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Robust Stability of Impulsive Large-Scale Delay Systems with Time-Varying Uncertainties *

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Abstract: The problem of robust stability for impulsive large scale delay systems with time-varying interval uncertainties is introduced and studied. Some explicit criteria of exponential robust stability in the large for such systems are established based on vector comparison theorem and the techniques of matrices and inequalities. An example is given to illustrate the effectiveness of results obtained.

Key words: robust stability; impulses; uncertainties; delay; time-varying systems; large-scale systems

具有时变不确定性的脉冲时滞大系统的鲁棒稳定性

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摘要: 提出并研究了具有时变不确定性的脉冲时滞大系统的鲁棒稳定性. 借助于向量比较原理和矩阵不等式等方法, 对该系统建立了若干大范围指数鲁棒稳定的判据, 并举例说明了结论的有效性.

关键词: 鲁棒稳定; 脉冲; 不确定性; 时滞; 时变系统; 大系统

1 Introduction

Many evolutionary processes, such as biological phenomena involving thresholds, bursting rhythm models in medicine and biology, optimal control models in economics, neural networks systems, frequency-modulated systems, and motion of missiles or airplanes are characterized by the fact that at certain moments of time they experience an abrupt change of state. These perturbations act instantaneously, in the form of impulses, which can be successfully described by the measure differential systems[1]. The stability theory of measure differential systems with impulses has been developed recently[2~6]. Up to now, most research on impulsive systems has been concentrated on the systems without delays. It is well known, however, that delays often occur in many systems, such as transportation systems, communication systems, chemical processing systems, metallurgical processing systems, environmental systems, power systems, and so on. This prompted us to investigate impulsive systems with delays. On the other hand, in practice, uncertainties are often encountered in various dynamical systems due to modelling errors, measurement errors, and linearization approximations. The problem of the robust stability analysis of the systems suffered from such uncertainties has attracted much attention in recent years[7~11]. But there are few results of delay uncertain dynamical systems[7~9] and no reports on impulsive delay dynamical systems with time-varying uncertainties. In view of above discussion, we first introduce the problem of the robust stability of impulsive large scale delay dynamical systems with time-varying interval uncertainties in the paper. Some analytical methods and techniques are employed to investigate the sufficient conditions for robust exponential stability. This paper is organized as follows. In Section 2, the problem to be tackled in this paper is stated. In Section 3, some explicit criteria of robust exponential stability for such systems are established. An example and conclusion are given in Section 4 and Section 5, respectively.

2 Problem formulation

Let $R_+ = [0, +\infty)$, $J = [t_0, +\infty)$ $(t_0 \geqslant 0)$, and $\mathbb{R}^n$ denote the n- Euclidean space with the norm $\| x \| = \sum_{i=1}^{n} | x_i |$, $x \in \mathbb{R}^n$. Correspondingly, for a matrix

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$A = (a_{ij})_{n\times n} \in \mathbb{R}^{n\times n}$, $\|A\| = \max_j \sum_{i=1}^{n} |a_{ij}|$, $|A| = (|a_{ij}|)_{n\times n}$. We use the notation $A(a_{ij}) \geqslant B(b_{ij})$ and $\mathrm{col}(x_1,\cdots,x_n) \geqslant \mathrm{col}(y_1,\cdots,y_n)$ to imply that $a_{ij} \geqslant b_{ij}$ and $x_i \geqslant y_i$, $i,j = 1,\cdots,n$, respectively.

Consider the following impulsive large scale interval dynamical system with delays

$$Dx_i = N[\underline{A}_i(t),\bar{A}_i(t)]x_i(t) + \sum_{j=1}^{r}\{N[\underline{A}_{ij}(t),\bar{A}_{ij}(t)]x_j(t)Du_j + N[\underline{B}_{ij}(t),\bar{B}_{ij}(t)]x_j(t-\tau(t))Dv_j\},\ i = 1,\cdots,r \tag{1}$$

with the initial-value conditions

$$x_i(t) = \phi_i(t),\ t_0 - \tau \leqslant t \leqslant t_0,\ i = 1,\cdots,r \tag{2}$$

where Dx_i, Du_j and Dv_j denote the distributional derivatives of the functions x_i, u_j and v_j respectively. $u_i, v_i: \mathbb{R}_+ \to \mathbb{R}$ are functions of bounded variation and right-continuous on every compact subinterval of J. This implies that Du_i and Dv_i can be identified with the Lebesgue-Stieltjes measure which has the effect of suddenly changing the state of the system at the points of discontinuity of u_i and v_i; $x_i \in \mathbb{R}^{n_i}$, $\underline{A}_i(t),\bar{A}_i(t) \in \mathbb{R}^{n_i\times n_i}$ are continuous on J, $\underline{A}_{ij}(t),\bar{A}_{ij}(t) \in \mathbb{R}^{n_i\times n_j}$ are bounded and Lebesgue-Stieltjes integrable with respect to u_j on J, $\underline{B}_{ij}(t),\bar{B}_{ij}(t) \in \mathbb{R}^{n_i\times n_j}$ are bounded and Lebesgue-Stieltjes integrable with respect to v_j on J. $\Phi(t) = \mathrm{col}(\phi_1(t),\cdots,\phi_r(t))$, $\phi_i:[t_0-\tau,t_0]\to\mathbb{R}^{n_i}$ are functions of bounded variation and right-continuous, $0 \leqslant \tau(t) \leqslant \tau$ for some constant τ.

$$N[\underline{A}_i(t),\bar{A}_i(t)] = \{A_i(t) \mid \underline{A}_i(t) \leqslant A_i(t) \leqslant \bar{A}_i(t)\},$$
$$N[\underline{A}_{ij}(t),\bar{A}_{ij}(t)] = \{A_{ij}(t) \mid \underline{A}_{ij}(t) \leqslant A_{ij}(t) \leqslant \bar{A}_{ij}(t)\},$$
$$N[\underline{B}_{ij}(t),\bar{B}_{ij}(t)] = \{B_{ij}(t) \mid \underline{B}_{ij}(t) \leqslant B_{ij}(t) \leqslant \bar{B}_{ij}(t)\}$$

where $A_i(t) \in \mathbb{R}^{n_i\times n_i}$ are continuous on J, $A_{ij}(t) \in \mathbb{R}^{n_i\times n_j}$ and $B_{ij}(t) \in \mathbb{R}^{n_i\times n_j}$ are integrable with respect to u_j and v_j on J respectively, $i,j = 1,\cdots,r$, $\sum_{j=1}^{r} n_j = n$.

In this paper, we assume that

$$u_i(t) = t + \sum_{k=1}^{\infty}\alpha_{ik}H_k(t),$$
$$v_i(t) = t + \sum_{k=1}^{\infty}\beta_{ik}H_k(t),\quad i = 1,\cdots,r \tag{3}$$

where the discontinuity points are $t_1 < t_2 < \cdots < t_k < \cdots$ and $\lim_{k\to\infty} t_k = \infty$, $t_1 > t_0$; α_{ik} and β_{ik} are constants, and $H_k(t)$ are Heaviside functions, i.e.,

$$H_k(t) = \begin{cases}0, & t < t_k,\\ 1, & t \geqslant t_k.\end{cases}$$

Definition 2.1 The impulsive and time-delay large scale interval dynamical system (1) is said to be exponentially stable in the large, if for any matrix

$$A_i(t) \in N[\underline{A}_i(t),\bar{A}_i(t)],$$
$$A_{ij}(t) \in N[\underline{A}_{ij}(t),\bar{A}_{ij}(t)],$$
$$B_{ij}(t) \in N[\underline{B}_{ij}(t),\bar{B}_{ij}(t)],$$

the zero solution of the impulsive and time-delay large scale system

$$Dx_i = A_i(t)x_i(t) + \sum_{j=1}^{r}[A_{ij}(t)x_j(t)Du_j + B_{ij}(t)x_j(t-\tau(t))Dv_j],\ i = 1,\cdots,r \tag{4}$$

is exponentially stable in the large.

3 Robust stability

In this section, we discuss the stability of impulsive and time-delay large scale interval system (1) and assume that $t_k - t_{k-1} \geqslant \delta\tau$, $\delta > 1$, and

$$\underline{A}_i(t) = (\underline{a}_{js}^{(i)}(t))_{n_i\times n_i},\ \bar{A}_i(t) = (\bar{a}_{js}^{(i)}(t))_{n_i\times n_i}.$$

For the following system

$$\dot{z}_i(t) = P_i(t)z_i(t) \tag{5}$$

where $z_i(t) \in \mathbb{R}^{n_i}$, $P_i(t) = (p_{js}^{(i)}(t))_{n_i\times n_i}$, $p_{jj}^{(i)}(t) = \bar{a}_{jj}^{(i)}(t)$, and

$$p_{js}^{(i)}(t) = \max\{|\underline{a}_{js}^{(i)}(t)|, |\bar{a}_{js}^{(i)}(t)|\},$$
$$j \neq s,\ j,s = 1,\cdots,n_i,$$

from Theorem 1 ([11]), we can easily obtain Lemma 3.1.

Lemma 3.1 Suppose that the trivial solution of system (5) is exponentially stable in the large, namely there exists an $\alpha_i > 0$ such that

$$\|z_i(t)\| \leqslant \|z_i(t_0)\|\exp[-\alpha_i(t-t_0)],\ t \geqslant t_0. \tag{6}$$

Then for any $A_i(t) \in N[\underline{A}_i(t),\bar{A}_i(t)]$, the trivial solution of system

$$\dot{x}_i(t) = A_i(t)x_i(t) \tag{7}$$

is exponentially stable in the large, moreover

$$\| x_i(t) \| \leqslant \| x_i(t_0) \| \exp[-\alpha_i(t-t_0)], t \geqslant t_0 \tag{8}$$

where $x_i(t) = X_i(t,t_0)x_i(t_0)$, $X_i(t,t_0)$ is the matrix of fundamental solution for system (7), $X_i(t_0,t_0) = I_i$ (identity matrix), $x_i(t_0) = x_{i0}$.

For all $A_i(t) \in N[\underline{A}_i(t), \bar{A}_i(t)]$, $A_{ij}(t) \in N[\underline{A}_{ij}(t), \bar{A}_{ij}(t)]$, and $B_{ij}(t) \in N[\underline{B}_{ij}(t), \bar{B}_{ij}(t)]$, if define the matrices

$$\begin{aligned} A_{ij}^{(0)}(t) &= \frac{1}{2}(\underline{A}_{ij}(t) + \bar{A}_{ij}(t)), \\ A_{ij}^{(m)}(t) &= \frac{1}{2}(\bar{A}_{ij}(t) - \underline{A}_{ij}(t)), \end{aligned} \tag{9}$$

$$\begin{aligned} B_{ij}^{(0)}(t) &= \frac{1}{2}(\underline{B}_{ij}(t) + \bar{B}_{ij}(t)), \\ B_{ij}^{(m)}(t) &= \frac{1}{2}(\bar{B}_{ij}(t) - \underline{B}_{ij}(t)), \end{aligned} \tag{10}$$

then

$$\begin{aligned} A_{ij}(t) &= A_{ij}^{(0)}(t) + \Delta A_{ij}(t), \\ B_{ij}(t) &= B_{ij}^{(0)}(t) + \Delta B_{ij}(t), \end{aligned} \tag{11}$$

where $\Delta A_{ij}(t)$ and $\Delta B_{ij}(t)$ are the uncertain matrices which satisfy

$$| \Delta A_{ij}(t) | \leqslant A_{ij}^{(m)}(t), \quad | \Delta B_{ij}(t) | \leqslant B_{ij}^{(m)}(t). \tag{12}$$

For convenience, we have the following notations

$$\begin{aligned} a_{ij} &= \sup_{t \geqslant t_0} \{ a_{ij}(t) \}, \\ a_{ij}(t) &= \| A_{ij}^{(0)}(t) \| + \| A_{ij}^{(m)}(t) \|, \end{aligned} \tag{13}$$

$$\begin{aligned} b_{ij} &= \sup_{t \geqslant t_0} \{ b_{ij}(t) \}, \\ b_{ij}(t) &= \| B_{ij}^{(0)}(t) \| + \| B_{ij}^{(m)}(t) \|, \end{aligned} \tag{14}$$

$$\begin{aligned} A_k(t) &= \mathrm{diag}(I_1, \cdots, I_r) - (A_{ij}^{(0)}(t)\alpha_{jk})_{r\times r}, \\ B_k(t) &= (B_{ij}^{(0)}(t)\beta_{jk})_{r\times r}, \end{aligned} \tag{15}$$

$$\begin{aligned} A_k^{(m)}(t) &= (A_{ij}^{(m)}(t) \, | \alpha_{jk} |)_{r\times r}, \\ B_k^{(m)}(t) &= (B_{ij}^{(m)}(t) \, | \beta_{jk} |)_{r\times r}, \end{aligned} \tag{16}$$

$$\begin{cases} \beta_k = \dfrac{1}{\left[\dfrac{\lambda_{\min}(A_k^{\mathrm{T}}(t_k)A_k(t_k))}{n}\right]^{1/2} - \| A_k^{(m)}(t_k) \|}, \\ \gamma_k = \dfrac{\| B_k(t_k) \| + \| B_k^{(m)}(t_k) \|}{\left[\dfrac{\lambda_{\min}(A_k^{\mathrm{T}}(t_k)A_k(t_k))}{n}\right]^{1/2} - \| A_k^{(m)}(t_k) \|}, \end{cases} \tag{17}$$

$$\begin{cases} \tilde{\beta}_k = \dfrac{\| A_k^{-1}(t_k) \|}{1 - \| A_k^{-1}(t_k) \| \, \| A_k^{(m)}(t_k) \|}, \\ \tilde{\gamma}_k = \dfrac{\| A_k^{-1}(t_k)B_k(t_k) \| + \| A_k^{-1}(t_k) \| \, \| B_k^{(m)}(t_k) \|}{1 - \| A_k^{-1}(t_k) \| \, \| A_k^{(m)}(t_k) \|}. \end{cases} \tag{18}$$

Theorem 3.1 For system (1), suppose that for $i = 1, \cdots, r$ and $k = 1, 2, \cdots$,

i) $\left[\dfrac{\lambda_{\min}(A_k^{\mathrm{T}}(t_k)A_k(t_k))}{n}\right]^{1/2} - \| A_k^{(m)}(t_k) \| > 0$;

ii) the trivial solution of system (5) is exponentially stable in the large and there exists a constant α such that

$$\eta = \max_{1\leqslant i\leqslant r}\left\{ \frac{1}{\alpha_i - \alpha} \sum_{j=1}^{r} (a_{ij} + b_{ij}\mathrm{e}^{\alpha\tau}) \right\} < 1$$

where $\alpha_i > 0$ is given by (6), $0 < \alpha < \min_{1\leqslant i\leqslant r}\{\alpha_i\}$;

iii) $\max\{\mathrm{e}^{\alpha\tau}, (\beta_k + \gamma_k\mathrm{e}^{\alpha\tau})\} \leqslant M_k \leqslant C$ for constant C.

Then $\dfrac{\ln(CM)}{\delta\tau} - \alpha < 0$ implies that system (1) is exponentially stable in the large, and the solution of system (1) has the following estimate

$$\| X(t) \| \leqslant \| \Phi \| M \cdot \exp\left[\left(\left(\frac{\ln(CM)}{\delta\tau} - \alpha\right)(t - t_0)\right)\right], t \geqslant t_0$$

where $M = \dfrac{1}{1-\eta}$.

Proof For any $A_i(t) \in N[\underline{A}_i(t), \bar{A}_i(t)]$, $A_{ij}(t) \in N[\underline{A}_{ij}(t), \bar{A}_{ij}(t)]$, and $B_{ij}(t) \in N[\underline{B}_{ij}(t), \bar{B}_{ij}(t)]$, consider the stability of system (4). In view of (3), we see that u_i' and v_i' exist on $[t_{k-1}, t_k)$. Thus (4) becomes that

$$x_i'(t) = A_i(t)x_i + \sum_{j=1}^{r}[A_{ij}(t)x_j(t) + B_{ij}(t)x_j(t-\tau(t))], t \in [t_{k-1}, t_k). \tag{19}$$

Let the initial value of system (19) be

$$x_i(t) = \phi_{ik-1}(t), \quad t \in [t_{k-1} - \tau, t_{k-1}]$$

and $\Phi_{k-1}(t) = \mathrm{col}(\phi_{1k-1}(t), \cdots, \phi_{rk-1}(t))$, where $\phi_{ik-1}(t)$ is the function of bounded variation and right-continuous on $[t_{k-1} - \tau, t_{k-1}]$. We have from (19) that, for $t \in [t_{k-1}, t_k)$

$$x_i(t) = X_i(t, t_{k-1})\phi_{ik-1}(t_{k-1}) + \int_{t_{k-1}}^{t} X_i(t,s)\sum_{j=1}^{r}[A_{ij}(s)x_j(s) + B_{ij}(s)x_j(s-\tau(s))]\mathrm{d}s \tag{20}$$

where $X_i(t, t_{k-1})$ is the matrix of fundamental solution for system (7), $X_i(t_{k-1}, t_{k-1}) = I_i$ (identity matrix). From (9) ~ (14), it follows that

$$\| A_{ij}(t) \| \leqslant \| A_{ij}^{(0)}(t) \| + \| A_{ij}^{(m)}(t) \| = a_{ij}(t) \leqslant a_{ij} \tag{21}$$

and

$$\| B_{ij}(t) \| \leqslant \| B_{ij}^{(0)}(t) \| + \| B_{ij}^{(m)}(t) \| = b_{ij}(t) \leqslant b_{ij}. \tag{22}$$

In view of Assumption ii) and Lemma 3.1, we see from (8) and (20) ~ (22) that

$$\| x_i(t) \| \leqslant \exp[-\alpha_i(t - t_{k-1})] \| \phi_{ik-1} \| + \int_{t_{k-1}}^{t} \exp[-\alpha_i(t-s)] \sum_{j=1}^{r} [a_{ij} \| x_j(s) \| + b_{ij} \| x_j(s - \tau(s)) \|] \mathrm{d}s, t \in [t_{k-1}, t_k) \tag{23}$$

where $\| \phi_{ik-1} \| = \sup\limits_{t_{k-1}-\tau \leqslant t \leqslant t_{k-1}} \| \phi_{ik-1}(t) \|$. Multiplying by $\exp[\alpha(t - t_{k-1})]$ on both sides of (23), where α is defined by assumption ii), then

$$\| x_i(t) \| \exp[\alpha(t - t_{k-1})] \leqslant \| \phi_{ik-1} \| + \int_{t_{k-1}}^{t} \exp[-(\alpha_i - \alpha)(t-s)] \cdot \sum_{j=1}^{r} \{ a_{ij} \| x_j(s) \| \exp[\alpha(s - t_{k-1})] + b_{ij} e^{\alpha\tau} \| x_j(s - \tau(s)) \| \cdot \exp[\alpha(s - \tau(s) - t_{k-1})] \} \mathrm{d}s. \tag{24}$$

Let

$$y_i(t) = \sup_{t_{k-1}-\tau \leqslant s \leqslant t} \{ \| x_i(s) \| \exp[\alpha(s - t_{k-1})] \}. \tag{25}$$

We get from (24), (25) and assumption ii) that

$$y_i(t) \leqslant \| \phi_{ik-1} \| + \eta y_j(t). \tag{26}$$

If rewrite

$$y(t) = \max_i \{ y_i(\xi), \xi \in [t_{k-1} - \tau, t], i = 1, \cdots, r \},$$

then we have from (26) that

$$y(t) \leqslant \| \phi_{ik-1} \| + \eta y(t),$$

namely,

$$y(t) \leqslant \frac{1}{1-\eta} \| \phi_{ik-1} \|, \quad t \in [t_{k-1}, t_k)$$

which reduce to

$$\| x_i(t) \| \leqslant \exp[-\alpha(t - t_{k-1})] y(t) \leqslant \frac{1}{1-\eta} \| \phi_{ik-1} \| \exp[-\alpha(t - t_{k-1})], t \in [t_{k-1}, t_k),$$

or

$$\| X(t) \| \leqslant \| \Phi_{k-1} \| M \exp[-\alpha(t - t_{k-1})], \quad t \in [t_{k-1}, t_k) \tag{27}$$

where $M = \frac{1}{1-\eta}$. On the other hand, system (4) implies that

$$x_i(t_k) - x_i(t_k - h) = \int_{t_k-h}^{t_k} A_i(s) x_i(s) \mathrm{d}s + \int_{t_k-h}^{t_k} \sum_{j=1}^{r} A_{ij}(s) x_j(s) \mathrm{d}u_j(s) + \int_{t_k-h}^{t_k} \sum_{j=1}^{r} B_{ij}(s) x_j(s - \tau(s)) \mathrm{d}v_j(s)$$

where $h > 0$ is sufficiently small, which reduces to

$$x_i(t_k) - x_i(t_k -) = \sum_{j=1}^{r} [A_{ij}(t_k) x_j(t_k) \alpha_{jk} + B_{ij}(t_k) x_j(t_k - \tau(t_k)) \beta_{jk}],$$

and further gives

$$A_k(t_k) X(t_k) = X(t_k -) + \Delta A_k(t_k) X(t_k) + [B_k(t_k) + \Delta B_k(t_k)] X(t_k - \tau(t_k)), \tag{28}$$

where $A_k(t)$ and $B_k(t)$ are vector-value functions and given by (15), and

$$\Delta A_k(t_k) = (\Delta A_{ij}(t_k) \alpha_{jk})_{r \times r},$$
$$\Delta B_k(t_k) = (\Delta B_{ij}(t_k) \beta_{jk})_{r \times r}.$$

It is easy to see that

$$\| \Delta A_k(t_k) X(t_k) \| \leqslant \| A_k^{(m)}(t_k) \| \| X(t_k) \| \tag{29}$$

and

$$\| \Delta B_k(t_k) X(t_k - \tau(t_k)) \| \leqslant \| B_k^{(m)}(t_k) \| \| X(t_k - \tau(t_k)) \|, \tag{30}$$

in which $A_k^{(m)}(t)$ and $B_k^{(m)}(t)$ are given by (16), and

$$\| A_k(t_k) X(t_k) \| \geqslant \| A_k(t_k) X(t_k) \|_2 \geqslant \left[\frac{\lambda_{\min}(A_k^{\mathrm{T}}(t_k) A_k(t_k))}{n} \right]^{1/2} \| X(t_k) \|. \tag{31}$$

Summing up (28) ~ (31), we get

$$\left[\frac{\lambda_{\min}(A_k^{\mathrm{T}}(t_k) A_k(t_k))}{n} \right]^{1/2} \| X(t_k) \| \leqslant \| X(t_k -) \| + \| A_k^{(m)}(t_k) \| \| X(t_k) \| + [\| B_k(t_k) \| + \| B_k^{(m)}(t_k) \|] \| X(t_k - \tau(t_k)) \|.$$

Because of assumption i), we immediately arrive at

$$\| X(t_k) \| \leqslant \beta_k \| X(t_k -) \| + \gamma_k \| X(t_k - \tau(t_k)) \| \tag{32}$$

where β_k and γ_k are defined by (17). By (27) and (32), we have the following inference

$$\| X(t) \| \leqslant \| \Phi \| M^k M_1 \cdots M_{k-1} \cdot \exp[-\alpha(t - t_0)], t \in [t_{k-1}, t_k) \tag{33}$$

where M_k is given by assumption iii). Since $M_k \leqslant C$, $t_k - t_{k-1} \geqslant \delta\tau$, $(\delta > 1)$ and $MC \geqslant 1$,

$$M^k M_1 \cdots M_{k-1} \leqslant \exp\left[\frac{\ln(CM)}{\delta\tau}(t_{k-1} - t_0)\right] \leqslant \exp\left[\frac{\ln(CM)}{\delta\tau}(t - t_0)\right], \tag{34}$$

where $t \in [t_{k-1}, t_k)$. From (33) and (34), we immediately arrive at

$$\| X(t) \| \leqslant \| \Phi \| M\exp\left[\left(\frac{\ln(CM)}{\delta\tau} - \alpha\right)(t - t_0)\right], \quad t \geqslant t_0$$

which implies that the conclusion of Theorem 3.1 holds. This completes the proof. Q.E.D.

If $A_k(t_k)$ is invertible, then from (28) that

$$X(t_k) = A_k^{-1}(t_k)[X(t_k -) + \Delta A_k(t_k)X(t_k) + (B_k(t_k) + \Delta B_k(t_k))X(t_k - \tau(t_k))]. \tag{35}$$

In view of (29) and (30), we have from (35) that

$$\| X(t_k) \| \leqslant \| A_k^{-1}(t_k) \| [\, \| X(t_k -) \| + \| A_k^{(m)}(t_k) \| \cdot \| X(t_k) \| \,] + (\| A_k^{-1}(t_k)B_k(t_k) \| + \| A_k^{-1}(t_k) \| \, \| B_k^{(m)}(t_k) \|) \| X(t_k - \tau(t_k)) \|. \tag{36}$$

Further, if $\| A_k^{-1}(t_k) \| \, \| A_k^{(m)}(t_k) \| < 1$ then (36) becomes

$$\| X(t_k) \| \leqslant \tilde{\beta}_k \| X(t_k -) \| + \tilde{\gamma}_k \| X(t_k - \tau(t_k)) \|.$$

Similar to the inference of Theorem 3.1, we can obtain the following conclusion.

Corollary 3.1 For system (1), suppose that

i) $A_k(t_k)$ is invertible and $\| A_k^{-1}(t_k) \| \, \| A_k^{(m)}(t_k) \| < 1, k \in N$;

ii) the assumption ii) of Theorem 3.1 holds;

iii) $\max\{e^{\alpha\tau}, (\tilde{\beta}_k + \tilde{\gamma}_k e^{\alpha\tau})\} \leqslant M_k \leqslant C$ with C constant, where $\tilde{\beta}_k$ and $\tilde{\gamma}_k$ are given by (18).

Then the conclusion of Theorem 3.1 holds.

Theorem 3.2 For system (1), suppose that

i) the assumption i) of Theorem 3.1 holds;

ii) for $i = 1, \cdots, r$, the trivial solution of system (5) is exponentially stable in the large and $\alpha \triangleq \alpha_0 - (\| A \| + \| B \| e^{\alpha_0 \tau}) > 0$, where $\alpha_0 = \min_{1 \leqslant i \leqslant r}\{\alpha_i\}$, $A = (a_{ij})_{r\times r}$, $B = (b_{ij})_{r\times r}$, and α_i, a_{ij}, b_{ij} are given by (6), (13) and (14) respectively;

iii) the assumption iii) of Theorem 3.1 holds, where α is defined by condition ii).

Then $\frac{\ln C}{\delta\tau} - \alpha < 0$ implies that system (1) is exponentially stable in the large and

$$\| X(t) \| \leqslant \| \Phi \| \exp\left[\left(\frac{\ln C}{\delta\tau} - \alpha\right)(t - t_0)\right], t \geqslant t_0.$$

Proof Similar to the inference of Theorem 3.1, we obtain the inequality (23), which reduces to

$$\| X(t) \| \leqslant \exp[-\alpha_0(t - t_{k-1})] \| \Phi_{k-1} \| + \int_{t_{k-1}}^{t} \exp[-\alpha_0(t - s)](\| A \| \, \| X(s) \| + \| B \| \, \| X(s - \tau(s)) \|)ds, t \in [t_{k-1}, t_k). \tag{37}$$

Let $w(t) = \sup_{t_{k-1} - \tau \leqslant \xi \leqslant t}\{\| X(\xi) \| \exp[\alpha_0(\xi - t_{k-1})]\}$.

Then (37) becomes

$$w(t) \leqslant \| \Phi_{k-1} \| + \int_{t_{k-1}}^{t} (\| A \| + \| B \| e^{\alpha_0 \tau})w(s)ds,$$

moreover

$$w(t) \leqslant \| \Phi_{k-1} \| \exp[(\| A \| + \| B \| e^{\alpha_0 \tau})(t - t_{k-1})]$$

i.e.,

$$\| X(t) \| \leqslant \| \Phi_{k-1} \| \exp[-(\alpha_0 - (\| A \| + \| B \| e^{\alpha_0 \tau}))(t - t_{k-1})] = \| \Phi_{k-1} \| \exp[-\alpha(t - t_{k-1})], t \in [t_{k-1}, t_k).$$

The rest of the argument is similar to that of Theorem 3.1, hence the details are omitted. Q.E.D.

Corollary 3.2 For system (1), suppose that

i) $A_k(t_k)$ is invertible, and

$$\| A_k^{-1}(t_k) \| \, \| A_k^{(m)}(t_k) \| < 1, \ k \in N;$$

ii) the assumption ii) of Theorem 3.2 holds;

iii) $\max\{e^{\alpha\tau}, (\tilde{\beta}_k + \tilde{\gamma}_k e^{\alpha\tau})\} \leqslant M_k \leqslant C$ with C constant, where $\tilde{\beta}_k$ and $\tilde{\gamma}_k$ are given by (18).

Then the conclusion of Theorem 3.2 holds.

4 An example

Consider the following impulsive and time-delay interval system

$$\begin{aligned} Dx_1 = {} & N[\underline{A}_1, \bar{A}_1]x_1(t) + N[\underline{A}_{11}(t), \bar{A}_{11}(t)]x_1(t)Du_1 + \\ & N[\underline{A}_{12}(t), \bar{A}_{12}(t)]x_2(t)Du_2 + \\ & N[\underline{B}_{11}(t), \bar{B}_{11}(t)]x_1(t - \tau(t))Dv_1 + \\ & N[\underline{B}_{12}(t), \bar{B}_{12}(t)]x_2(t - \tau(t))Dv_2, \\ Dx_2 = {} & N[\underline{A}_2, \bar{A}_2]x_2(t) + N[\underline{A}_{21}(t), \bar{A}_{21}(t)]x_1(t)Du_1 + \\ & N[\underline{A}_{22}(t), \bar{A}_{22}(t)]x_2(t)Du_2 + \\ & N[\underline{B}_{21}(t), \bar{B}_{21}(t)]x_1(t - \tau(t))Dv_1 + \end{aligned}$$

$$N[\underline{B}_{22}(t),\bar{B}_{22}(t)]x_2(t-\tau(t))Dv_2 \quad (38)$$

with the initial-value condition (2), where

$$[\underline{A}_1,\bar{A}_1]=[-16,-10],$$

$$[\underline{A}_2,\bar{A}_2]=[-12,-10],$$

$$0\leqslant\tau(t)\leqslant\tau=\frac{1}{10},$$

$$(\underline{A}_{ij}(t))_{2\times 2}=\begin{pmatrix}-1-\frac{\sin t}{2} & -2\frac{\cos^2 t}{3}\\ -\frac{\sin^2 t}{2} & -1-\frac{\cos t}{3}\end{pmatrix},$$

$$(\bar{A}_{ij}(t))_{2\times 2}=\begin{pmatrix}\frac{\sin t}{2} & 1+2\frac{\cos^2 t}{3}\\ \frac{\sin^2 t}{2} & \frac{\cos t}{3}\end{pmatrix},$$

$$(\underline{B}_{ij}(t))_{2\times 2}=\begin{pmatrix}0 & -\frac{\sin^2 t}{e}\\ -\frac{\cos^2 t}{e} & \frac{1}{2e}\end{pmatrix},$$

$$(\bar{B}_{ij}(t))_{2\times 2}=\begin{pmatrix}0 & \frac{\sin^2 t}{e}\\ \frac{\cos^2 t}{e} & \frac{1}{2e}\end{pmatrix},$$

and u_i, v_i are given by (3) and (4) respectively, where

$$t_k-t_{k-1}\geqslant\delta\tau,$$

$$\alpha_{1k}=\frac{1}{6}[1+(-1)^k],\alpha_{2k}=\frac{1}{8}[1-(-1)^k],$$

$$\beta_{1k}=-\frac{e}{4},\beta_{2k}=\frac{e}{6}(-1)^k.$$

It is not difficult to get from (6), (9), (10), (13) ~ (16), and (18) that

$$A_1^{(0)}=-13,A_2^{(0)}=-11,A_1^{(m)}=3,A_2^{(m)}=1,$$

$$\alpha_1=\alpha_2=10,$$

$$(A_{ij}^{(0)}(t))_{2\times 2}=\begin{pmatrix}-\frac{1}{2} & \frac{1}{2}\\ 0 & -\frac{1}{2}\end{pmatrix},$$

$$(A_{ij}^{(m)}(t))_{2\times 2}=\begin{pmatrix}\frac{1+\sin t}{2} & \frac{1}{2}+\frac{2\cos^2 t}{3}\\ \frac{\sin^2 t}{2} & \frac{1}{2}+\frac{\cos t}{3}\end{pmatrix},$$

$$(B_{ij}^{(0)}(t))_{2\times 2}=\begin{pmatrix}0 & 0\\ 0 & -\frac{1}{2e}\end{pmatrix},$$

$$(B_{ij}^{(m)}(t))_{2\times 2}=\begin{pmatrix}0 & \frac{\sin^2 t}{e}\\ \frac{\cos^2 t}{e} & \frac{1}{2e}\end{pmatrix},$$

$$A=(a_{ij})_{2\times 2}=\begin{pmatrix}3/2 & 5/2\\ 1/2 & 4/3\end{pmatrix},$$

$$B=(b_{ij})_{2\times 2}=\begin{pmatrix}0 & 1/e\\ 1/e & 1/e\end{pmatrix},$$

$$A_k(t_k)=\begin{pmatrix}1+\frac{\alpha_{1k}}{2} & -\frac{\alpha_{2k}}{2}\\ 0 & 1+\frac{\alpha_{2k}}{2}\end{pmatrix},$$

$$A_k^{(m)}(t_k)=\begin{pmatrix}\frac{1+\sin t_k}{2}\|\alpha_{1k}\| & \left(\frac{1}{2}+\frac{2\cos^2 t_k}{3}\right)\|\alpha_{2k}\|\\ \frac{\sin^2 t_k}{2}\|\alpha_{1k}\| & \left(\frac{1}{2}+\frac{\cos t_k}{3}\right)\|\alpha_{2k}\|\end{pmatrix},$$

$$B_k(t_k)=\begin{pmatrix}0 & 0\\ 0 & \frac{\beta_{2k}}{2e}\end{pmatrix},$$

$$B_k^{(m)}(t_k)=\begin{pmatrix}0 & \frac{\sin^2 t_k}{e}\|\beta_{2k}\|\\ \frac{\cos^2 t_k}{e}\|\beta_{1k}\| & \frac{1}{2e}\|\beta_{2k}\|\end{pmatrix}.$$

It is easy to compute that

$$A_k^{-1}(t_k)=\frac{1}{\left(\frac{1+\alpha_{1k}}{2}\right)\left(1+\frac{\alpha_{2k}}{2}\right)}\begin{pmatrix}1+\frac{1}{2}\alpha_{2k} & \frac{1}{2}\alpha_{2k}\\ 0 & 1+\frac{1}{2}\alpha_{1k}\end{pmatrix},$$

$$\|A_k^{-1}(t_k)\|=1,\ \|A_k^{(m)}(t_k)\|\leqslant\frac{1}{2},$$

$$\|B_k(t_k)\|=\frac{1}{12},\ \|B_k^{(m)}(t_k)\|\leqslant\frac{1}{4},$$

$$\tilde{\beta}_k\leqslant 2,\ \tilde{\gamma}_k\leqslant\frac{2}{3}.$$

Next, we verify that the conditions of Corollary 3.2 hold. In fact

i) $A_k^{-1}(t_k)$ exists and $\|A_k^{-1}(t_k)\|\ \|A_k^{(m)}(t_k)\|\leqslant\frac{1}{2}\leqslant 1, k\in N$;

ii) $\alpha\triangleq\alpha_0-(\|A\|+\|B\|e^{\alpha_0\tau})=10-(3+\frac{2}{e}e)=5>0$;

iii) $\max\{e^{\alpha\tau},(\tilde{\beta}_k+\tilde{\gamma}_k e^{\alpha\tau})\}\leqslant\max\{e^{1/2},2+\frac{2}{3}e^{1/2}\}=2+\frac{2}{3}e^{1/2}\triangleq c$. If $\delta>2.3$, then $\frac{\ln C}{\delta\tau}-\alpha<0$, which implies from Corollary 3.2 that system (38) is exponentially stable in the large.

5 Conclusion

In this paper, the problem of robust stability for impulsive and time-delay large scale dynamical systems with time-varying interval uncertainties is formulated and studied for the first time. Some sufficient conditions, expressed in terms of norms as well as comparison theorem, are derived in order for impulsive large scale interval dynamical systems with delays to be robustly stable against interval perturbations. It is believed that the results obtained will be helpful to the analysis and synthesis of robust control systems with impulsive effects.

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