

# Controlling Integrator Processes with Long Delay

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**Abstract:** A control scheme is proposed in this paper for integrator processes with long delay. Three separate controllers are configured in the scheme for different objectives. With the control scheme, the closed-loop set-point response and load response are decoupled from each other and consequently can be independently optimized. A simple and clear design procedure is presented for the proposed scheme. Simulations show that the proposed scheme has fast set-point tracking, good load rejection and improved robustness to variations in process dynamics over existing control schemes.

**Key words:** integrator process; time delay; control

## 控制大时滞积分过程

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**摘要:** 提出了一种包含三个控制器的大时滞积分过程控制方案, 给出了简单的设计和整定方法. 该方案能分离闭环设定值响应和扰动响应, 从而能独立优化这些响应. 仿真表明, 所提出的控制方案具有快速的设定值跟踪和良好的抗扰动能力, 同时系统鲁棒性也比现有的控制方案大为增强.

**关键词:** 积分过程; 时滞; 控制

## 1 Introduction

A non-self-regulating integrator process is one of the representative industrial processes. A typical example is the liquid-level system of a cylindrical storage/feed tank with a pump attached to the outflow line. Also, self-regulating processes with large time constant can be approximated by an integrator plus delay model for control purpose. This approximation was studied in detail by Chien and Fruehauf through industrial distillation columns<sup>[1]</sup>. The control of integrator processes is, therefore, important in industry.

The conventional PID control is effective for an integrator process without delay or with small delay. But PID control behaves poorly for the process with long delay. The Smith predictor, a popular long delay compensator<sup>[2]</sup>, is ineffective for the integrator process with long delay as it will lead to a steady-state offset for a load disturbance<sup>[3]</sup>. Many efforts have been made to improve the performance of the Smith predictor<sup>[3~6]</sup>. The modified scheme of Watanabe and Ito does not have steady-state offset if the process delay time is exactly

known<sup>[3]</sup>. But the resulting set-point response is too slow, and the steady-state offset can not be removed if the estimate of the process delay time deviates from its real value<sup>[4~6]</sup>. These drawbacks can be overcome by the new Smith predictor of Astrom, Hang and Lim (AHL)<sup>[4]</sup>. AHL scheme also separates the closed-loop set-point response from the load responses. However, the controller tuning of AHL scheme is complicated and there lacks a clear tuning method. The extensions of AHL scheme by Matausek and Micic (MM)<sup>[5]</sup> and Zhang and Sun (ZS)<sup>[6]</sup> have clear tuning procedures. Losing the separation nature of the closed-loop set-point and load responses, MM scheme is simple with only two proportional controllers. Retaining the separation nature mentioned above, ZS scheme contains a local positive feedback loop which is a potential instability source. Either MM scheme or ZS scheme has limited robustness, as will be shown later in simulations.

The objective of this paper is to propose a control scheme with a clear design procedure for integrator processes with long delay. The proposed scheme is expected

to retain the separation nature of the closed-loop set-point and load responses and to have fast set-point tracking, good load rejection, and improved robustness over the existing schemes.

## 2 Control scheme

The proposed control scheme for integrator processes with long delay is shown in Fig. 1. The process dynamics are governed by  $P(s) = G_p(s) e^{-ds}$ , where  $G_p = K_p/s$  does not contain any delay,  $d$  is delay time.  $R$  and  $L$  denote set-point and load respectively.  $y$  denotes the process output. Three controllers  $K_0$ ,  $G_r(s)$  and  $G_c(s)$  are configured respectively for process pre-stabilization, set-point tracking and load rejection. These three controllers,  $K_0$ ,  $G_r(s)$  and  $G_c(s)$ , are thus referred to as pre-stabilizer, set-point controller and load controller respectively by the author.

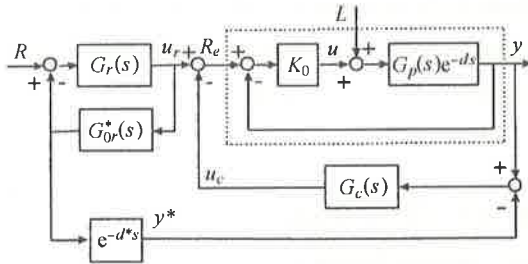


Fig. 1 The proposed control scheme

Once  $K_0$  is determined, the dashed box part of Fig. 1 can be viewed as one block, which is referred to as pre-stabilized process in this paper. Fig. 1 can then be simplified into Fig. 2 with:

$$G_{0r}(s) = \frac{K_0 K_p}{s + K_0 K_p e^{-ds}}, \quad (1)$$

$$G_{0l}(s) = \frac{K_p}{s + K_0 K_p e^{-ds}}.$$

The set-point controller  $G_r(s)$  and the load controller  $G_c(s)$  are then synthesized for the pre-stabilized process. This is the reason why a pre-stabilized process model  $G_{0r}^*(s)e^{-d^*s}$  is contained in the control scheme. The superscript  $*$  indicates that the corresponding variable/parameter is an estimate.

From Fig. 2, we have the transfer functions from  $R$  to  $y$  and from  $L$  to  $y$ , respectively, as:

$$H_r(s) = \frac{Y(s)}{R(s)} = \frac{G_r(s) G_{0r}(s) e^{-ds}}{1 + G_r(s) G_{0r}^*(s) e^{-d^*s}} \cdot \frac{1 + G_c(s) G_{0r}^*(s) e^{-d^*s}}{1 + G_c(s) G_{0r}(s) e^{-ds}}, \quad (2)$$

$$H_l(s) = \frac{Y(s)}{L(s)} = \frac{G_{0l}(s)}{1 + G_c(s) G_{0r}(s) e^{-ds}} e^{-ds}. \quad (3)$$

$H_l(s)$  is determined by  $G_c(s)$  and is independent of  $G_r(s)$ . If the process dynamics are well modeled, i. e.,  $K_p^* \approx K_p$ ,  $d^* \approx d$  and consequently  $G_{0r}^*(s) \approx G_{0r}(s)$ , equation (2) becomes:

$$H_r(s) = \frac{Y(s)}{R(s)} \approx \frac{G_r(s) G_{0r}(s)}{1 + G_r(s) G_{0r}^*(s) e^{-d^*s}} e^{-ds}. \quad (4)$$

$H_r(s)$  is thus determined by  $G_r(s)$  and becomes independent of  $G_c(s)$ . Hence, with a good process model, Fig. 2 can be simplified into Fig. 3.

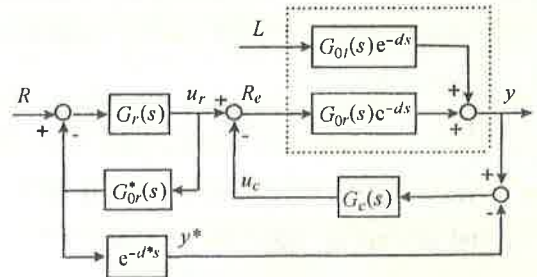


Fig. 2 A simplified structure of Fig. 1

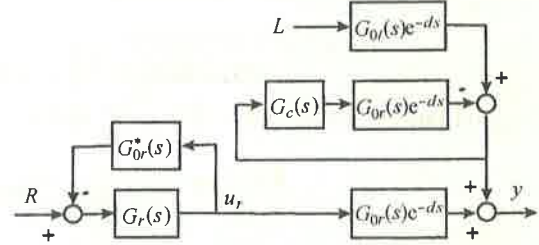


Fig. 3 A simplified structure of Fig. 2 with a good process model

Two feedback loops are clearly seen from Fig. 3. These loops, the lower one and the upper one, are used for set-point tracking and load rejection respectively, resulting in the separation of the closed-loop set-point and load responses. With a well tuned pre-stabilizer, a fast set-point tracking and a good load rejection can thus be simultaneously obtained by independently designing the set-point controller and the load controller. Notice that  $H_r(s) \rightarrow 1$  and  $H_l(s) \rightarrow 0$  as  $s \rightarrow 0$ , implying that there is no steady-state offset in set-point if the closed-loop system is stable.

## 3 Controller tuning

To tune the pre-stabilizer  $K_0$ , a proportional controller, consider the local feedback loop in the dashed box of Fig. 1. The characteristic equation of this local feedback loop is expressed by:

$$1 + W_0(s) = 0, W_0(s) = (K_0 K_p / s) e^{-ds}. \quad (5)$$

The phase margin,  $\Phi_0$ , of the local feedback loop can be computed from the following relations:

$$|W_0(j\omega)| = K_0 K_p / \omega = 1,$$

$$\Phi_0 = \pi + \arg\{W_0(j\omega)\} = \pi/2 - d\omega. \quad (6)$$

Corresponding to  $\Phi_0 = 0$ , the ultimate value of  $K_0$  is  $\pi/(2K_p d)$ . The author recommends designing:

$$K_0 = 1/(2K_p^* d^*). \quad (7)$$

The resulting  $\Phi_0 \approx 61^\circ$ . A relatively large  $\Phi_0$  is taken as  $K_0$  addresses to pre-stabilize the process.

The direct synthesis method is adopted to design the set-point controller  $G_r(s)$ . Considering Fig. 3 and equation (4), assume that the desired transfer function from  $R$  to  $y$ ,  $H_{rd}(s)$ , is:

$$H_{rd}(s) = G_{rd}(s) e^{-ds} = \frac{1}{T_r s + 1} e^{-ds}, \quad (8)$$

where  $T_r$  is the desired time constant. Letting  $H_r(s) = H_{rd}(s)$  and accounting equations (1) and (7), we have:

$$H_l(s) = \frac{sK_p}{s(s + K_0 K_p e^{-ds}) + K_c K_0 K_p (s + K_0 K_p^* e^{-d^* s})(1 + T_{cd}s) e^{-ds}} e^{-ds}. \quad (11)$$

With a good process model, i.e.,  $K_p^* \approx K_p$  and  $d^* \approx d$ , the above equation becomes:

$$H_l(s) = \frac{sK_p}{(s + K_0 K_p e^{-ds})[s + K_c K_0 K_p (1 + T_{cd}s) e^{-ds}]} e^{-ds}. \quad (12)$$

The characteristic equation of the load rejection loop is obtained by letting the denominator of  $H_l(s)$  be equal to zero. There are two factors in this characteristic equation. The first factor has been considered previously for determining the pre-stabilizer  $K_0$ . The second factor reads:

$$1 + W_c(s) = 0, \quad (13)$$

$$W_c(s) = \frac{K_c K_0 K_p}{s} (1 + T_{cd}s) e^{-ds}.$$

This relation can be viewed as a characteristic equation of a unity feedback system with a transfer function  $W_c(s)$  in its forward path. The phase margin  $\Phi_c$  of this imaginary system is determined by:

$$\begin{cases} |W_c(j\omega)| = K_c K_0 K_p \sqrt{1 + (T_{cd}\omega)^2} / \omega = 1, \\ \Phi_c = \pi + \arg\{W_c(j\omega)\} = \\ \pi/2 + \tan^{-1}(T_{cd}\omega) - d\omega. \end{cases} \quad (14)$$

$$G_r(s) = \frac{1}{G_{0r}^*} \cdot \frac{G_{rd}(s)}{1 - G_{rd}(s)} = \frac{2d^*}{T_r} \left(1 + \frac{1}{2d^* s} e^{-d^* s}\right). \quad (9)$$

The resulting  $G_r(s)$  is a nonlinear PI controller with a gain of  $K_r = 2d^*/T_r$  and a variable integral time of  $T_{ri} = 2d^* e^{d^* s}$ . The integral action decreases with the increase of the frequency of the input signal.

The load controller  $G_c(s)$  is designed to be a PID controller with a variable integral time:

$$G_c(s) = K_c \left(1 + \frac{K_0 K_p^*}{s} e^{-d^* s}\right) (1 + T_{cd}s) = K_c \left(1 + \frac{1}{2d^* s} e^{-d^* s}\right) (1 + T_{cd}s). \quad (10)$$

The integral time of the controller is  $T_{ci} = \frac{1}{K_0 K_p^*} e^{d^* s} = 2d^* e^{d^* s}$ . The controller gain  $K_c$  and derivative time  $T_{cd}$  remain to be determined. Substituting equation (10) into equation (3) and considering the expressions of  $G_{0l}(s)$  and  $G_{0r}(s)$  of equation (1), we have:

Unknown  $K_c$ ,  $T_{cd}$  and  $\omega$  can not be derived from the above relations that contain only two independent equations. To overcome this difficulty, the author suggests determining the derivative time  $T_{cd}$  in advance through the following experience relation:

$$T_{cd} = \alpha d^*, \quad (15)$$

where  $\alpha$  is a constant coefficient with the value of  $1/3 \sim 1/2$ . The controller gain  $K_c$  can thus be obtained from the relations of equation (14) with a specified  $\Phi_c$ . The author recommends taking  $\Phi_c = 60^\circ$  for long delay.

In summary, a three-step controller tuning procedure has been presented. The pre-stabilizer  $K_0$  is determined first through equation (7). Then, the set-point controller  $G_r(s)$  and the load controller  $G_c(s)$  can be designed independently.  $G_r(s)$  is derived from equation (9) with a desired  $T_r$  and  $G_c(s)$  is obtained through equations (10), (14) and (15) together with  $\alpha = 1/3 \sim 1/2$  and  $\Phi_c = 60^\circ$  for long delay. The estimates of the system

parameters are used in all these steps.

#### 4 Examples

The following two integrator processes with long delay are considered as examples:

$$P_1(s) = \frac{1}{s}e^{-5s}, P_2(s) = \frac{1}{5s}e^{-7.4s}. \quad (16)$$

The process  $P_1(s)$  can be found in [4 ~ 6]. The process  $P_2(s)$  results from an approximation of the bottom level control of an industrial distillation column<sup>[1]</sup>. It was adopted as an example by Zhang and Sun<sup>[6]</sup>. The controller settings for the above two processes are as follows. For  $P_1(s)$ ,  $K_0 = 0.1$ ,  $G_{0r}^*(s) = 1/(10s + e^{-5s})$ ,  $T_r = 1/0.6$ ,  $G_r(s) = 6(1 + \frac{1}{10s}e^{-5s})$ ,  $G_c(s) = 1.7(1 + \frac{1}{10s}e^{-5s})(1 + 2.5s)$  and  $\Phi_c = 61^\circ$ . For  $P_2(s)$ ,  $K_0 = 5/14.8$ ,  $G_{0r}^*(s) = 1/(14.8s + e^{-7.4s})$ ,  $T_r = 1$ ,  $G_r(s) = 14.8(1 + \frac{1}{14.8s}e^{-7.4s})$ ,  $G_c(s) = 1.51(1 + \frac{1}{14.8s}e^{-7.4s})(1 + 2.5s)$  and  $\Phi_c = 60^\circ$ . A unit step set-point change at  $t = 0$  and a step load change  $L = -0.1$  at  $t = 70$  are respectively introduced, as in [4 ~ 6]. The proposed scheme is compared with MM and ZS schemes.

Figs. 4 and 5 show the simulation results of the first process  $P_1(s)$ . The ideal case with a good process model is depicted in Fig. 4. All the three control schemes have similar set-point tracking responses. The load response of the proposed scheme is slightly faster than that of ZS scheme and has a smaller overshoot than MM scheme. The good robustness of the proposed scheme can be observed from Fig. 5 that corresponds to a 30% estimating error in process delay time. ZS scheme becomes unstable. The proposed scheme has smaller oscillations than MM scheme.

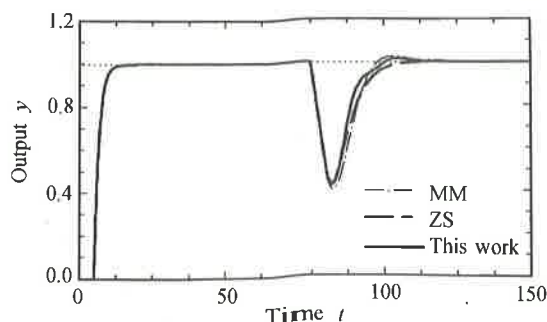


Fig. 4 Responses of the three schemes for  $P_1$  with a good process model

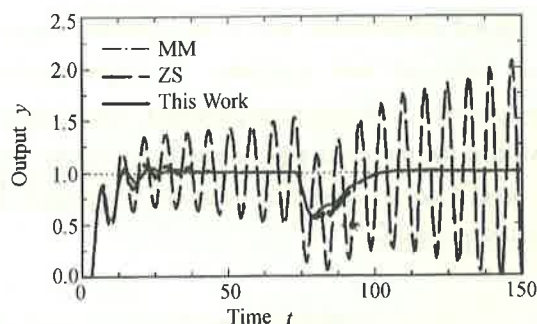


Fig. 5 Responses of the three schemes for  $P_1$  with a 30% estimating error in process delay time,  $d^*=5$ ,  $d=3.5$

The simulation results of the second process  $P_2$  are depicted in Figs. 6 and 7. Fig. 6 shows the ideal case with a good process model. All the three control schemes have similar set-point tracking performances. In load responses, ZS scheme is too slow and MM scheme is slightly oscillatory. The proposed scheme is superior to other two schemes. Fig. 7 corresponds to a 40% deviation of the process delay time from its normal value. ZS scheme is strongly oscillatory and MM scheme is slightly oscillatory. The proposed scheme outperforms other two schemes.

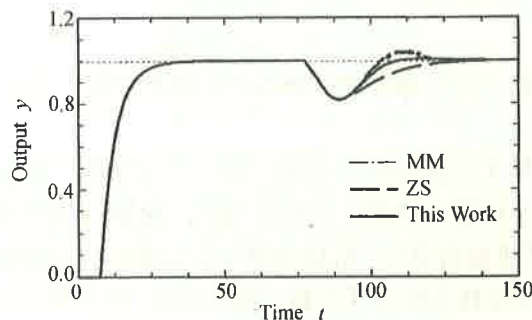


Fig. 6 Responses of the three schemes for  $P_2$  with a good process model

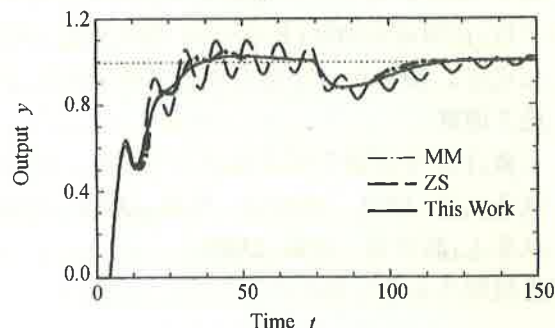


Fig. 7 Responses of the three schemes for  $P_2$  with a 40% deviation of the process delay time from 7.4 to 4.44

#### 5 Conclusion

A control scheme has been proposed for integrator processes with long delay. It contains three separate controllers. Each controller has a clear physical interpretation: The set-point controller and the load controller can



be designed independently due to the separation nature of the set-point and load responses. A simple controller tuning procedure has also been presented. Simulation has shown that the proposed scheme has fast set-point tracking, good load rejection and improved robustness over the existing schemes.

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## 第三届全球智能控制与自动化大会(WCICA'2000)

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