Convergence of an Algorithm for Optimal Control of Nonlinear Continuous-Time Systems with Model-Reality Differences *

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Abstract: This paper presents an extension algorithm of dynamic integrated system optimization and parameter estimation based on time-variant linear-quadratic problem for nonlinear optimal control where model-reality differences exist. Convergence of the algorithm is inverstigated. A sufficient condition is derived for the convergence and optimality of the algorithm. It is shown that limit point of the algorithm solution satisfies the maximum principle for optimal control. The implementable conditions of the algorithm are emphasized, simulation example denotes the efficiency and the applicability of the algorithm.

Key words: nonlinear system optimal control; model-reality difference; convergence; optimality

具有模型和实际差异的非线性系统最优控制算法及其收敛性

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摘要: 针对模型和实际之间的差异,提出了一种基于时变线性二次型问题的动态系统优化和参数估计集成的算法,该算法能逼近实际问题最优解.给出了该算法收敛的一个充分条件,分析了它的最优性.仿真例子说明了该算法的有效性和实用性.

关键词: 非线性系统最优控制; 模型与实际差异; 收敛性; 最优性

1 Introduction

In the industrial process control, the steady-state optimization control has made considerable headway[1]. However, the dynamic optimizing control methods are still based on the exact model[2]. Because it is difficult to identify or it is not permitted to introduce the external excitation for systems, the model-reality differences exist for many industrial processes, and so, the model based solution is not only no optimal, but also possibly violates the constraints for the real system. P.D. Roberts[3,4] first presents an algorithm of dynamic integratd system optimization and parameter estimation (DISOPE) for nonlinear system where the model-reality differences exist. The algorithm has been successful in solving many simulation problems[3~5]. However, convergence analysis of the algorithm has never been published and to the best of the authors' knowledge the not trivial sufficient conditions for convergence have never been derived. In this paper, we present an extension algorithm of dynamic integrated system optimization and parameter estimation based on the time-variant linear quadratic problem for nonlinear system where the model-reality differences exist. We give a sufficient condition , under this condition, the solution of the algorithm converges to the real optimal solution in spite of model-reality differences. The implementable conditions of the algorithm are emphasized. Simulation example illustrates the efficiency and applicability of the algorithm.

2 DISOPE approach

Consider the real optimal control problem (ROP), with given initial conditions

$$\begin{cases} \min\limits_{u(t)} \{\phi(x(t_f)) + \int_{t_0}^{t_f} L^*(x(t), u(t), t)\mathrm{d}t\} \\ \text{s.t.}\ \dot{x}(t) = f^*(x(t), u(t), t), x(t_0) = x_0 \end{cases} \tag{1}$$

defined over the fixed time horizon $t \in [t_0, t_f]$, where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the continuous state and

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control vectors respectively. $\phi(x(t_f))$ is a given terminal meausure, $L^*(\cdot,\cdot,\cdot)$ is the real performance measure function and $f^*(\cdot,\cdot,\cdot)$ represents reality. It is assumed that the mapping f^* and L^* are rather complex, and as a consequence, we have at our disposal the time-variant linear model and a quadratic performance function. So, the following model based optimal control problem(MOP) is considered.

$$\begin{cases}\min\{\frac{1}{2}x^\tau(t_f)\Phi x(t_f)+\frac{1}{2}\int_{t_0}^{t_f}[u^\tau(t)R(t)u(t)+\\ \quad x^\tau(t)Q(t)x(t)+2\gamma(t)]\mathrm{d}t\}\\ \text{s.t.}\begin{cases}\dot{x}(t)=A(t)x(t)+B(t)u(t)+\alpha(t),\\ x(t_0)=x_0\end{cases}\end{cases} \tag{2}$$

where $a(t)\in\mathbb{R}^n,\gamma(t)\in\mathbb{R}^l$ are considered as continuous shift parameters.

Define an expanded optimal control problem (EOP) which is equivalent to ROP:

$$\min\{\frac{1}{2}x^\tau(t_f)\Phi x(t_f)+\frac{1}{2}\int_{t_0}^{t_f}[u^\tau(t)R(t)u(t)+$$
$$x^\tau(t)Q(t)x(t)+2\gamma(t)+$$
$$r_1\|u(t)-v(t)\|^2+r_2\|x(t)-z(t)\|^2]\mathrm{d}t\},$$

$$\text{s.t.}\begin{cases}\dot{x}(t)=A(t)x(t)+B(t)u(t)+\alpha(t),\\ x(t_0)=x_0,\\ f^*(z(t),v(t),t)=\\ A(t)z(t)+B(t)v(t)+\alpha(t),\\ L^*(z(t),v(t),t)=\\ \frac{1}{2}(v^\tau(t)R(t)v(t)+\\ z^\tau(t)Q(t)z(t)+2\gamma(t)),\\ u(t)=v(t),\ x(t)=z(t)\end{cases} \tag{3}$$

where $z(t)$ and $v(t)$ are introducted to separate the state and control variables between the optimization and parameter estimation problems. The terms proportional to r_1 and r_2 are introduced as convexification terms.

Application of the maximum principle for optimal control to (3), produces the following subsets of the necessary optimality conditions:

$$\begin{cases}\dot{x}(t)=A(t)x(t)+B(t)u(t)+\alpha(t),\\ \dot{p}(t)=-\bar{Q}(t)x(t)-A^\tau(t)p(t)+\beta(t),\\ u(t)=-\bar{R}^{-1}(t)(B^\tau(t)p(t)-\lambda(t)),\\ x(t_0)=x_0,\quad p(t_f)=\Phi x(t_f),\end{cases} \tag{4}$$

$$\alpha(t)=f^*(z(t),v(t),t)-A(t)z(t)-B(t)v(t), \tag{5}$$

$$\begin{cases}\lambda(t)=\left[B(t)-\frac{\partial f^*}{\partial v(t)}\right]^\tau\bar{p}(t)+R(t)v(t)-\frac{\partial L^*}{\partial v(t)},\\ \beta(t)=\left[A(t)-\frac{\partial f^*}{\partial z(t)}\right]^\tau\bar{p}(t)+Q(t)z(t)-\frac{\partial L^*}{\partial z(t)},\end{cases} \tag{6}$$

$$v(t)=u(t),\quad z(t)=x(t) \tag{7}$$

where $\bar{Q}=Q+r_2I_n,\bar{R}=R+r_1I_m,p(t)\in\mathbb{R}^n$ is the costate vector, and $\lambda(t)\in\mathbb{R}^m,\beta(t)\in\mathbb{R}^n$ are modifiers. $\bar{P}(t)$ is introduced to separate the model-based optimal control problem and the modifiers calculation. Optimality conditions (4) are satisfied by solving the modified model based optimal control problem (MMOP):

$$\begin{cases}\min_{u(t)}\{\frac{1}{2}x^\tau(t_f)\Phi x(t_f)+\int_{t_0}^{t_f}\frac{1}{2}[u^\tau(t)R(t)u(t)+\\ x^\tau(t)Q(t)x(t)+2\gamma(t)+\\ r_1\|u(t)-v(t)\|^2+r_2\|x(t)-z(t)\|^2-\\ 2\lambda^\tau(t)u(t)-2\beta^\tau(t)x(t)]\mathrm{d}t\}\\ \text{s.t.}\begin{cases}\dot{x}(t)=A(t)x(t)+B(t)u(t)+\alpha(t),\\ x(t_0)=x_0\end{cases}\end{cases} \tag{8}$$

under the specified parameters $\alpha(t),\gamma(t)$, specified modifiers $\lambda(t),\beta(t)$, and specified $z(t),v(t)$. Let

$$p(t)=G(t)x(t)+g(t). \tag{9}$$

From (4) and (9), the solution of (4) can be obtained by the following Riccati equations:

$$\begin{cases}\dot{G}(t)=G(t)B(t)R^{-1}(t)B^\tau(t)G(t)-\\ \qquad A^\tau(t)G(t)-G(t)A(t)-Q(t),\\ \dot{g}(t)=-A^\tau(t)g(t)-A^\tau(t)\alpha(t)+\\ \qquad G(t)B(t)R^{-1}(t)B^\tau(t)g(t)+\\ \qquad G(t)B(t)\lambda(t),\\ G(t_f)=\Phi,\quad g(t_f)=0\end{cases} \tag{10}$$

where $G(t)\in\mathbb{R}^{n\times n},g(t)\in\mathbb{R}^n$.

The above analysis gives rise to the following algorithm of DISOPE based on the time-variant linear quadratic problem.

Assume that $A(t),B(t),Q(t),R(t)$ and $R^{-1}(t)$ are known and L^*,f^* and their derivatives are computable.

Step 0^0 Compute a nominal solution $u^0(t),x^0(t)$, $p^0(t)$, let $z^0(t)=x^0(t),v^0(t)=u^0(t),\bar{P}^0(t)=p^0(t)$ and $i=0$.

Step 1^0 Compute the parameters $a^i(t)$ which satisfy

(5).

Step 2^0 Use (6) to compute the modifiers $\lambda^i(t)$, $\beta^i(t)$.

Step 3^0 With specified $\alpha^i(t), \gamma^i(t), \lambda^i(t), \beta^i(t)$, $t \in [t_0, t_f]$, solve the MMOP defined by (8) to obtain $\hat{u}^i(t), \hat{x}^i(t), \hat{p}^i(t)$.

Step 4^0 A simple relaxation method below is employed to satisfy (7):

$$\begin{cases} v^{i+1}(t) = v^i(t) + k_v(\hat{u}^i(t) - v^i(t)), \\ z^{i+1}(t) = z^i(t) + k_z(\hat{x}^i(t) - z^i(t)), \\ \bar{p}^{i+1}(t) = \bar{p}^i(t) + k_p(\hat{p}^i(t) - \bar{p}^i(t)) \end{cases} \quad (11)$$

where $k_v, k_z, k_p \in (0,1]$ are scalar gains. If $v^{i+1}(t) = v^i(t)$, $t \in [t_0, t_f]$, within a defined tolerance, stop, else let $i = i + 1$, continue from Step 1^0.

3 Convergence and optimization analysis of the algorithm

Fristly, we make the following assumptions:

A1 The optimal solution for ROP given by $x^*(t)$, $u^*(t)$ exists and is unique in the time interval $[t_0, t_f]$, where t_f is a specified constant.

A2 $f^*(x(t), u(t), t)$ is continuously differentiable function of $x(t), u(t)$. $L^*(x(t), u(t), t)$ is continuously differentiable function of $x(t), u(t)$.

A3 $A(t), B(t), Q(t), R(t)$ are all continuous on the interval $t \in [t_0, t_f]$, and $R^{-1}(t)$ are all bounded.

Remark 1 Assumption A3 guarantees the existence of the solution of MMOP(8)[6].

3.1 Optimization

Theorem 1 Under the assumption A1 ~ A3 and assuming the convergence of the algorithm, the converged solution of the algorithm satisfies the optimality necessary conditions of the real optimal control prolbem defined by (1).

Proof From the Maximum principle[5], and the above derivatives of the algorithm, the conclusion is readily obtained.

3.2 The algorithm mapping

Without loss of generality, we will consider the special case of $\Phi(\cdot) = 0$ and $\Phi = 0$. The transition from iteration i to iteration $i+1$ of the algorithm can be expressed in terms of the equations of (4), (5), (6) and (11).

From (4), we have

$$\begin{bmatrix} \dot{\hat{x}}^i(t) \\ \dot{\hat{p}}^i(t) \end{bmatrix} = \begin{bmatrix} A(t) & -B(t)\bar{R}^{-1}(t)B^{\mathrm{T}}(t) \\ -\bar{Q}(t) & -A^{\mathrm{T}}(t) \end{bmatrix} \cdot \begin{bmatrix} \hat{x}^i(t) \\ \hat{p}^i(t) \end{bmatrix} + H_1 y^i(t) + g(y^i(t)) \quad (12)$$

where $y^i(t) = [v^i(t)^{\mathrm{T}}, z^i(t)^{\mathrm{T}}, \bar{p}^i(t)^{\mathrm{T}}]^{\mathrm{T}}$ and

$$\begin{cases} H_1 = \begin{bmatrix} r_1 B(t)\bar{R}(t)^{-1} & 0 & 0 \\ 0 & r_2 I_n & 0 \end{bmatrix}, \\ g(y^i(t)) = \begin{bmatrix} g_1(y^i(t)) \\ g_2(y^i(t)) \end{bmatrix}, \\ g_1(y^i(t)) = B(t)\bar{R}(t)^{-1}\lambda^i(t) + \alpha^i(t), \\ g_2(y^i(t)) = \beta^i(t). \end{cases} \quad (13)$$

Here $g(y^i(t))$ represents the model-reality differences.

The solution of equation (12) gives as:

$$\begin{bmatrix} \hat{x}^i(t) \\ \hat{p}^i(t) \end{bmatrix} = \Psi(t, t_0) \begin{bmatrix} x_0 \\ \hat{p}^i(t_0) \end{bmatrix} + \int_{t_0}^{t} \Psi(t, \tau)(H_1 y^i(\tau) + g(y^i(\tau))\mathrm{d}\tau, \quad \hat{p}^i(t_f) = 0 \quad (14)$$

where $\Psi(t, t_0) = \begin{bmatrix} \varphi_{11}(t, t_0) & \varphi_{12}(t, t_0) \\ \varphi_{21}(t, t_0) & \varphi_{22}(t, t_0) \end{bmatrix} = \begin{bmatrix} \varphi_1(t, t_0) \\ \varphi_2(t, t_0) \end{bmatrix}$ is the transition matrix relative to the vecto $\begin{bmatrix} \hat{x}^i(t) \\ \hat{p}^i(t) \end{bmatrix}$.

By using the transversality conditions, from (14), w obtain

$$\begin{bmatrix} \hat{x}^i(t) \\ \hat{p}^i(t) \end{bmatrix} = \begin{bmatrix} \mu_x(t, t_f, t_0) \\ \mu_p(t, t_f, t_0) \end{bmatrix} x_0 - \begin{bmatrix} \varphi_{12}(t, t_0) \\ \varphi_{22}(t, t_0) \end{bmatrix} \varphi_{22}(t_f, t_0)^{-1} \cdot \int_{t_0}^{t_f} \varphi_2(t_f, \tau)(H_1 y^i(\tau) + g(y^i(\tau))\mathrm{d}\tau + \int_{t_0}^{t} \Psi(t, \tau)(H_1 y^i(\tau) + g(y^i(\tau)))\mathrm{d}\tau \quad (15)$$

where

$$\mu_x(t, t_f, t_0) = \varphi_{11}(t, t_0) - \varphi_{12}(t, t_0)\varphi_{22}^{-1}(t_f, t_0)\varphi_{21}(t_f, t_0),$$

$$\mu_p(t, t_f, t_0) = \varphi_{21}(t, t_0) - \varphi_{22}(t, t_0)\varphi_{22}^{-1}(t_f, t_0)\varphi_{21}(t_f, t_0).$$

From (4) we get

$$\hat{u}^i(t) = \mu_u(t,t_f,t_0)x_0 + \bar{R}^{-1}B^{\mathrm{T}}(t)\varphi_{22}(t,t_0)\varphi_{22}^{-1}(t_f,t_0)\int_{t_0}^{t_f}\varphi_2(t_f,\tau)(H_1y^i(\tau) + g(y^i(\tau)))\mathrm{d}\tau - \bar{R}^{-1}B^{\mathrm{T}}(t)\int_{t_0}^{t}\varphi_2(t,\tau)(H_1y^i(\tau) + g(y^i(\tau)))\mathrm{d}\tau \tag{16}$$

where $\mu_u(t,t_f,t_0) = -\bar{R}^{-1}(t)B^{\mathrm{T}}(t)\mu_p(t,t_f,t_0)$.

Combining (15), (16), we obtain

$$\hat{y}^i(t) = \mu(t,t_f,t_0)x_0 - \int_{t_0}^{t_f}\Omega(t,t_f,\tau,t_0)(H_1y^i(\tau) + g(y^i(\tau)))\mathrm{d}\tau + H_2y^i(t) + d(y^i(t)) \tag{17}$$

where

$$\begin{cases} \mu(t,t_f,t_0) = \begin{bmatrix}\mu_u(t,t_f,t_0)\\ \mu_x(t,t_f,t_0)\\ \mu_p(t,t_f,t_0)\end{bmatrix}, \quad \eta(t_f,t_0,\tau) = \varphi_{22}^{-1}(t_f,t_0)\varphi_2(t_f,\tau), \\ \Omega(t,t_f,\tau,t_0) = \begin{cases}\begin{bmatrix}-\bar{R}^{-1}B^{\mathrm{T}}(t)\varphi_{22}(t,t_0)\eta(t_f,t_0,\tau) + \bar{R}^{-1}B^{\mathrm{T}}(t)\varphi_2(t,\tau)\\ \varphi_{12}(t,t_0)\eta(t_f,t_0,\tau) - \varphi_1(t,\tau)\\ \varphi_{22}(t,t_0)\eta(t_f,t_0,\tau) - \varphi_2(t,\tau)\end{bmatrix}, & t_0 \leqslant \tau \leqslant t,\\ \begin{bmatrix}-\bar{R}^{-1}B^{\mathrm{T}}(t)\varphi_{22}(t,t_0)\eta(t_f,t_0,\tau)\\ \varphi_{12}(t,t_0)\eta(t_f,t_0,\tau)\\ \varphi_{22}(t,t_0)\eta(t_f,t_0,\tau)\end{bmatrix}, & t \leqslant \tau \leqslant t_f,\end{cases} \\ H_2 = \begin{bmatrix} r_1\bar{R}^{-1}(t) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, \quad d(y^i(t)) = \begin{bmatrix}\bar{R}^{-1}(t)\lambda^i(t)\\ 0\\ 0\end{bmatrix}. \end{cases} \tag{18}$$

From (11) and (17), the algorithm mapping is obtained as follows

$$y^{i+1}(t) = K\hat{y}^i(t) + (I_{2n+m} - K)y^i(t) = K\mu(t,t_f,t_0) - K\int_{t_0}^{t_f}\Omega(t,t_f,\tau,t_0)(H_1y^i(\tau) + g(y^i(\tau)))\mathrm{d}\tau + (KH_2) + I_{2n+M} - K)y^i(t) + Kd(y^i(t)) \tag{19}$$

where $K = \text{block diag}\{k_vI_m, k_zI_n, k_pI_n\}$.

3.3 Convergence of the algorithm

The following additional assumption is required.

A4 The function $g(y(t))$ and $d(y(t))$ defined by (13), (18) and they are Lipschitz continuous for all $y(t), t \in [t_0, t_f]$, with Lipschitz constants h_1 and h_2 respectively, that is

$$\begin{cases}\| g(y^i(t)) - g(y^{i-1}(t)) \| \leqslant h_1 \| y^i(t) - y^{i-1}(t) \|,\\ \| d(y^i(t)) - d(y^{i-1}(t)) \| \leqslant h_2 \| y^i(t) - y^{i-1}(t) \|,\end{cases} \tag{20}$$

where $\| y(t) \| = \sup_{t\in[t_0,t_f]} \| y(t) \|_2$, $\| y(t) \|_2 = \sqrt{y(t)^{\mathrm{T}}y(t)}$.

Theorem 2 A sufficient condition of the convergence for the algorithm mapping (19) is given by the following expression

$$(\omega_1(t_f,t_0) + h_2\omega_2(t_f,t_0))(t_f - t_0) + k_vh_2 + \| KH_2 + I - K \| = \sigma < 1 \tag{21}$$

where h_1 and h_2 are defined by (20), K and H_2 are defined by (19), (18), and

$$\begin{cases}\omega_1(t_f,t_0) = \sup_{t\in[t_0,t_f]}\sup_{t\in[t_0,t_f]} \| K\Omega(t,t_f,\tau,t_0)H_1 \|,\\ \omega_2(t_f,t_0) = \sup_{t\in[t_0,t_f]}\sup_{\tau\in[t_0,t_f]} \| K\Omega(t,t_f,\tau,t_0) \|.\end{cases} \tag{22}$$

Proof From (19) we have

$$y^{i+1}(t) - y^i(t) = \int_{t_0}^{t_f}K\Omega(t,t_f,\tau,t_0)(H_1(y^i(\tau) - y^{i-1}(\tau)) + g(y^i(\tau)) - g(y^{i-1}(\tau)))\mathrm{d}\tau + (KH_2 + I_{2n+M} - K)(y^i(t) - y^{i-1}(t)) + K(d(y^i(t)) - d(y^{i-1}(t))). \tag{23}$$

Taking the norm for (23), and using the properties of norm, we obtain

$$\| y^{i+1}(t) - y^{i}(t) \| \leqslant$$

$$\| \int_{t_0}^{t_f} K\Omega(t,t_f,\tau,t_0)H_1(y^i(\tau) - y^{i-1}(\tau))\mathrm{d}\tau \| +$$

$$\| \int_{t_0}^{t_f} K\Omega(t,t_f,\tau,t_0)(g(y^i(\tau)) - g(y^{i-1}(\tau)))\mathrm{d}\tau \| +$$

$$\| (KH_2 + I_{2n+M} - K)(y^i(t) - y^{i-1}(t)) \| +$$

$$\| K(d(y^i(t)) - d(y^{i-1}(t))) \| .$$

From (20), (21), and (17), (19), we get

$$\| y^{i+1}(t) - y^{i}(t) \| \leqslant$$

$$\{(\omega_1(t_f,t_0) + h_2\omega_2(t_f,t_0))(t_f - t_0) +$$

$$k_v h_2 + \| KH_2 + I - K \| \} \| y^i(t) - y^{i-1}(t) \| .$$

Therefore, if condition (21) is satisfied the algorithm mapping of (19) is contractive mapping, and then $\{y^i(t)\}$ converges uniformly.

4 Simulation study

Consider the real optimal control problem as follows

$$\frac{1}{2}\int_0^1 (\| x(t) \|_Q^2 + \| u(t) \|_R^2)\mathrm{d}t$$

$$\text{s.t.}\ \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2-5t & -3-2t \end{bmatrix} x(t) +$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + 0.5x(t)u(t) + \begin{bmatrix} x_1^2(t)x_2(t) \\ x_1(t)x_2^2(t) \end{bmatrix},$$

$$x(0) = [1 \quad 1]^{\mathrm{T}}.$$

Model-based optimal is chosen as the following form

$$\frac{1}{2}\int_0^1 (\| x(t) \|_Q^2 + \| u(t) \|_R^2)\mathrm{d}t$$

$$\text{s.t.}\ x(t) = \begin{bmatrix} 0 & 1 \\ -2-5t & -3-2t \end{bmatrix} x(t) +$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \alpha(t),$$

$$x(0) = [1 \quad 1]^{\mathrm{T}}.$$

Using MATLAB to make simulation test, choose erro precision as eps = 10^{-4}, sample period as 0.02, $Q = I_2$, $R = 1$ and relaxation factors as $k_v = 0.5$, $k_p = k_z =$ 1. By 12 iterations the algorithm converged, the optima state and control curves, error curve and performance in dex are shown in Fig. 1. The iterative number of the al gorithm is 5 times less than of [4]. Therefore, the algo rithm can reduce the number of iteration.

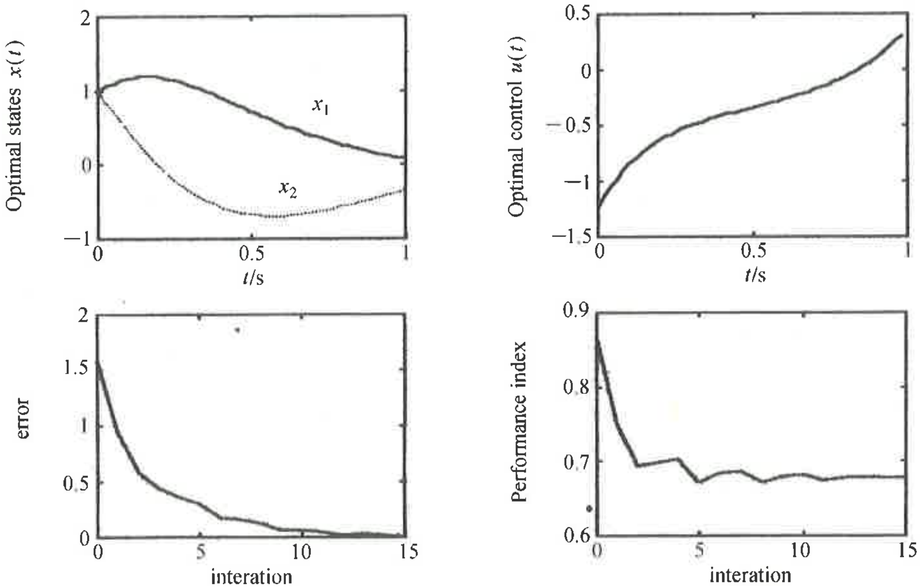

Fig. 1 Optimal states, optimal control, error and performance index curves

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