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Delay-Independent Stability Criteria for Single and Composite Linear Systems with Multiple Time-Varying Delays *

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Abstract: This paper presents the sufficient stability criteria for single and composite linear systems with multiple time-varying delays respectively. The results are derived by using Razumikhin-type theorem together with an algebraic inequality technique. It also provides some approaches for determining the free parameter matrices and scalars in the established criteria. The obtained criteria do not depend on the quantity or derivative of delays. Therefore, they are suitable for systems either with time-varying delays or with constant delays or without delay. Finally, some examples and remarks are given to illustrate the applications of the proposed methods and to compare the obtained results with the existing ones in the literature.

Key words: single and composite systems; multiple time-varying delays; stability criteria optimization

具有多时变时滞的单一和复合线性系统的时滞无关稳定性判据

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摘要: 针对单一和复合多时变时滞系统分别建立了稳定性的充分条件.结果的推导采用了拉什密辛(Razumikhin)定理结合代数不等式的方法.同时,本文提供了一些优化稳定性判据中自由参数的方法.所建判据既不依赖于时滞的大小也不依赖时滞的导数,故既适用于时变时滞系统也适用于常时滞系统或无时滞系统.最后,给出了一些例子说明所提方法的应用,并比较文献中存在的结果.

关键词: 单一和复合系统; 多时变时滞; 稳定性判据优化

Nomenclature

$\mathbb{R}^n$: real vector space of dimension n,

$\mathbb{R}^{n\times n}$: real matrix space of dimension $n\times n$,

A^{T} : transpose of $A \in \mathbb{R}^{n\times n}$,

$\lambda(A)$: eigenvalue of $A \in \mathbb{R}^{n\times n}$,

$\lambda_m(A)$: minimum eigenvalue of $A \in \mathbb{R}^{n\times n}$,

$\lambda_M(A)$: maximum eigenvalue of $A \in \mathbb{R}^{n\times n}$,

$A > 0$: symmetric and positive definite matrix,

$\| x \|$: euclidean norm of $x \in \mathbb{R}^n$: $\| x \| = (x^{\mathrm{T}}x)^{1/2}$,

$\| A \|$: induced norm from $\| x \|$: $\| A \| = [\lambda_M(A^{\mathrm{T}}A)]^{1/2}$,

$\mu(A)$: matrix measure of $A \in \mathbb{R}^{n\times n}$: $\mu(A) = 0.5\lambda_M(A^{\mathrm{T}} + A)$,

$\mathrm{Re}(\lambda(A))$: real part of eigenvalue of $A \in \mathbb{R}^{n\times n}$,

I_n: $n\times n$ identity matrix,

$\forall$: for all.

1 Introduction

In recent years, the stability of time-delay systems has attracted the attention of many researchers. There have been many sufficient stability criteria for various time-delay systems in the literature. All of the existing sufficient criteria usually include some free parameter matrices and/or scalars. Generally speaking, the different choices of these free parameter matrices and/or scalars in the criteria may give different results. The problem is how to choose these free parameter matrices and/or scalars so that the least conservative results can be obtained. So far, many researchers have been seeking various methods of reducing the conservativeness for their sufficient stability criteria, see, for example, [1 ~ 11] and references therein.

In this paper, we establish delay-independent stability criteria for single and composite linear systems with

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multiple arbitrary unknown and time-varying delays by using the Razumikhin-type theorem[12] together with an algebriac inequality technique (see Lemma 1 in the next section). We also discuss how to choose appropriately the free parameter matrices and scalars in the established stability criteria and provide some methods for determining these free parameter matrices and scalars so that as less conservative results as possible can be obtained. The paper is organized as follows: the stability criteria for the systems under consideration are derived and the methods of choosing the free parameter matrices and scalars are provided in Section 2, four examples and some remarks are given in Section 3 to illustrate the proposed methods and to compare the obtained results with those in the literature. The conclusion is given in Section 4.

2 Stability criteria

2.1 Single system with multiple time-varying delays

Let us consider a single linear system with multiple time-varying delays as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + \sum_{k=1}^{m} B_k x(t - \tau_k(t)), \ t \geqslant 0, \\ x(t) = \phi(t), \ t \in [-r, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $A \in \mathbb{R}^{n\times n}$ and $B_k \in \mathbb{R}^{n\times n}$ are constant matrices, $0 \leqslant \tau_k(t) \leqslant r < \infty$ denotes arbitrary unknown and time-varying delay, $r > 0$ is a positive number, and $\phi(t)$ denotes a continuous vector-valued initial function on $t \in [-r, 0]$. We assume that A is stable. Then the Lyapunov equation

$$PA + A^{\mathrm{T}}P = -Q \tag{2}$$

has the unique $n \times n$ symmetric and positive difinite solution P, where Q is an $n \times n$ symmetric and positive definite matrix.

Lemma 1[10] For any positive constant $\varepsilon > 0$ and any constant matrix $T \in \mathbb{R}^{n\times m}$, we have

$$2u^{\mathrm{T}}Tv \leqslant \varepsilon u^{\mathrm{T}}TD^{-1}T^{\mathrm{T}}u + \varepsilon^{-1}v^{\mathrm{T}}Dv, \quad u \in \mathbb{R}^n, v \in \mathbb{R}^m \tag{3}$$

where $D \in \mathbb{R}^{m\times m}$ is a symmetric and positive definite matrix.

Theorem 1 System (1) is asymptotically stable if

$$\lambda_m(\varepsilon Q - \varepsilon^2 mP\sum_{k=1}^{m} B_k P^{-1} B_k^{\mathrm{T}} P - P) > 0 \tag{4}$$

where $P > 0$ and $Q > 0$ satisfy the Lyapunov equation (2) and $\varepsilon > 0$ is a positive number.

Proof Let

$$V(x(t)) = \varepsilon x^{\mathrm{T}}(t)Px(t) \tag{5}$$

where P and ε are defined as in (4). Along the trajectory of system (1) and by Lemma 1 and (2), we obtain

$$\begin{aligned} &\dot{V}(x(t)) = \\ &2\varepsilon x^{\mathrm{T}}(t)PAx(t) + 2\varepsilon x^{\mathrm{T}}(t)P\sum_{k=1}^{m} B_k x(t - \tau_k(t)) \leqslant \\ &-\varepsilon x^{\mathrm{T}}(t)Qx(t) + \varepsilon^2 m x^{\mathrm{T}}(t)P\sum_{k=1}^{m} B_k P^{-1} B_k^{\mathrm{T}} Px(t) + \\ &m^{-1}\sum_{k=1}^{m} x^{\mathrm{T}}(t - \tau_k(t))Px(t - \tau_k(t)). \end{aligned} \tag{6}$$

Note that (5) and the following Razumikhin condition[12]:

$$V(x(s+\theta)) < qV(x(s)), \ s \in [0, \infty), \quad \forall \theta \in [-r, 0], q > 1 \tag{7}$$

imply

$$0 < qx^{\mathrm{T}}(s)Px(s) - m^{-1}\sum_{k=1}^{m} x^{\mathrm{T}}(s - \tau_k(s))Px(s - \tau_k(s)) \tag{8}$$

where $q > 1$ is a constant. By condition (7) and (8), we further obtain at $t = s$

$$\begin{aligned} &\dot{V}(x(s)) < \\ &-x^{\mathrm{T}}(s)(\varepsilon Q - \varepsilon^2 mP\sum_{k=1}^{m} B_k P^{-1} B_k^{\mathrm{T}} P - qP)x(s) \leqslant \\ &-\lambda_m(\varepsilon Q - \varepsilon^2 mP\sum_{k=1}^{m} B_k P^{-1} B_k^{\mathrm{T}} P - qP)\,\| x(s) \|^2. \end{aligned} \tag{9}$$

If condition (4) holds, then there exists a constant $q > 1$ satisfying

$$\rho = \lambda_m(\varepsilon Q - \varepsilon^2 mP\sum_{k=1}^{m} B_k P^{-1} B_k^{\mathrm{T}} P - qP) > 0 \tag{10}$$

such that $\dot{V}(x(s)) < -\rho \| x(s) \|^2$. By Razumikhin-type theorem[12, Chapter 5, Theorem 4.2], we complete the proof.

Remark 1 As mentioned in the introduction, the different choices of the free parameter matrix Q and the scalar ε in the criterion (4) may give the different upper bounds. Therefore, it is more meaningful to find an approach of choosing appropriate Q and ε so that the maximum robustness bound can be obtained. To do so, we propose an approach by solving following minimization problem:

$$\min_{Q>0,\varepsilon>0}\{\lambda_m(\varepsilon Q - \varepsilon^2 mP\sum_{k=1}^{m} B_k P^{-1} B_k^{\mathrm{T}} P - P)\} \geqslant$$

$$e > 0 \quad (11)$$

where $P > 0$ and $Q > 0$ satisfy the Lyapunov equation (2), $\varepsilon > 0$ is a positive number and e is a fixed positive number. But as we know, $\lambda_m(U_1) = \lambda_m(U_2)$ does not certainly mean $U_1 = U_2$ for $U_1 > 0$ and $U_2 > 0$. In fact, an $n \times n$ symmetric and positive definite matrix $U \in \mathbb{R}^{n\times n}$ has $n(n+1)/2$ free parameters and the problem of $\eta^* = \lambda_m(U)$ with one fixed number η^* has infinite solution $U^* > 0$ for $n \geqslant 2$ case. Obviously, the choice of Q^* and ε^* to minimize the problem (11) is not unique[10]. Besides, as problem (11) provides $n(n+1)/2+1$ free parameters, this implies that if there exist solutions Q^* and ε^* to problem (11), we must have

$$\lambda_m(\varepsilon^* Q^* - (\varepsilon^*)^2 mP\sum_{k=1}^{m} B_k P^{-1} B_k^{\mathrm{T}} P - P) = e > 0,$$

where $P > 0$ is the solution of the Lyapunov equation (2) with $Q^* > 0$. Now, Let $e = 0.0001$ denote the tolerance. Then, criterion (11) is better than (4) because the conservativeness in criterion (4) has been reduced as less as possible and (11) also provides an approach for determining Q^* and ε^*. The problems like (11) can be solved by using some well-known optimization tools, such as MATLAB Optimization Toolbox which have been used worldwide.

If considering system (1) with the structured perturbations as follows:

$$B_k = \beta_k(t) E_k,\ k = 1,2,\cdots,m \quad (12)$$

where $E_k \in \mathbb{R}^{n\times n}$ is a constant matrix and $\beta_k(t)$ is the time-varying uncertain parameter. We have the following corollary for robustness stability bounds.

Corollary 1 System (1) with the structured perturbations (12) is asymptotically stable if

$$\sum_{k=1}^{m}\beta_k^2(t) < \max_{Q>0,\varepsilon>0}\{m\lambda_m(\varepsilon P^{-1/2} Q P^{-1/2} - \varepsilon^2 mP^{1/2}\sum_{k=1}^{m} E_k P^{-1} E_k^{\mathrm{T}} P^{1/2})\} \quad (13a)$$

for all $t \geqslant 0$, or

$$|\beta_k(t)| < \max_{Q>0,\varepsilon>0}\{\lambda_m^{1/2}(\varepsilon P^{-1/2} Q P^{-1/2} - \varepsilon^2 mP^{1/2}\sum_{k=1}^{m} E_k P^{-1} E_k^{\mathrm{T}} P^{1/2})\},\ k = 1,2,\cdots,m \quad (13b)$$

for all $t \geqslant 0$, where $P > 0$ and $Q > 0$ satisfy the Lyapunov equation (2) and $\varepsilon > 0$ is a positive number.

Proof With a similar proof of Theorem 1, we start from (6) and obtain

$$\dot V(x(t)) \leqslant -\varepsilon x^{\mathrm{T}}(t) Q x(t) + \varepsilon^2 m x^{\mathrm{T}}(t) P\sum_{k=1}^{m} E_k P^{-1} E_k^{\mathrm{T}} P x(t) + m^{-1}\sum_{k=1}^{m}\beta_k^2(t) x^{\mathrm{T}}(t-\tau_k(t)) P x(t-\tau_k(t)). \quad (14)$$

Then, condition (7) implies

$$0 < qm^{-1}\sum_{k=1}^{m}\beta_k^2(s) x^{\mathrm{T}}(s) P x(s) - m^{-1}\sum_{k=1}^{m}\beta_k^2(s) x^{\mathrm{T}}(s-\tau_k(s)) P x(s-\tau_k(s)) \quad (15)$$

where $q > 1$ is a constant. By condition (7) and (15), we can further obtain at $t = s$

$$\begin{aligned}\dot V(x(s)) <\ & -\varepsilon x^{\mathrm{T}}(s) Q x(s) + \varepsilon^2 m x^{\mathrm{T}}(s) P\sum_{k=1}^{m} E_k P^{-1} E_k^{\mathrm{T}} P x(s) + qm^{-1}\sum_{k=1}^{m}\beta_k^2(s) x^{\mathrm{T}}(s) P x(s) \leqslant \\ & -\lambda_m[P^{1/2}(\varepsilon P^{-1/2} Q P^{-1/2} - \varepsilon^2 mP^{1/2}\sum_{k=1}^{m} E_k P^{-1} E_k^{\mathrm{T}} P^{1/2} - qm^{-1}\sum_{k=1}^{m}\beta_k^2(s) I_n) P^{1/2}]\,\| x(s)\|^2 = \\ & -\lambda_m[P(\varepsilon P^{-1/2} Q P^{-1/2} - \varepsilon^2 mP^{1/2}\sum_{k=1}^{m} E_k P^{-1} E_k^{\mathrm{T}} P^{1/2} - qm^{-1}\sum_{k=1}^{m}\beta_k^2(s) I_n)]\,\| x(s)\|^2 \leqslant \\ & -\lambda_m(P)[\lambda_m(\varepsilon P^{-1/2} Q P^{-1/2} - \varepsilon^2 mP^{1/2}\sum_{k=1}^{m} E_k P^{-1} E_k^{\mathrm{T}} P^{1/2}) - qm^{-1}\sum_{k=1}^{m}\beta_k^2(s)]\,\| x(s)\|^2. \end{aligned} \quad (16)$$

If (13a) holds for all $t \geqslant 0$, it also holds at $t = s$. Then, there exists a constant $q > 1$ satisfying

$$\omega = \lambda_m(\varepsilon^* P^{-1/2} Q^* P^{-1/2} - (\varepsilon^*)^2 mP^{1/2}\sum_{k=1}^{m} E_i P^{-1} E_i^{\mathrm{T}} P^{1/2}) - qm^{-1}\sum_{k=1}^{m}\beta_k^2(s) > 0 \quad (17)$$

such that $\dot V(x(s)) < -\lambda_m(P)\omega\,\| x(s)\|^2$, $\lambda_m(P)\omega > 0$, where Q^* and ε^* are the solution to the maximization problem (13a). Condition (13b) can be easily proved by the same procedure as above. According to the proof of Theorem 1, we complete the proof of this corollary.

2.2 Composite system with multiple time-varying delays

Now, we consider a composite linear time-delay system consisting of N interconnected subsystems as follows:

$$\dot{x}_i(t) = A_i x_i(t) + \sum_{j=1}^{N} A_{ij} x_j(t - \tau_{ij}(t)),$$
$$i = 1,2,\cdots,N \tag{18}$$

where $x_i(t) \in \mathbb{R}^{ni}$, $\sum_{i=1}^{N} n_i = n$, A_i and A_{ij} are constant matrices with appropriate dimensions, and $0 \leqslant \tau_{ij}(t) \leqslant r < \infty$ denotes arbitrary unknown and time-varying delay. We assume that A_i is stable for each i. Then the Lyapunov equation

$$P_i A_i + A_i^{\mathrm{T}} P_i = -Q_i \tag{19}$$

has the unique $n_i \times n_i$ symmetric and positive definite solution P_i for each i, where Q_i is an $n_i \times n_i$ symmetric and positive definite matrix.

Theorem 2 System (18) is asymptotically stable if

$$\lambda_m(\varepsilon_i Q_i - \varepsilon_i^2 N P_i \sum_{j=1}^{N} A_{ij} P_j^{-1} A_{ij}^{\mathrm{T}} P_i - P_i) > 0,$$
$$\forall i = 1,2,\cdots,N \tag{20}$$

where $P_i > 0$ and $Q_i > 0$ satisfy the Lyapunov equation (19) and $\varepsilon_i > 0$ is a positive number.

Proof Let

$$V(x(t)) = \sum_{i=1}^{N} \varepsilon_i x_i^{\mathrm{T}}(t) P_i x_i(t) \tag{21}$$

where $x(t) \in \mathbb{R}^n$, P_i is the solution of (19) and $\varepsilon_i > 0$ is a positive number. Along the trajectory of system (18) and by Lemma 1 and (19), we obtain

$$\begin{aligned}\dot{V}(x(t)) = &\sum_{i=1}^{N}[2\varepsilon_i x_i^{\mathrm{T}}(t) P_i A_i x_i(t) + \\ &2\varepsilon_i x_i^{\mathrm{T}}(t) P_i \sum_{j=1}^{N} A_{ij} x_j(t - \tau_{ij}(t))] \leqslant \\ &\sum_{i=1}^{N}[-\varepsilon_i x_i^{\mathrm{T}}(t) Q_i x_i(t) + \\ &\varepsilon_i^2 N x_i^{\mathrm{T}}(t) P_i \sum_{j=1}^{N} A_{ij} P_j^{-1} A_{ij}^{\mathrm{T}} P_i x_i(t) + \\ &N^{-1} \sum_{j=1}^{N} x_j^{\mathrm{T}}(t - \tau_{ij}(t)) P_j x_j(t - \tau_{ij}(t))].\end{aligned} \tag{22}$$

Note that (7) and (21) imply

$$\begin{aligned}V(x(s+\theta)) = &\sum_{j=1}^{N} \varepsilon_j x_j^{\mathrm{T}}(s+\theta) P_j x_j(s+\theta) < \\ qV(x(s)) = &\\ q\sum_{j=1}^{N} \varepsilon_j &x_j^{\mathrm{T}}(s) P_j x_j(s), s \in [0, \infty), \\ \forall \theta \in &[-r, 0], q > 1\end{aligned} \tag{23}$$

which yields

$$\begin{aligned}0 < qN^{-1} &\sum_{i=1}^{N} x_j^{\mathrm{T}}(s) P_j x_j(s) - N^{-1} \sum_{j=1}^{N} x_j^{\mathrm{T}}(s - \\ &\tau_{ij}(s)) P_j x_j(s - \tau_{ij}(s)), q > 1\end{aligned} \tag{24}$$

for all i, where $q > 1$ is a constant. By using (23) and (24), we further obtain at $t = s$

$$\begin{aligned}\dot{V}(x(s)) < &\sum_{i=1}^{N}[-x_i^{\mathrm{T}}(s)(\varepsilon_i Q_i - \\ &\varepsilon_i^2 N P_i \sum_{j=1}^{N} A_{ij} P_j^{-1} A_{ij}^{\mathrm{T}} P_i) x_i(s) + \\ &qN^{-1} \sum_{j=1}^{N} x_j^{\mathrm{T}}(s) P_j x_j(s)] = \\ &\sum_{i=1}^{N}[-x_i^{\mathrm{T}}(s)(\varepsilon_i Q_i - \\ &\varepsilon_i^2 N P_i \sum_{j=1}^{N} A_{ij} P_j^{-1} A_{ij}^{\mathrm{T}} P_i) \cdot \\ &x_i(s) + q x_i^{\mathrm{T}}(s) P_i x_i(s)] \leqslant \\ &-\sum_{i=1}^{N}[\lambda_m(\varepsilon_i Q_i - \varepsilon_i^2 N P_i \cdot \\ &\sum_{j=1}^{N} A_{ij} P_j^{-1} A_{ij}^{\mathrm{T}} P_i - q P_i) \| x_i(s) \|^2].\end{aligned} \tag{25}$$

If (20) holds, then there exists a constant $q > 1$ satisfying

$$\mu_i = \lambda_m(\varepsilon_i Q_i - \varepsilon_i^2 N P_i \sum_{j=1}^{N} A_{ij} P_j^{-1} A_{ij}^{\mathrm{T}} P_i - q P_i) > 0 \tag{26}$$

for all i such that $\dot{V}(x(s)) < -\min_{i=1,\cdots,N}\{\mu_i\} \| x(s) \|^2$. According to the proof of Theorem 1, we complete the proof of this theorem.

Remark 2 The free parameter matrices Q_i and scalars ε_i in condition (20) can be chosen by solving the following minimization problem:

$$\begin{cases}\min\limits_{Q_i>0,\varepsilon_i>0} \{\lambda_1 + \lambda_2 + \cdots + \lambda_N\} \geqslant 0.0001 \times N \\ \text{subject to}: \lambda_1 = \lambda_m(\varepsilon_i Q_i - \varepsilon_i^2 N P_i \cdot \\ \qquad \sum_{j=1}^{N} A_{ij} P_j^{-1} A_{ij}^{\mathrm{T}} P_i - P_i) \geqslant 0.0001, \\ i = 1,2,\cdots,N\end{cases} \tag{27}$$

where $P_i > 0$ and $Q_i > 0$ satisfy the Lyapunov equation (19) and $\varepsilon_i > 0$ is a positive number.

Remark 3 It can be seen that the results established in Theorem 2 have been derived without using the techniques of the M-matrix or quasi-diagonal dominance.

Remark 4 It can be also seen that from the proofs of Theorem 1, Corollary 1 and Theorem 2 all the results established in this section do not depend on the quantity or derivate of delays. Therefore, the obtained stability criteria are delay-independent and they are also suitable for systems either with time-varying delays or with constant delays or without delay.

3 Examples and Remarks

To show the superiority of our methods, we give in this section four illustrative examples. All the free parameter matrices and scalars in the examples are chosen by using the MATLAB Optimization Toolbox. Some remarks are also given to compare the obtained results with the results in the literature.

Example 1 Consider a 2×2 time-delay system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-\tau(t)),\ t \geqslant 0, \\ x(t) = \phi(t),\ t \in [-r, 0] \end{cases} \tag{28}$$

with

$$A = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0.7313 \\ 0.7313 & 1 \end{bmatrix}.$$

By (11), we obtain $\lambda_m(\varepsilon^* Q^* - (\varepsilon^*)^2 PBP^{-1}B^{\mathrm{T}}P - P) = 0.0001 >$ with

$$Q^* = \begin{bmatrix} 1.7443 & 1.6808 \\ 1.6808 & 1.6256 \end{bmatrix},\ \varepsilon^* = 0.6512.$$

By Theorem 1, the system is asymptotically stable.

Remark 5 As mentioned in Remark 1, there are many different choices of Q^* and ε^* in Example 1. For example, $\lambda_m(\varepsilon^* Q^* - (\varepsilon^*)^2 PBP^{-1} B^{\mathrm{T}} P - P) = 0.0001 > 0$ with

$$Q^* = \begin{bmatrix} 0.9989 & 0.9962 \\ 0.9962 & 0.9949 \end{bmatrix},\ \varepsilon^* = 0.9988.$$

In the following examples, we only give one choice for Q^* and ε^*.

Remark 6 For system (28), the following stability conditions are established in [1,5,10]

$$\text{i) } \|B\| < \frac{[\lambda_m(Q)/\lambda_M(P)]}{2\|T\|\,\|T^{-1}\|\,[\lambda_M(P)/\lambda_m(P)]^{1/2}},$$
$$PT^{-1}AT + (T^{-1}AT)^{\mathrm{T}}P = -Q \tag{29}$$

by Cheres et al.[1].

$$\text{ii) } \|B\| < \frac{\alpha}{\|T\|\,\|T^{-1}\|\,[\lambda_M(P)/\lambda_m(P)]},$$
$$P(T^{-1}AT + \alpha I_n) + (T^{-1}AT + \alpha I_n)^{\mathrm{T}}P = -Q \tag{30}$$

where $0 < \alpha < \min_i\{\,|\,\mathrm{Re}(\lambda_i(A))\,|\,\}$, by Wu and Mizukami[5];

$$\text{iii) } \begin{cases} \|B\| < \dfrac{\lambda_m^{1/2}(Q-P^2)}{\|T\|\,\|T^{-1}\|\,[\lambda_M(P)/\lambda_m(P)]^{1/2}}, \\ PT^{-1}AT + (T^{-1}AT)^{\mathrm{T}}P = -Q \end{cases} \tag{31}$$

by Xu[10]. In (29) ~ (31), T denotes a similarity transformation matrix. The maximum upper bound for $\|B\|$ is $\|B\|^* = 0.3246$ by (29) with

$$Q^* = \begin{bmatrix} 1.6543 & 0.3095 \\ 0.3095 & 0.5787 \end{bmatrix},$$

$$T^* = \begin{bmatrix} 1.5164 & -0.2854 \\ -0.5536 & 1.1382 \end{bmatrix},$$

$\|B\|^* = 0.3246$ by (30) with

$$Q^* = \begin{bmatrix} 2.0754 & 0.4453 \\ 0.4453 & 0.0956 \end{bmatrix},$$

$$T^* = \begin{bmatrix} 2.0818 & -0.5049 \\ -0.6875 & 1.6215 \end{bmatrix},\ \alpha^* = 0.6660,$$

and $\|B\|^* = 0.3246$ by (31) with

$$Q^* = \begin{bmatrix} 3.0851 & 0.2399 \\ 0.2399 & 0.9151 \end{bmatrix},$$

$$T^* = \begin{bmatrix} 0.7735 & -0.2730 \\ -0.1989 & 0.6432 \end{bmatrix},$$

respectively. All of (29) ~ (31) give the same maximum upper bound for $\|B\|$. Note that in Example 1 we have $\|B\| = 1.7313$ and (29) ~ (31) do not hold. This shows that condition (4) is less conservative than (29) ~ (31) when more structure information of B can be taken into account.

Example 2 Consider system (28) with

$$A = \begin{bmatrix} -2 & -1 \\ 0 & -2 \end{bmatrix},$$

$$B = \beta(t)E = \beta(t)\begin{bmatrix} 0 & 1.2 \\ 1 & 1.1 \end{bmatrix}.$$

By (13b), we obtain the robust stability bound $|\beta(t)| < 1.3838$ with

$$Q^* = \begin{bmatrix} 11.0167 & 2.1584 \\ 2.1584 & 6.9280 \end{bmatrix},\ \varepsilon^* = 0.7828.$$

If $\tau(t) = \tau \geqslant 0$, τ is an arbitrary constant delay, a better result is $|\beta(t)| < 1.39297$ obtained by Xu in[10].

Example 3 Consider system (1) with $m = 2$ and

$$A = \begin{bmatrix} -2 & 0 & -1 \\ 0 & -3 & 0 \\ -1 & -1 & -4 \end{bmatrix},$$

$$B_1 = k_1(t)E_1 = k_1(t)\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix},$$

$$B_2 = k_2(t)E_2 = k_2(t)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

By (13a) and (13b), we obtain $k_1^2(t) + k_2^2(t) < 2.8594$ and $|k_i(t)| < 1.1957, i = 1,2$, with

$$Q^* = \begin{bmatrix} 0.2170 & -0.0258 & 0.1061 \\ -0.0258 & 100.0003 & -0.0859 \\ 0.1061 & -0.0859 & 2.1056 \end{bmatrix},$$

$\varepsilon^* = 0.57$.

Remark 7 In [7], Chen et al. derive the following stability criteria for system (1) with multiple constant delays:

i) $\| B_1 \cdots B_m \| + \mu(A)/\sqrt{m} < 0$; (32)

ii) $\| P[B_1 \cdots B_m] \| - 1/\sqrt{m} < 0$; (33)

iii) $\lambda_M[(|R|^{\mathrm{T}} + |R|)/2] - 1 < 0$,
$R = [I_n \cdots I_n]^{\mathrm{T}} P[B_1 \cdots B_m]$; (34)

where $PA + A^{\mathrm{T}}P = -2I_n$. Now let $\tau_i(t) = \tau_i, i = 1,2$, and

$$B_1 = \begin{bmatrix} -1.1956 & 0 & -1.1956 \\ 0 & 0 & 0 \\ -1.1956 & 0 & -1.1956 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1.1956 & 0 \\ 0 & -1.1956 & 0 \end{bmatrix},$$

in Example 3. We have $\| B_1 B_2 \| + \mu(A)/\sqrt{2} = 1.4984 > 0$, $\| P[B_1 B_2] \| - 1/\sqrt{2} = 0.0582 > 0$ and $\lambda_M[(|R|^{\mathrm{T}} + |R|)/2] - 1 = -0.0622 < 0$. Condition (32) and (33) do not hold but condition (34) holds. According to Chen et al.[7], the system with the constant delays is asymptotically stable. By (11), we obtain $\lambda_m(\varepsilon^* Q^* - 2(\varepsilon^*)^2 P \sum_{k=1}^{2} B_k P^{-1} B_k^{\mathrm{T}} P - P) = 2.2257 \times 10^{-6} > 0$ with

$$Q^* = \begin{bmatrix} 0.1866 & -0.0009 & 0.0791 \\ -0.0009 & 100.0000 & -0.1002 \\ 0.0791 & -0.1002 & 1.8261 \end{bmatrix},$$

$\varepsilon^* = 0.4019$.

According to Theorem 1, the system is also asymptotically stable for the case where the delays are time-varying.

Example 4 Consider system (18) with $N = 2$ and

$$A_1 = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}, \quad A_{11} = \begin{bmatrix} 0.2 & 0.0 \\ 0.1 & 0.1 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 0.27 & 0 \\ 0 & 0.25 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -7 & 2 \\ 1 & -5 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 0.1 & 0.7 \\ 1 & 1 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}.$$

This example has been studied by Xu in[6]. By (28), we obtain

$$\begin{cases} \lambda_1^* + \lambda_2^* = 0.0002 > 0, \\ \lambda_1^* = \lambda_m(\varepsilon_1^* Q_1^* - (\varepsilon_1^*)^2 N P_1 \cdot \\ \qquad \sum_{j=1}^{2} A_{1j} P_j^{-1} A_{1j}^{\mathrm{T}} P_1 - P_1) = 0.0001 > 0, \\ \lambda_2^* = \lambda_m(\varepsilon_2^* Q_2^* - (\varepsilon_2^*)^2 N P_2 \cdot \\ \qquad \sum_{j=1}^{2} A_{2j} P_j^{-1} A_{2j}^{\mathrm{T}} P_2 - P_2) = 0.0001 > 0 \end{cases}$$

with

$$Q_1^* = \begin{bmatrix} 1.4729 & 1.1797 \\ 1.1797 & 1.0144 \end{bmatrix}, \quad \varepsilon_1^* = 0.6869,$$

$$Q_2^* = \begin{bmatrix} 2.0094 & -0.3938 \\ -0.3938 & 1.4048 \end{bmatrix}, \quad \varepsilon_2^* = 0.1303.$$

According to Theorem 2, the composite system is asymptotically stable.

4 Conclusion

The sufficient stability criteria for single and composite linear systems with multiple time-varying delays have been derived by using the Razuminkhin-type theorem together with an algebraic inequality technique. The methods of choosing the free parameter matrices and scalars in the established criteria appropriately have been provided. All of the obtained criteria are independent of delays and suitable for the case where delays are time-varying or constant ones. The illustrative examples and remarks are given to show the applications of the methods and to compare the obtained results with the existing ones in the literature.

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本文作者简介

胥布工 见本刊 1999 年第 2 期第 305 页.

(Continued on page 360)

δ- modification algorithm. The similar results are obtained, using fewer restrictive assumptions compared with [4].

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