Vibration Control of a Flexible Smart Beam

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Abstract: In this paper, the vibration control of a flexible cantilevered beam with collocated piezoelectric sensor and actuator is studied. A dynamic governing equation of motion for the smart beam is derived and a linear feedback control law is presented. By using LaSall's invariant principle in infinite dimensional space and linear semigroup theory, it is shown that implementation of the control results in vibration suppression provided that the distribution of the collocated sensor and actuator make the stabilizable condition hold.

Key words: flexible structure; smart structure; vibration control

挠性智能梁的振动控制

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摘要: 研究采用共位配置的压电敏感器和致动器的挠性悬臂梁的振动控制问题,建立了智能梁的模型,设计了一种线性反馈控制律,并应用无穷维空间的 LaSalle 不变原理和线性半群理论证明了当敏感器和控制器的分布使得系统能镇条件成立时,所设计的控制抑制了梁的振动.

关键词: 挠性结构; 智能结构; 振动控制

1 Introduction

In recent years, to fulfill the requirement of precise pointing accuracy for large spacecraft, much effort has been directed toward looking for smart structure, i.e., structures with highly integrated sensors and actuators, which can be used to change the mechanical properties of the structure. As of now, piezoelectric materials are most often employed as sensors or actuators for smart structure applications.

Research on smart structure systems using piezoelectric materials was first undertaken by Bailey and Hubbard[1]. By utilizing a uniform layer of the material bonded to one face of a cantilevered beam as the actuator, they implemented a control strategy using Lyapunov's direct method, and showed that vibrational mode of the beam could be controlled based upon the measurement of the angular velocity at the beam's tip. Burke et. al.[2] showed that spatially-varying piezoelectric film actuators distributions can be applied to control all vibrational modes of flexible beams with nearly arbitrary boundary condition. Piezoelectric actuators were also used as elements of intelligent structures by Crawly and de Luis[3]. As is evident from previous studies, numerous researchers have only demonstrated the stability of the proposed control algorithm by simulation and experimental results without severe theoretical analysis for system stability.

In this paper, a dynamic governing equation of motion for a smart cantilevered beam is derived by applying Hamilton principle, and a linear feedback control for the smart beam is presented. The main result of the paper is that it is shown that implementation of the control results in vibration suppression provided that the distribution of the collocated sensor and actuators make the stabilizable condition hold.

2 System models

Fig. 1 shows the structure of the smart cantilevered beam. The actuator and sensor are layers made of piezoelectric ceramic (PZT) and piezoelectric polymer polyvinylidene fluoride (PVDF) materials, respectively, collocated to both sides of the beam. In Fig. 1, h stands for the thickness of the different layers of the composite

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beam. The subscripts s, b and a denote sensor, beam and actuator respectively.

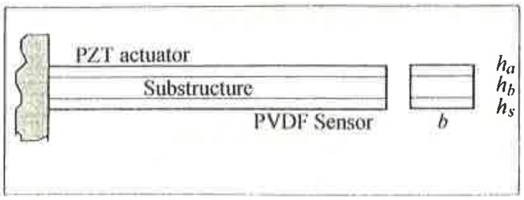

Fig. 1 The composite beam

Piezoelectric actuator is made based on the constitutive property of the PZT material. When control voltage $V(x,t)$ is applied to the PZT material, the induced strain ε_p in the PZT material is given by

$$\varepsilon_p(x,t) = \frac{d_{31}}{h_a}V(x,t), \tag{1}$$

where $d_{31} > 0$ is the piezoelectric field strain field constant.

Suppose the bonding between the actuator layer and the beam is perfect, i.e., there is no shear lag layer effects on the beam. Then the bending moment produced from the PZT actuator can be expressed as (for detail, see[1]):

$$M(x,t) = k_a\varepsilon_p(x,t), \tag{2}$$

where $k_a > 0$ is a constant depending on the geometry and materials of the beam.

For the development work, we assume that

$$V(x,t) = c_1V(t)F(x), \tag{3}$$

where function $F(x)$, being the local width of the electrodes covering both sides of the actuator, denotes the weight function of the actuator. $V(t)$ is the input voltage to the PZT actuator layer as the control variable. c_1 is a positive constant.

Substituting equation (1), (3) into (2) yields

$$M(x,t) = cV(t)F(x), \tag{4}$$

where $c = c_1k_a\dfrac{d_{31}}{h_a} > 0$. Equation (4) describes relationship between the applied voltage and actuation of the PZT actuator.

Piezoelectric polymer polyvinylidene fluoride (PVDF) is also strain sensitive and relies on the applied strain to produce electrical charge. This process is the reverse to piezoelectric actuation. The amount of electrical charge is proportional to the amount of strain induced by the structure.

For the case that sensor and actuator are collocated, we have that the function $F(x)$ is also the weight function of the sensor. Then the ouput of the sensor is given by

$$I(t) = k_s\int_0^l F(x)u_{xxt}(x,t)\mathrm{d}x, \tag{5}$$

where $u(x,t)$ denotes the transverse displacement of the beam at time t and position x along the beam length direction. $k_s > 0$ is a constant determined by the sensor geometry, material and the capacitance between the electrodes of both side of the sensor surfaces. The subscripts $(\cdot)_x$ and $(\cdot)_t$ denote spatial and time derivatives, respectively.

We use the Euler-Bernoulli beam model to describe the dynamical behavior of the beam. The kinetic energy T and potential energy V of the structure including the piezoelectric actuator and sensor are expressed as

$$T = \frac{1}{2}\int_0^l \rho u_t^2(x,t)\mathrm{d}x,$$

$$V = \frac{1}{2}\int_0^l \frac{1}{EI}[EIu_{xx}(x,t) - cV(t)F(x)]^2\mathrm{d}x,$$

where l is the length of the beam, EI is the effective bending stiffness of the smart flexible beam and ρ is the mass per length of the composite beam.

By applying the Hamilton principle, we derive that the governing equation of motion for transverse vibration $u(x,t)$ can be written as

$$\begin{cases} \rho u_{tt}(x,t) + EIu_{xxxx}(x,t) - cV(t)F_{xx}(x) = 0, \\ 0 < x < l,\ t > 0; \\ u(0,t) = 0, \quad u_x(0,t) = 0; \\ EIu_{xx}(l,t) = cV(t)F(l), \\ EIu_{xxx}(l,t) = cV(t)F_x(l). \end{cases} \tag{6}$$

3 Control law: design and analysis

Our goal is to suppress the structural vibration of the system described by equation (6) using the input voltage $V(t)$ to the PZT as the control variable. In order to complete the mission, we design the following control law:

$$V(t) = -kI(t), \tag{7}$$

where k is a positive constant.

Define the function space $\mathscr{H}$ as

$$\mathscr{H} = \{(u,v)^{\mathrm{T}} \mid u \in H_0^2(0,l), v \in L^2(0,l)\},$$

where the spaces $L^2(0,l)$ and $H_0^k(0,l)$ are defined as follows

$$L^2(0,l) = \{f:[0,l] \to \mathbb{R} \mid \int_0^l f^2\mathrm{d}x < \infty\},$$

$$H_0^k(0,l) = \{f \in L^2(0,l) \mid f, f', \cdots, f^{(k)} \in L^2(0,l), f(0) = f'(0) = 0\},$$

for all positive integer k.

In $\mathscr{H}$, we define the energy inner product as follows:

$$\langle z, \hat{z} \rangle_{\mathscr{H}} = \frac{1}{2}\int_0^l [\rho u_t \hat{u}_t + EI u_{xx} \hat{u}_{xx}] dx, \quad (8)$$

where $z = (u, u_t)^T \in \mathscr{H}, \hat{z} = (\hat{u}, \hat{u}_t)^T \in \mathscr{H}$. The corresponding energy norm is given by

$$(\| z \|_{\mathscr{H}})^2 = \frac{1}{2}\int_0^l (\rho u_t^2 + EI u_{xx}^2) dx. \quad (9)$$

We note that $\mathscr{H}$ together with the energy inner product derived by (8) becomes a Hibert space ([4]).

From (6) and (7), we see that the closed-loop system can be written as follows:

$$\dot{z} = \mathscr{A} z, \quad (10)$$

where $z = (u, u_t)^T$. The linear operator $\mathscr{A}: \mathscr{H} \to \mathscr{H}$ is defined as

$$\mathscr{A} z = (u_t, -\frac{EI}{\rho} u_{xxxx} - \frac{ckk_s}{\rho} F_{xx}(x) \int_0^l F(x) u_{xxt} dx)^T.$$

The domain of the operator $\mathscr{A}$ is defined as

$$D(\mathscr{A}) := \{(u, v)^T \mid u \in H_0^4(0,l), v \in H_0^2(0,l)$$
$$EI u_{xx}(l) = -ckk_s F(l) \int_0^l F(x) u_{xxt} dx,$$
$$EI u_{xxx}(l) = -ckk_s F_x(l) \int_0^l F(x) u_{xxt} dx\}.$$

The energy function associated with (10) is defined by

$$E(t) = \| z(t) \|_{\mathscr{H}}^2 = \frac{1}{2}\int_0^l (\rho u_t^2 + EI u_{xx}^2) dx. \quad (11)$$

Lemma 1 Consider the system given by (10). Then for $t \geq 0$, $E(t)$ is a nonincreasing function of time t along the classical solution of equation (10).

Proof Differentiating (11) with respect to t and using equation (10), we obtain that

$$\dot{E}(t) = \int_0^l (\rho u_t u_{tt} + EI u_{xx} u_{xxt}) dx =$$
$$\int_0^l [u_t(-EI u_{xxxx} - ckk_s F_{xx}(x) \cdot \int_0^l F(\tau) u_{\tau\tau t} d\tau) + EI u_{xx} u_{xxt}] dx =$$
$$EI(-\int_0^l u_t du_{xxx} + \int_0^l u_{xx} u_{xxt} dx) - ckk_s \int_0^l F(x) u_{xxt} dx \int_0^l u_t dF_x =$$
$$-ckk_s [\int_0^l F(x) u_{xxt} dx]^2 \leq 0. \quad (12)$$

Thus, we conclude that for $t \geq 0$, $E(t)$ is a nonincreasing function of t along the classical solution of equation (10).

Define an operator A on $L^2(0,l)$ by

$$A\phi(x) = \phi''''(x),$$
$$D(A) = \{\phi \in L^2(0,l) \mid \phi(0) = \phi'(0) = \phi''(l) = \phi'''(l) = 0\}.$$

Let $\{\lambda_n, \phi_n(x)\}_{n=1}^{\infty}$ be the eigenpairs of operator A. Then from [5], we have

i) $\lambda_n = \beta_n^4$ with β_n satisfying

$$1 + \cos\beta_n l \cosh\beta_n l = 0,$$

$\beta_n > 0$, and $\beta_n \to \infty$ as $n \to \infty$.

ii) $\phi_n(x) = (\cosh\beta_n x - \cos\beta_n x) - \frac{\cosh\beta_n l + \cos\beta_n l}{\sinh\beta_n l + \sin\beta_n l}(\sinh\beta_n x - \sin\beta_n x)$

forms an orthogonal basis on $L^2(0,l)$.

Assumption 1 (Stabilizable condition) The weight function $F(x)$ satisfies

$$\begin{cases} F(x) \in H^2(0,l)\{f \mid f, f', f'' \in L^2(0,l)\}, \\ \int_0^l F(x) \phi''_n(x) dx \neq 0, \quad n = 1, 2, \cdots. \end{cases} \quad (13)$$

Lemma 2 Consider the system given by (10). Suppose (13) holds. Let S denote the subspace of $\mathscr{H}$ defined by $S := \{z(t) \in \mathscr{H} \mid dE(t)/dt = 0, t \geq 0\}$. Then $z(\cdot)$, the classical solution of equation (10) which lies in S, satisfies $z(t) \equiv 0, t \geq 0$.

Proof Let $z = (u, u_t)^\tau$ be the classical solution of equation (10) which lies in S. Then, from (12), we have

$$\begin{cases} \rho u_{tt}(x,t) + EI u_{xxxx}(x,t) = 0, \quad 0 < x < l, \\ u(0,t) = u_x(0,t) = u_{xx}(l,t) = u_{xxx}(l,t) = 0, \\ \int_0^l F(x) u_{xxt}(x,t) dx = 0. \end{cases} \quad (14)$$

Define $$A_1 = \begin{bmatrix} 0 & I \\ -\frac{EI}{\rho} A & 0 \end{bmatrix}.$$

The domain of the linear operator A_1 is

$$D(A_1) := \{(u, u_t)^\tau \mid u \in H_0^4(0,l), u_t \in H_0^2(0,l), u_{xx}(l) = u_{xxx}(l) = 0\}.$$

Then the system (14) can be rewritten as follows:

$$z(t) = A_1 z(t), \quad z(0) = z_0 \quad (15)$$

with $\int_0^l F(x)u_{xxt}(x,t)\mathrm{d}x = 0$, where $z = (u, u_t)^{\mathrm{T}}$.

It is well known that A_1 generates an exponentially decaying C_0 semigroup $T_1(t)$ on $\mathscr{H}$. For $\Phi = (\zeta, \eta)^{\mathrm{T}} \in \mathscr{H}$, denote $\alpha_n = \langle \zeta, \phi_n \rangle_{\mathscr{H}}$, $\beta_n = \langle \eta, \phi_n \rangle_{\mathscr{H}}$, then we have

$$T_1(t)\Phi = \sum_{n=1}^{\infty}\begin{bmatrix}(\alpha_n \cos\sqrt{\frac{EI}{\rho}\lambda_n}t + \frac{\beta_n}{\sqrt{\frac{EI}{\rho}\lambda_n}}\sin\sqrt{\frac{EI}{\rho}\lambda_n}t)\phi_n \\ (\beta_n\cos\sqrt{\frac{EI}{\rho}\lambda_n}t - \sqrt{\frac{EI}{\rho}\lambda_n}\alpha_n\sin\sqrt{\frac{EI}{\rho}\lambda_n}t)\phi_n\end{bmatrix}.$$

Then, from (15), we deduce that

$$u(x,t) = \sum_{n=1}^{\infty}(\langle u(x,0), \phi_n(x)\rangle_{\mathscr{H}}\cos\sqrt{\frac{EI}{\rho}\lambda_n}t + \frac{\langle u_t(x,0), \phi_n(x)\rangle_{\mathscr{H}}}{\sqrt{\frac{EI}{\rho}\lambda_n}}\sin\sqrt{\frac{EI}{\rho}\lambda_n}t)\phi_n(x),$$

$$u_t(x,t) = \sum_{n=1}^{\infty}(\langle u_t(x,0), \phi_n(x)\rangle_{\mathscr{H}}\cos\sqrt{\frac{EI}{\rho}\lambda_n}t - \sqrt{\frac{EI}{\rho}\lambda_n}\langle u(x,0), \phi_n(x)\rangle_{\mathscr{H}}\sin\sqrt{\frac{EI}{\rho}\lambda_n}t)\phi_n(x).$$

This implies that

$$0 = \int_0^l F(x)u_{xxt}(x,t)\mathrm{d}x = \sum_{n=1}^{\infty}(\int_0^l F(x)\phi_n''(x)\mathrm{d}x\langle u_t(x,0), \phi_n(x)\rangle_{\mathscr{H}}\cdot \cos\sqrt{\frac{EI}{\rho}\lambda_n}t - \sqrt{\frac{EI}{\rho}\lambda_n}\int_0^l F(x)\phi_n''(x)\mathrm{d}x \cdot \langle u(x,0), \phi_n(x)\rangle_{\mathscr{H}}\sin\sqrt{\frac{EI}{\rho}\lambda_n}t),$$

which means that $\int_0^l F(x)\phi_n''(x)\mathrm{d}x\langle u_t(x,0), \phi_n(x)\rangle_{\mathscr{H}}$ and $-\sqrt{\frac{EI}{\rho}\lambda_n}\int_0^l F(x)\phi_n''(x)\mathrm{d}x\langle u(x,0), \phi_n(x)\rangle_{\mathscr{H}}$ are the Fourier coefficients of the uniformly almost periodic function 0. Hence for $n \geqslant 1$,

$$\int_0^l F(x)\phi_n''(x)\mathrm{d}x\langle u(x,0), \phi_n(x)\rangle_{\mathscr{H}} = -\sqrt{\frac{EI}{\rho}\lambda_n}\int_0^l F(x)\phi_n''(x)\mathrm{d}x\langle u_t(x,0), \phi_n(x)\rangle_{\mathscr{H}} = 0.$$

But from assumption (13), we know that $\int_0^l F(x)\phi_n''(x)\mathrm{d}x \neq 0$. Then we have

$$\langle u(x,0), \phi_n(x)\rangle_{\mathscr{H}} = \langle u_t(x,0), \phi_n(x)\rangle_{\mathscr{H}} = 0,$$

which implies $u(x,0) = u_t(x,0) = 0$ and $E(0) = 0$. Thus we have $z(t) \equiv 0$ for $t \geqslant 0$.

From Lemma 1, by the Sobolev embedding theorem and the argument similar to that used in [6], we can obtain the following lemma

Lemma 3 The operator $\mathscr{A}$: $D(\mathscr{A}) \subset \mathscr{H} \rightarrow \mathscr{H}$ generates a C_0 semigroup of contraction on $\mathscr{H}$ with $(\lambda I - \mathscr{A})^{-1}$ being a compact operator for $\lambda > 0$.

By Lemma 1 ~ Lemma 3, using LaSalle's invariance principle ([7]) and linear semigroup theory ([8]), we immediately have the following result.

Theorem 1 Consider the system given by ([10]). Then for $z(0) \in D(\mathscr{A})$, equation (10) has a unique classical solution $z(t)$. Moreover, if (13) holds, we have $\lim_{t\to\infty} \| z(t) \|_{\mathscr{H}} = 0$.

Consider two special cases of the expression of $F(x)$, we can obtain the following corollarys by verifying (13) holds for these cases.

Corollary 1 Consider the system (10) with $F(x) \equiv 1$. Then for $z(0) \in D(\mathscr{A})$, the system has a unique classical solution $z(t)$, which satisfies $\lim_{t\to\infty} \| z(t) \|_{\mathscr{H}} = 0$.

Corollary 2 Consider the system (10) with $F(x) = \frac{(l-x)^2}{l^2}$. Then for $z(0) \in D(\mathscr{A})$, the system has a unique classical solution $z(t)$, which satisfies $\lim_{t\to\infty} \| z(t) \|_{\mathscr{H}} = 0$.

4 Conclusions

In this paper, an active distributed damper for a cantilever beam is designed and evaluated. A linear feedback control for the smart beam is developed using Lyapunov's second method and the stabilizable condition for the system is presented. The main result of the paper is that based upon LaSall's invariant principle in infinite dimensional space and linear semigroup theory, it is shown that implementation of the control algorithm results in vibration suppression.

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together (3.3) with (3.4), one gets

$$V(t_0,\infty;u_\infty^*) = \lim_{T\to\infty} V(t_0,T;u_\infty^*) =$$

$$\lim_{T\to\infty} V(t_0,T;u_\infty^*) = x_0'\bar{P}x_0. \tag{3.5}$$

We proceed to prove $V(t_0,\infty;u_\infty^*) = V^*(t_0,\infty)$. If not, then

$$V^*(t_0,\infty) < V(t_0,\infty;u_\infty^*), \tag{3.6}$$

thus, there exists a control u_1, such that

$$\lim_{T\to\infty} V(t_0,T;u_1) = V^*(t_0,\infty).$$

But from (3.5),

$$\lim_{T\to\infty} V^*(t_0,T) = V(t_0,\infty;u_\infty^*),$$

so (3.6) implies

$$\lim_{T\to\infty} V(t_0,T;u_1) < \lim_{T\to\infty} V^*(t_0,T),$$

which demands that for sufficiently large T,

$$V(t_0,T;u_1) < V^*(t_0,T),$$

by optimality, this is not possible. So we have proved

$$V^*(t_0,\infty) = \mathrm{E}\int_{t_0}^{\infty}(x^{*\prime}Qx^* + u^{*\prime}Ru^*)\mathrm{d}t =$$

$$x_0'\bar{P}x_0 < \infty.$$

By lemma 2.3, $\lim_{t\to\infty}\mathrm{E}\,|\,x^*(t)\,|^2 = 0$, i.e. if we take $u(t) = K_0x(t) = -(R + D'\bar{P}D)^{-1}(B'\bar{P} + D'\bar{P}C)x(t)$ in (1.1), then system (1.1) is stabilizable, Theorem 1 is complete.

Remark 3.1 One can easily give an example to illustrate that stabilizability doesn't imply exact controllability. Consider one dimensional stochastic system

$$\mathrm{d}x = (ax + bu)\mathrm{d}t + cx\mathrm{d}W,$$

$$x(t_0) = x_0,$$

when $b \neq 0$, this system is always stabilizable (this can be proved by simple computation), but by Theorem 3.1 of [1], it is not exactly controllable because $d = 0$.

Remark 3.2 In order to prove Theorem 1, we relate it with an optimal control problem, this method is very useful in many problems, much application of optimal control ideas can be found in [5].

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