s) = \left[\begin{array}{c|c} A & B_1 \\ \hline C_2 & D_{21} \end{array}\right] \left[\begin{array}{c|c} A_q & B_q \\ \hline C_q & D_q \end{array}\right] + \left[\begin{array}{c|c} A & B_2 \\ \hline C_2 & D_{22} \end{array}\right] = \left[\begin{array}{cc|c} A & B_1 C_q & B_1 D_q + B_2 \\ 0 & A_q & B_q \\ \hline C_2 & D_{21} C_q & D_{21} D_q + D_{22} \end{array}\right].$$

On the other hand, $K(s) = \mathrm{CSD}(\Pi(s), Q(s)) = (\Pi_{11}(s) Q(s) + \Pi_{12}(s))(\Pi_{21}(s) Q(s) + \Pi_{22}(s))^{-1}$. From above equations and (6) we complete the proof of first part of theorem 1. The rest follows by substituting $A_q = B_q = C_q = 0$ and eliminating uncontrollable modes.

3.3 The order of controller and $Q(s)$

In Section 2, we obtain the state space realization of $\Theta(s)$ and $\Pi(s)$. Given chain scattering matrix $S(s)$, $\deg(\Pi(s)) \leqslant \deg(S(s))$. (Generally realizations of $\Theta(s)$ and $\Pi(s)$ in Section 2 are not necessary minimal).

From state space parameterization of controller in Section 3.2, we have:

$$\deg(K(s)) \leqslant \deg(\Pi(s)) + \deg(Q(s)) \leqslant \deg(S(s)) + \deg(Q(s)). \tag{7}$$

If $\deg(Q(s)) = 0$, we have: $\deg(K(s)) \leqslant \deg(S(s))$, i.e., the order of controller is no larger than the order of generalized plant. Sometimes when we carry out minimal realization of $\Pi(s)$, we can get: $\deg(\Pi(s)) < \deg(S(s))$, choose $Q(s) = D_q, \bar{\sigma}(D_q) < 1$, thus we can get controller $K(s)$: $\deg(K(s)) < \deg(S(s))$, i.e., the order of controller is lower than that of generalized plant. In fact we obtain such an example, see Appendix.

It is clear from Theorem 1 that when we set $Q(s) = D_q$ the order of controller does not increase, where D_q is an constant matrix, $\bar{\sigma}(D_q) < 1$. Maybe we could use such parameterization to satisfy other specifications such as reduced order controller, stable controller, etc..

If $\deg(Q(s)) > 0$, generally the order of controller will increase.

Remark Because the order of minimal realization of $\Pi(s)$ may be less than that of generalized plant, sometimes the order of our controller is lower than that of DGKF's[8]. It is interesting that even then our method is different from that of DGKF, we find that the poles and zeros of resulted controller are consistent ($Q(s) = 0$) in our numerical design examples.

4 Design stable controller by $Q(s)$

If we set parameter $Q(s) = 0$, sometimes we get unstable controller (both in our method and DGKF method). In fact, we obtain such an example (see Appendix).

To obtain stable and lower order controller, we may use constant parameter

$$Q(s) = D_q, \bar{\sigma}(D_q) < 1, D_q \in \mathbb{R}^{p\times q}. \quad (8)$$

From Theorem 1, controller $K(s)$ is stable if and only if

$$A_k = A - (B_1 D_q + B_2)(D_{21} D_q + D_{22})^{-1} C_2 \quad (9)$$

is stable, equivalently, to get a stable controller $K(s)$ is to find a constant $D_q \in \mathbb{R}^{p\times q}$ such that

$$\max_{\bar{\sigma}(D_q)<1} \lambda(A - (B_1 D_q + B_2)(D_{21} D_q + D_{22})^{-1} C_2) < 0. \quad (10)$$

If $p = q = 1$, (10) could be efficiently solved by dichotomizing search in set $(-1, 1)$.

When p, q are not too large we may find an appropriate D_q through search. For large p, q how to numerically solve (10) is now being investigated.

5 Design reduced controller by $Q(s)$

5.1 Introduction

Generally, H_∞ controller can be represented as follows:

$$K(s) = \mathrm{CSD}(\Pi(s), Q(s)) = (\Pi_{11}(s)Q(s) + \Pi_{12}(s)) \cdot (\Pi_{21}(s)Q(s) + \Pi_{22}(s))^{-1} = N(s)D(s)^{-1}, \quad (11)$$

$$N(s) = \Pi_{11}(s)Q(s) + \Pi_{12}(s),$$
$$D(s) = \Pi_{21}(s)Q(s) + \Pi_{22}(s)$$

where $Q(s) \in \mathbb{R}\mathrm{H}^{p\times q}$ and $\| Q(s) \|_\infty < 1$.

In this section we propose a method to get reduced order controller by $Q(s)$. A reduced order controller design example is given in Appendix.

5.2 SISO case

We investigate the problem to get reduced order controller by $Q(s)$ for SISO system ($p = q = 1$). We want to find a $Q(s)$ to cancel common zeros of denominator $D(s)$ and numerator $N(s)$ of $K(s)$. If there exists a $Q(s) \in \mathbb{R}\mathrm{H}^{p\times q}$, $\| Q(s) \|_\infty < 1$ such that equations (12), (13):

$$N(s) = \Pi_{11}(s)Q(s) + \Pi_{12}(s) = 0, \quad (12)$$
$$D(s) = \Pi_{21}(s)Q(s) + \Pi_{22}(s) = 0 \quad (13)$$

have common zeros, the common zeros of $D(s)$ and $N(s)$ can be cancelled. On the other hand from the discussion in Section 3 we have: if $\deg(Q(s)) = 0$, the order of $K(s)$ will not increase when $Q(s)$ is nonzero. So if we take $Q(s)$ a constant parameter, when equations (12), (13) have common zeros, we shall get reduced order controller. Since at least one zero of denominator of $K(s)$ is cancelled, a n-th order controller will reduce to at least a $(n-1)$-th order controller.

Theorem 2 If s_i is a common zero of equations (12), (13), we must have:

$$\Pi_{11}(s_i)\Pi_{22}(s_i) - \Pi_{12}(s_i)\Pi_{21}(s_i) = 0, \quad (14)$$
$$Q(s_i) = -\Pi_{12}(s_i)/\Pi_{11}(s_i). \quad (15)$$

Corollary 1 The common zeros of $D(s)$, $N(s)$ are all in open left half plane.

From Theorem 2 we have the following algorithm to compute $Q(s)$ and $K(s)$ (SISO case):

Step 1) Solve algebraic equation (14) we get s_i.

Step 2) From equation (15) we get $Q(s_i)$.

Step 3) Choose: $Q(s) = Q(s_i)$, where s_i is a solution of equation (14) with the largest multiplicity and $Q(s_i)$ is a real number whose absolute value is less than 1.

Step 4) From Theorem 1 we get $K(s)$. Computing minimal realization of $K(s)$ we get reduced order controller.

5.3 MIMO case

In this case the problem is more difficult than SISO case. Here we give partial solution to this problem. If there exist $0 \neq \xi_1 \in \mathbb{R}^q$, $0 \neq \xi_2 \in \mathbb{R}^q$ and $Q(s) \in \mathbb{R}\mathrm{H}^{p\times q}$, $\| Q(s) \|_\infty < 1$ such that equations (16), (17):

$$N(s)\xi_1 = (\Pi_{11}(s)Q(s) + \Pi_{12}(s))\xi_1 = 0, \quad (16)$$

$$D(s)\xi_2 = (\Pi_{21}(s)Q(s) + \Pi_{22}(s))\xi_2 = 0 \quad (17)$$

have common zeros, we can cancel these common zeros of $D(s)$ and $N(s)$. Thus the order of controller, i.e., the number of zeros of $D(s)$, will decrease at least by 1. From discussion in Section 4 we know that when $\deg(Q(s)) = 0$, the order of controller will not increase. So if we choose a constant matrix which makes equations (16), (17) have common zeros as $Q(s)$, the order of controller must decrease at least by 1, i.e., n-th order controller must reduce to at least $(n-1)$-th order controller.

Theorem 3 If s_i is a common zero of equations (16), (17) and $\xi = \xi_1 = \xi_2 \not\equiv 0$, we have:

$$\det(\Pi(s_i)) = 0, \tag{18}$$

where $\Pi(s)$ is defined in Theorem 1.

Theorem 4 Supposing $\xi \in \mathbb{R}^q, \eta \in \mathbb{R}^p$, there exists $Q(s_i) \in \mathbb{R}^{p\times q}, \bar{\sigma}(Q(s_i)) < 1, \eta = Q(s_i)\xi$ if and only if $\eta^T\eta < \xi^T\xi$.

From Theorem 3 and 4 we have the following algorithm to compute $Q(s)$ and $K(s)$ (MIMO case):

step 1) Solving algebraic equation (18) we get n zeros s_i of $\Pi(s)$. One can use Matlab function tzero(). Ordering zeros s_i by their multiplicity. Let $i = 0$.

step 2) If $i < n$, solve linear equations: $\Pi(s_i)\zeta = 0$, where $\zeta \in \mathbb{R}^{p+q}$; if $i = n$, return.

step 3) Partition ζ as: $\zeta = [\eta^T \quad \xi^T]^T$, where $\xi \in \mathbb{R}^q, \eta \in \mathbb{R}^p$. If $\eta^T\eta < \xi^T\xi$, from Theorem 4 we get $Q(s) = Q(s_i)$; If there exists no such $\xi, \eta, i = i+1$, go to Step 2).

step 4) From Theorem 1 we get $K(s)$. Compute minimal realization of $K(s)$ we get reduced order controller.

6 Conclusion

In this paper we discuss numerical computation method and Q-parameter utilization in H$_\infty$ controller based on CSD of transfer function and (J, J')-loseless factorization. We propose methods to design stable controller and reduced order controller by $Q(s)$.

We give a numerical method to compute (J, J')-loseless factorization in $\mathbb{RL}_\infty$. Then we outline the procedure to compute H$_\infty$ controller based on CSD of generalized plant and (J, J')-loseless factorization.

We give state space parameterization of controller and the order of controller is discussed. How to obtain lower order stable controller through constant matrix parameter is discussed. Finally we give a method to get reduced order controller by parameter $Q(s)$.

Our method has been implemented in Matlab environment. Design examples are given to illustrate the procedure. How to use our controller parameterization to satisfy more specifications is worthy of further investigating.

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Appendix H$_\infty$ Controller design based on chain scattering description

We demonstrate the design procedure of ordinary controller, stable controller (Section 4), and reduced order controller (Section 5) through a simple imaginary plant. (adopt from[6])

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \begin{bmatrix} x \\ w \\ u \end{bmatrix}$$

where

$$A = \begin{bmatrix} -3 & -2 \\ -2 & -7 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, B_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}, C_2^T = \begin{bmatrix} -1 \\ -3 \end{bmatrix},$$

$$D_{11} = D_{22} = 0, D_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, D_{21} = 1.$$

Step 1 compute chain scattering description matrix $S(s)$:

In this case, D_{21} is invertible, we can directly get $S(s)$:

$$S=\left[\begin{array}{c|cc} A-B_1D_{21}^{-1}C_2 & B_2-B_1D_{21}^{-1}D_{22} & B_2 \\ \hline C_1-D_{11}D_{21}^{-1}C_2 & D_{12}-D_{11}D_{21}^{-1}D_{22} & D_{11}D_{21}^{-1} \\ -D_{21}^{-1}C_2 & -D_{21}^{-1}D_{22} & D_{21}^{-1} \end{array}\right]=$$

$$\left[\begin{array}{cc|cc} -2 & 0 & -1 & 1 \\ 0 & -1 & -1 & 2 \\ \hline 2 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right].$$

Step 2 Find (J, J')-loseless factorization:

In this case, $m = 2, r = p = q = 1$. First, we solve (2) to get a $D_\pi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then by solving two Riccati equations we get $X = \mathrm{Ric}(Hx) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \geqslant 0, Y = \mathrm{Ric}(H_y) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \geqslant 0$. Thus we get (J, J')-loseless factorization:

$$\Theta(s)=\left[\begin{array}{c|cc} -2 & -1 & 1 \\ \hline 2 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{array}\right], \Pi(s)=\left[\begin{array}{c|cc} -5 & 1 & -2 \\ \hline 2 & 1 & 0 \\ 3 & 0 & 1 \end{array}\right].$$

Step 3 Compute H_∞ controller.

a) Compute stable H_∞ controller.

From Theorem 2, take $Q(s) = D_q$, where D_q is a constant number $(p = q = 1)$. We have: $K(s) = \left[\begin{array}{c|c} 1-3D_q & -D_q+2 \\ \hline -2+3D_q & D_q \end{array}\right]$, $A_k = 1-3D_q$. If $Q(s) = D_q = 0$, $A_k = 1$, we get a unstable controller. Clearly we can get lower order (order less than 2) stable controller by taking any $1/3 < D_q < 1$. For example if we take $Q(s) = D_q = 5/6$, we have $K(s) = (5s+11)/(6s+9)$, which is a 1-st order stable controller.

b) Compute reduced order H_∞ controller.

$\Pi_{11}(s) = (s+7)/(s+5), \Pi_{12}(s) = -4/(s+5), \Pi_{21}(s) = 3/(s+5), \Pi_{22}(s) = (s-1)/(s+5)$. Solve equation (15) we get common zero: $s = -1$. From equation (16) we have $Q(-1) = 2/3$. Taking $Q(s) = 2/3$ we get reduced order controller $K(s) = 2/3$, which is a zero-th order proportional controller.

Remark For the above plant, the controller designed by DGKF method in Robust Toolbox in Matlab is $(Q(s) = 0)$: $K(s) = \left[\begin{array}{cc|c} -2 & 3 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 0 \end{array}\right]$. Cleary $K(s)$ is unstable and its order is higher than that of our controller.

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